## Stock Valuation Based on Earnings

The PV approach to stock valuation discounts dividends $D_{t}$, not earnings $E_{t}$ :

$$
P_{0}=\sum_{t} \frac{D_{t}}{(1+r)^{t}}
$$

If we were to discount earnings $\sum_{t} \frac{E_{t}}{(1+r)^{t}}$ instead, we would overestimate the value of the stock by the PV of retained earnings $\sum_{t} \frac{E_{t}-D_{t}}{(1+r)^{t}}$.

Of course, if the entire earnings are distributed as dividends each year $\left(E_{t}=D_{t}\right)$, then $P_{0}=\sum_{t} \frac{E_{t}}{(1+r)^{2}}$. In this case, the firm does not invest any of its own capital to grow; it is thus plausible to assume that dividends=earnings stay constant over time. Call this SCENARIO A. Then, applying the perpetuity formula,

$$
P_{0}=\frac{D_{1}}{r}=\frac{E_{1}}{r} .
$$

Thus, in this case, the Price-Earnings ratio is given by

$$
\begin{equation*}
\frac{P_{0}}{E_{1}}=\frac{1}{r} . \tag{1}
\end{equation*}
$$

- Yardeni's "Fed Model" is similar, as it says:

$$
\frac{P_{0}}{E_{1}}=\frac{1}{r_{10}},
$$

where $r_{10}$ is the interest rate on 10-year government bonds.

## SCENARIO B:

## Reinvestment of retained earnings at required rate of return.

Now assume that the firm retains some earnings, but invests them at the "required rate of return" $r$; in other words, the firm invests its earnings in $0-N P V$ investments. Clearly, this cannot change the value of stock, that is: the PV of the dividends. Thus, the formulas

$$
\begin{equation*}
P_{0}=\frac{E_{1}}{r} \tag{2}
\end{equation*}
$$

and $\frac{P_{0}}{E_{1}}=\frac{1}{r}$ still apply. Of course, now both earnings and dividends will grow.

## Example.

$E_{1}=\$ 10, r=0.1$. (I am using always per-share values).

- In scenario A, $P_{0}=\frac{\$ 10}{0.1}=\$ 100$.
- In scenario B, applying the formula (2), once again $P_{0}=\frac{\$ 10}{0.1}=\$ 100$. Magically, this will be true however much the firm retains and whatever the resulting dividend pattern is.

To check this, we need to verify that the fundamental value is indeed the PV of all future dividends (that is always true). That is, assuming constant growth, we need to verify that $P_{0}=\frac{D_{1}}{r-g}$. The key is to compute $g$, the growth rate of earnings and dividends.

- $g$ is determined by two factors:

1. the retention ratio $\rho=\frac{E_{t}-D_{t}}{E_{t}}$
2. and the return on retained earnings $\pi$.

Assuming that the retention ration and return on retained earnings is constant over time, the growth rate $g$ is constant over time and given as

$$
\begin{equation*}
g=\rho \pi . \tag{3}
\end{equation*}
$$

- See section 5.5 of the textbook for further explanation. You don't need to understand equation (3) fully, but you should understand the basic intuition behind it: earnings (and hence dividends) will grow the more rapidly, the more the firm invests (here, for simplicity all investment is assumed to come from retained earnings), and the more profitable those investments are.

Continuing the above example, suppose that the firm retains always $50 \%$ of its earnings, and pays out $50 \%$ as its dividends. Thus, $\rho=0.5$ and $D_{1}=\$ 5$.

By assumption, the firm reinvests earnings at its required rate of return, i.e. $\pi=r=0.1$. Hence equation (3) yields

$$
g=0.5 \times 0.1=0.05
$$

Plugging this into the Gordon growth formula, we get

$$
P_{0}=\frac{D_{1}}{r-g}=\frac{\$ 5}{0.1-0.05}=\$ 100 ; \text { voila! }
$$

## SCENARIO C:

## Reinvestment in positive NPV projects

Suppose now that the firm keeps reinvesting its earnings in positive NPV projects. Specifically, suppose that the return on its retained earnings is $12 \%$ rather than $10 \%$ as in B. Then its stock price will be accordingly higher. Indeed, by equation (3), the constant growth rate will now be

$$
g=0.5 \times 0.12=0.06
$$

hence

$$
P_{0}=\frac{D_{1}}{r-g}=\frac{\$ 5}{0.1-0.06}=\$ 125 .
$$

The difference $P_{0}-\frac{E_{1}}{r}=\$ 125-\$ 100=\$ 25$ is the Net Present Value of Growth Opportunities (NPVGO).

## SCENARIO D:

## Reinvestment in positive NPV projects

If the firm keeps making bad investments, the NPVGO may be negative. For example, if the return on its investments is only $8 \%$ (less than investor's required rate of return of $10 \%$ ), it destroys value. Indeed, the constant growth rate will now be

$$
g=0.5 \times 0.08=0.04
$$

hence

$$
P_{0}=\frac{D_{1}}{r-g}=\frac{\$ 5}{0.1-0.04}=\$ 83.33
$$

yielding an NPVGO of $\$-16.67$.

- Scenarios C and D contain an important general lesson: earnings growth justifies higher P/E-ratios only if it is based on genuinely profitable (i.e. positive NPV) investments. A firm's growth is however often profit neutral (zero NPV); for example, this is typically the case if the firm grows by acquiring other companies.

