# Network Formation in a Society with Fragmented Knowledge and Awareness

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**Abstract**

In modern society knowledge is extremely fragmented and production typically requires the linking of knowledge of many specialists. To study the incentives for linking knowledge across society, we combine a model of knowledge and awareness with a model of strategic network formation. We highlight the role of specialists and generalists. Specialization allows for "deep" knowledge that is potentially beneficial for society. Yet specialists with their "tunnel" view often fail to realize the benefit of linking knowledge across specialists. In contrast, generalists although they "know nothing" see many connections. They are in the center of resulting social networks and "talk a lot" despite having just superficial knowledge. When structuring hierarchies in a social network, it is efficient to put generalists in "positions of power" where they can impose communication on specialists. Finally, the generalist will emerge as an entrepreneur by employing specialists and retaining the residual profit.

**Keywords:** Social networks, knowledge, awareness, discovery, innovation, interdisciplinary work, R&D, hierarchies, employment, entrepreneurship, leadership, theory of the firm.

**JEL-Classifications:** C72, D82, D83, D85.

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*"If Siemens only knew what Siemens knows ..."*
*Heinrich von Pierer, CEO, Siemens AG, 1995*

# 1 Introduction

In modern society knowledge is extremely fragmented and production typically requires the linking of knowledge of many specialists. For instance, while about 60 years ago we believe it was possible to find an engineer in a car manufacturer who essentially knew how to design and build a car from scratch, nowadays it requires the combined knowledge of many specialists to design and build a car. In a sense, Adam Smith's "Division of Labor" became associated with a "Division of Knowledge". Consequently, when we refer to the knowledge of society, an organization, or of a firm, it is not enough to just consider all of the knowledge distributed among its members but we must also take into account the social structure that allows distributed knowledge to be accessed and utilized. Such social networks may form endogenously when agents benefit from gaining knowledge. One crucial impediment is that knowledge distributed in an organization is not transparent to all agents. An agent may not even conceive of the existence of some knowledge that other agents possess (i.e., unawareness). This is relevant for the formation of links between agents since an agent won't be even able to ask other agents for information that she is unaware of. In this paper, we study the incentives for linking knowledge and awareness across society and the resulting networks of knowledge in society. We ask given a distribution of knowledge and awareness in society, which social networks will form? Will these social networks allow for efficient use of knowledge in society? How to optimally design social networks for best accessibility of knowledge in society? Whom to put in a position of power to force communication upon others? Can we explain stylized facts on the organization of firms? Who will emerge as an entrepreneur?

In terms of methodology, we combine models of knowledge that originated in epistemic logic (so called Kripke frames or Aumann structures; Fagin et al., 1995) with game theoretic models of network formation (e.g., Jackson, 2008). In particular, we make use of unawareness structures (Heifetz, Meier, and Schipper 2006, 2008) that allow not only for the formalization of knowledge but also awareness. We consider a finite number of agents with different knowledge and awareness as players in a network formation game in which they can form links to share knowledge and awareness. One challenge is that the literature on network formation games focuses mostly on games with complete information while the games we consider are naturally games with asymmetric incomplete information (and unawareness). Standard notions of solutions to network formation games such as pairwise stability (Jackson and Wolinsky, 1996) or Nash networks (Bala and Goyal, 2000) do not readily apply. Even if we extend these notions to our setting, we wouldn't necessarily be able to interpret them as "stable" networks. The reason is that after some supposedly stable network is formed and knowledge and awareness is communicated via the formed links, the knowledge and awareness of players is changed. Because of such change, players may now form new links or reconsider existing links. Since we are interested in predicting which links are form at any *given* distribution of knowledge and awareness in society (including beliefs about other players knowledge and awareness, their beliefs about our beliefs etc.), we extend interim correlated rationalizability (Dekel, Fudenberg, and Morris, 2007) to network formation games with unawareness. We are not aware of a prior use of interim correlated rationalizability to either network formation games or games with unawareness. Yet, as argued elsewhere (Heifetz, Meier, and Schipper, 2013a, 2012), rationalizability concepts are very natural under unawareness (because of lack of stability of extended definitions of equilibrium in games with unawareness) and bears some resemblance of solutions to matching games under incomplete information (Liu et al. 2014, Pomatto, 2015). This is relevant since matching games could be understood as restricted network formation games. Interim correlated rationalizability can be understood as the incomplete information analogue to rationalizability (Bernheim, 1984, Pearce, 1984) and iterated elimination of strictly dominated strategies in normal-form games.

