Econ 134 - Financial Economics

Nehring

Problem Set # 7

Three out of fine problems will be graded for serious effort, for a maximum of 100 points. In addition, one part of each of the three chosen questions will be graded for success, for an additional 30 points. Maximal total score: 100+30=130 points.

Reminder: The general formula for the variance of the return on a portfolio P_w with "weight" (="proportion", percentage of amount invested) of X $=$ w and weight of Y $=$ 1-w is: $\sigma^2_{(P_w)} = w^2\sigma^2_X + 2w(1-w)\rho_{XY}\sigma_X\sigma_Y + (1-w)^2\sigma^2_Y$.

1. (30) You are now given the following parameters about Supertech X and Slowpoke Y: $\mu_X = 0.20, \sigma_X = 0.4; \mu_Y = 0.10, \sigma_Y = 0.3, \rho_{XY} = 0$. There is no risk-free asset.

Graph, as a function of the proportion w invested in Supertech,

i) the expected rate of return on a portfolio,

ii) the variance of the rate of return on a portfolio,

iii) the standard deviation of the rate of return on a portfolio.

To do the graphs, compute 5 points based on the following values for the proportion of Supertech w: 0, 0.25, 0.5, 0.75, 1.

iv) Would a rational, risk-averse investor ever put *all* his money into Supertech? Explain. (*Hint: to do iv) and v), you need to consider the risks and returns resulting from different portfolios P_w*).

v) Would a rational, risk-averse investor ever put *all* his money into Slowpoke? Explain.

2. (30) Supertech X and Slowpoke Y, again, same parameter values: $\mu_X = 0.20, \sigma_X = 0.4; \mu_Y = 0.10, \sigma_Y = 0.3$, but now ρ_{XY} varies. There is no risk-free asset.

i) Compute, for $\rho_{XY} = 1, 0, -1, 0.9$ the minimum-variance portfolio MVP and the minimum variance achieved (you thus need to do computations for four different cases).

ii) As ρ_{XY} increases, does the proportion of Supertech in the MVP go up or down? Explain in ordinary language why the pattern you find makes sense.

iii) Plot for $\rho_{XY} = 1, 0, -1$ the standard deviation of the rate of return on a portfolio as a function of the proportion w invested in Supertech. *Hint: copy the plot for $\rho_{XY} = 0$ from 2,iii) ; for $\rho_{XY} = 1, -1$, this function is linear between the proportion of Supertech in the MVP and the extreme values for w, $w = 0$ and $w = 1$.*

iv) For $\rho_{XY} = 1, 0, -1$, graph the risk-return combination resulting from investing half of the money in Supertech and half of the money in Slowpoke.

v) For $\rho_{XY} = 1, 0, -1$, graph the set of *all* available risk-return combinations (*hint: no additional computations are required*).

3. (20) You are now given the following parameters about Supertech X and Slowpoke Y: $\mu_X = 0.20, \sigma_X = 0.18; \mu_Y = 0.14, \sigma_Y = 0.22$.

You are told nothing about how the returns on X and Y are correlated. Explain briefly your answer to each of the following questions.

i) Might a rational, risk-averse investor *ever* (i.e. for some conceivable correlation coefficient ρ_{XY}) invest all of his money in Supertech ?

ii) Might a rational, risk-averse investor ever invest all of his money in Slowpoke?

iii) Might a rational, risk-averse investor ever invest some of his money in Slowpoke?

iv) Might a rational, risk-neutral investor ever invest some of his money in Slowpoke?

4. (40) The expected return on all stocks in the market is 22%. The return on a riskless investment, such as treasury bills, is 10%. Assume that all stocks are priced according to the CAPM.

i) If you hold a portfolio that has an expected rate of return of 34%, what is the beta of your portfolio?

ii) The stock of company X has a total risk of $\sigma_X = 0.4$ and $\beta_X = 0$. What is its expected return?

iii) If a company's stock has a $\beta = -0.5$, sells for $ 28 per share and won't pay dividends over the next year, what do you expect it to sell for one year from now?

iv) Company Z, a very stable corporation, has the same total risk as the market $\sigma_Z = \sigma_M = s$. If $s=0.24$ and $\mu_Z = 0.16$, what are β_Z and ρ_{ZM}? Do you in fact need the value for s?

5. (30) Ray has inherited from a deceased grand-uncle a portfolio P of investments in two mutual funds X and Y; $60,000 are invested in X, $90,000 in Y. Ray doesn't know anything about either the stockmarket or portfolio theory but suspects that one can invest more wisely than his grand-uncle did. You are his best friend; he admires your command of such esoteric matters and is happy to let you profit from it, provided that he is not made worse off by your advice, after deducting your friendly "consulting fee" C. You have the following information at your disposal:

$R_F = 0.02, \mu_X = 0.08, \mu_Y = 0.2, \mu_M = 0.12, \beta_X = 0.6, \beta_Y = 1.8, \sigma_X = 0.4, \sigma_Y = 0.5, \sigma_M = 0.2, \rho_{XY} = 0.$

(As usual, F denotes the risk-free asset, and M the market-portfolio; M contains also assets other than X and Y).

i) What are the expected return and standard deviation of the portfolio that Roy has inherited?

ii) By how much can you reduce Ray's risk without reducing his expected return; illustrate your answer to this and the next question with appropriate graphs.

iii) You don't know Ray's risk preferences; all you know is that he is risk-averse and only concerned with expected value and standard deviation of his portfolio's return. You therefore want to propose to him a portfolio Q and a fee C that won't make Ray worse off than he is under P, regardless of his preferences. What portfolio Q should you propose to Ray to extract as large a fee C from Ray as you can? How large is that fee (in $s per annum) you can get?