While our epistemic model is rather general (i.e., it allows for any configuration of knowledge and awareness among players), we explore in this initial work mainly results using different configurations of a society of specialists and generalists. Specialists have "deep" precise knowledge about particular propositions but lack awareness of many topics. Generalists "know nothing" (i.e., have only superficial knowledge) but are aware of many propositions. Recent empirical studies showed that recombination of knowledge of specialists leads can lead novelty (Catalini, 2015) and that generalist may be instrumental in producing novel research (Teodoridis, 2014, Melero and Palomeras, 2015). We consider three models of network formation. In the first model, players are assumed to freely access knowledge and awareness of all players in their component of the network. The individual benefit is monotone increasing in knowledge and awareness. Moreover, there are arbitrary small costs of initiating communication borne only to the player who initiates a link (unilateral cost of initiating communication). While our solution concepts are rather weak, it turns out that they allow us to fully characterize social networks in a society of specialists and generalists. We show that specialists with their "tunnel" view often fail to realize the benefit of linking of their knowledge across specialists. In contrast, generalists see many connections and are eager to form links. They are in a center of resulting social networks (i.e., a generalist-sponsored star) and "talk a lot" despite having just superficial knowledge.

We also consider a model of bilateral cost of communication borne to both players involved in a link. Such models can be tricky under incomplete information because the mere offer to communicate with some player in some sense signals already private information of the offering player (for instance his information about the other's agreeableness to such communication). We sidestep this issue completely because we are more interested in a design question. Often in organizations some members are placed in a position of power in which the can compel others to listen to them (and pay the cost of communication) quasi by conferred authority. We show that it is inefficient to place specialists in a position of power. Instead, generalists should be in the position of power.

Finally, we consider a model that should be even more relevant to economics and allows us to study who will emerge as an entrepreneur and how a firm may be formed. In this model, players can make wage offers or wage demands (but not both simultaneously) to others. A player is employed by the highest "bidder" at the highest "bid" but only if the "bidder's" wage offer exceeds the player's wage demand. This forms an employment network. We show that diverse specialists never employ others and that the the generalist emerges an entrepreneur, paying wages to specialists and retaining the residual. This resonates with an early literature on entrepreneurship and innovation. Schumpeter (1911, p. 100, p. 112, respectively) considered innovation as recombination of factors and portrait the entrepreneur as someone characterized by imaginative foresight. Since then foresight and imagination have been repeatedly featured as characteristics of the entrepreneur (see Knight, 1921, p. 270, Kirzner, 1973, Witt, 1997). In our model, discoveries are made by combining knowledge of specialists and it requires someone with general imaginative foresight (but not superior knowledge) to initiate the combination of knowledge. This literature is in contrast to existing formal models of leadership in organizations. Traditionally the argument is that the manager directs subordinates by virtue of having better knowledge than workers (Demsetz, 1988). Similarly, the knowledge-based hierarchy approach yields the conclusion that managers (i.e., actors higher up in the hierarchy) possess more specialized knowledge than workers (see Garciano and Rossi-Hansberg, 2015, for a review). Our approach is to complement this literature with a more explicit model of knowledge and awareness. In contrast to the standard notion of knowledge, the notion of awareness capture to a large extent Schumpeter's imaginative foresight, Knight's superior foresight, Kirzner's natural alertness to possible opportunities, and Witt's visions that develop into business conceptions. It makes these verbal notions amicable to formal analysis and allows us to formally prove how imaginative foresight may lead to entrepreneurship.

Unawareness structures allow us also to address some aspects of knowledge that researchers in knowledge management and organizational knowledge creation theory found to be missing in traditional models of information (see for instance Nonaka and Takeuchi, 1995). Based on Polanyi (1966), researchers in this field distinguish between tacit and explicit knowledge. Explicit knowledge is knowledge that can be formulated in a language. In contrast, tacit knowledge is embodied in unarticulated mental models that are frequently inaccessible to consciousness. While we do not go here as far as providing formal models of explicit and tacit knowledge[1], we do model what we believe is the crucial point behind the distinction of tacit from explicit knowledge, namely, that not all members of an organization may be aware of all knowledge (or lack of knowledge) distributed within organization. Lack of awareness differs from lack of information or knowledge. Lack of knowledge refers to a situation in which the player considers possible that some proposition is the case or is not the case. In contrast, lack of awareness or simply unawareness refers to a situation in which the player cannot even conceive of such a proposition. To emphasize again, unawareness posses an important constraint on communication. When a player is aware of a proposition but does not know whether or not the proposition is the case, then she could try to ask someone in the society who could know about the proposition. Yet, when a player is unaware of the proposition, she cannot even formulate such a question.

The paper is organized as following. In Section 2 we outline the model, in particular unawareness structures, networks, network games with unawareness, our notions of specialists and generalists. Section 3 analyzes the model of unilateral costs of initiating communication. The model of bilateral costs of communication and positions of power are analyzed in Section 4. In Section 5 we study the question of who becomes an entrepreneur by considering employments networks.

[1]Our model could be easily extended to formally model explicit and tacit knowledge. Formally, in our model we let explicit knowledge be defined by the possibility correspondence. Inverse images of possibility sets in higher spaces would correspond to implicit knowledge that seems to captures relevant aspects of tacit knowledge in the literature on knowledge management.

# 2 Model

## 2.1 Knowledge and Awareness

We model knowledge as in epistemic logic like in Kripke frames or Aumann structures (see Fagin et al, 1995). Yet, we require an extension since we also want to model (un-)awareness. As mentioned in the introduction, the difference between lack of knowledge and lack of awareness is that former refers to situations in which a player does not know whether or not some proposition is the case. In contrast, lack of awareness refers to situations in which the player cannot even conceive of some propositions.

A standard Kripke frame or Aumann structure features a space of states $S$. A proposition corresponds simply to a subset of states, the states in which the proposition is the case. A state can be considered as a complete description of propositions that are the case at that state. The problem is that if a player can reason about one such complete description of propositions then he can reason about all, making modeling unawareness infeasible (Dekel, Lipman, and Rustichini, 1998). That's why we introduce a lattice of disjoint state-spaces $\mathcal{S} = \langle \{S_\alpha\}_{\alpha \in \mathcal{A}}, \succeq \rangle$. To avoid technicalities, we assume that the lattice is nonempty and finite and that each space $S \in \mathcal{S}$ is nonempty and finite. Any finite lattice is complete, i.e., each subset of spaces has a least upper bounded (supremum) and a greatest lower bound (infinum). If $S_\alpha$ and $S_\beta$ are such that $S_\alpha \succeq S_\beta$ we say that "$S_\alpha$ is more expressive than $S_\beta$ – states of $S_\alpha$ describe situations with a richer vocabulary than states of $S_\beta$".[2] Spaces in the lattice can be more or less "rich" in terms of facts that may or may not obtain in them. The partial order relates to the "richness" of spaces. Denote by $\Omega = \bigcup_{\alpha \in \mathcal{A}} S_\alpha$ the disjoint union of these spaces. Each $S \in \mathcal{S}$ is assumed to be finite.

For every $S$ and $S'$ such that $S' \succeq S$, there is a surjective projection $r_S^{S'} : S' \longrightarrow S$, where $r_S^S$ is the identity. We interpret $r_S^{S'}(\omega)$ as "the restriction of the description $\omega$ to the more limited vocabulary of $S$." Projections "translate" states from "more expressive" spaces to states in "less expressive" spaces by "erasing" facts that can not be expressed in a lower space. Note that the cardinality of $S$ is smaller than or equal to the cardinality of $S'$. We require the projections to commute: If $S'' \succeq S' \succeq S$ then $r_S^{S''} = r_S^{S'} \circ r_{S'}^{S''}$. If $\omega \in S'$, denote $\omega_S = r_S^{S'}(\omega)$. If $D \subseteq S'$, denote $D_S = \{\omega_S : \omega \in D\}$.

For $D \subseteq S$, denote $D^\uparrow = \bigcup_{S' \in \{S' : S' \succeq S\}} \left(r_S^{S'}\right)^{-1}(D)$. ("All the extensions of descriptions in $D$ to at least as expressive vocabularies.") Clearly, $D^\uparrow$ is a subset of $\Omega$.

[2]Here and in what follows, phrases within quotation marks hint at intended interpretations, but we emphasize that these interpretations are not part of the definition of the set-theoretic structure.

A *propositions* is a pair $(E, S)$, where $E = D^{\uparrow}$ with $D \subseteq S$, where $S \in \mathcal{S}$. $D$ is called the *base* and $S$ the *base-space* of $(E, S)$, denoted by $S(E)$. If $E \neq \emptyset$, then $S$ is uniquely determined by $E$ and, abusing notation, we write $E$ for $(E, S)$. Otherwise, we write $\emptyset^S$ for $(\emptyset, S)$. Note that not every subset of $\Omega$ represents a proposition. Some proposition may be the case in a subset of a space. Then this proposition should be also "expressible" in "more expressive" spaces. Therefore the proposition contains not only the particular subset of states but also its inverse images in "more expressive" spaces.

Let $\Sigma$ be the set of *propositions* of $\Omega$, i.e., $D^{\uparrow}$ such that $D \in 2^S$, for some state-space $S \in \mathcal{S}$. The structure of the set of propositions works "almost" like an algebra. First consider negation of a proposition. If $(D^{\uparrow}, S)$ is an proposition where $D \subseteq S$, the negation $\neg(D^{\uparrow}, S)$ of $(D^{\uparrow}, S)$ is defined by $\neg(D^{\uparrow}, S) := ((S \setminus D)^{\uparrow}, S)$. Note that, by this definition, the negation of a proposition is a proposition. Abusing notation, we write $\neg D^{\uparrow} := \neg(D^{\uparrow}, S)$. By our notational convention, we have $\neg S^{\uparrow} = \emptyset^S$ and $\neg\emptyset^S = S^{\uparrow}$, for each space $S \in \mathcal{S}$. $\neg D^{\uparrow}$ is typically a proper subset of the complement $\Omega \setminus D^{\uparrow}$, that is, $(S \setminus D)^{\uparrow} \subsetneqq \Omega \setminus D^{\uparrow}$. Intuitively, there may be states in which the description of the proposition $D^{\uparrow}$ is both expressible and valid – these are the states in $D^{\uparrow}$; there may be states in which its description is expressible but invalid – these are the states in $\neg D^{\uparrow}$; and there may be states in which neither its description nor its negation are expressible – these are the states in $\Omega \setminus \left(D^{\uparrow} \cup \neg D^{\uparrow}\right) = \Omega \setminus S\left(D^{\uparrow}\right)^{\uparrow}$.

Further consider conjunctions of propositions. If $\left\{\left(D_{\lambda}^{\uparrow}, S_{\lambda}\right)\right\}_{\lambda \in L}$ is a collection of propositions (with $D_{\lambda} \subseteq S_{\lambda}$, for $\lambda \in L$), their conjunction $\bigwedge_{\lambda \in L}\left(D_{\lambda}^{\uparrow}, S_{\lambda}\right)$ is defined by $\bigwedge_{\lambda \in L}\left(D_{\lambda}^{\uparrow}, S_{\lambda}\right) := \left(\left(\bigcap_{\lambda \in L} D_{\lambda}^{\uparrow}\right), \sup_{\lambda \in L} S_{\lambda}\right)$. Note, that since $\mathcal{S}$ is a *complete* lattice, $\sup_{\lambda \in L} S_{\lambda}$ exists. If $S = \sup_{\lambda \in L} S_{\lambda}$, then we have $\left(\bigcap_{\lambda \in L} D_{\lambda}^{\uparrow}\right) = \left(\bigcap_{\lambda \in L}\left(\left(r_{S_{\lambda}}^{S}\right)^{-1}(D_{\lambda})\right)\right)^{\uparrow}$. Again, abusing notation, we write $\bigwedge_{\lambda \in L} D_{\lambda}^{\uparrow} := \bigcap_{\lambda \in L} D_{\lambda}^{\uparrow}$ (we will therefore use the conjunction symbol $\wedge$ and the intersection symbol $\cap$ interchangeably).

Intuitively, to take the intersection of events $(D_{\lambda}^{\uparrow}, S_{\lambda})_{\lambda \in L}$, we express them "most economically in the smallest language" in which they are all expressible $S = \sup_{\lambda \in L} S_{\lambda}$, take the intersection, and then the union of inverse images obtaining the event $\left(\bigcap_{\lambda \in L}((r_{S_{\lambda}}^{S})^{-1}(D_{\lambda}))\right)^{\uparrow}$ that is based in $S$.

The disjunction of $\left\{D_{\lambda}^{\uparrow}\right\}_{\lambda \in L}$ is defined by the de Morgan law $\bigvee_{\lambda \in L} D_{\lambda}^{\uparrow} = \neg\left(\bigwedge_{\lambda \in L} \neg\left(D_{\lambda}^{\uparrow}\right)\right)$. Typically $\bigvee_{\lambda \in L} D_{\lambda}^{\uparrow} \subsetneqq \bigcup_{\lambda \in L} D_{\lambda}^{\uparrow}$, and if all $D_{\lambda}$ are nonempty we have that $\bigvee_{\lambda \in L} D_{\lambda}^{\uparrow} = \bigcup_{\lambda \in L} D_{\lambda}^{\uparrow}$ holds if and only if all the $D_{\lambda}^{\uparrow}$ have the same base-space.

For more on the event structure, see Schipper (2015).

To formalize knowledge and awareness of players, we introduce for each player a possibility correspondence. The use of possibility correspondences for modeling knowledge is quite standard in game theory (e.g., see Chapter 5 in Osborne and Rubinstein, 1994). Yet, since we also model awareness, we need to extend possibility correspondences suitably across the lattice structure as in Heifetz, Meier, and Schipper (2006, 2008). Let $N = \{1, ..., n\}$ be a finite set of players. For each player $i \in N$ there is a *possibility correspondence* $\Pi_i : \Omega \to 2^{\Omega}$ with the following properties:

*Confinement:* If $\omega \in S$ then $\Pi_i(\omega) \subseteq S'$ for some $S' \preceq S$.

*Generalized Reflexivity:* $\omega \in \Pi_i^{\uparrow}(\omega)$ for every $\omega \in \Omega$.

*Stationarity:* $\omega' \in \Pi_i(\omega)$ implies $\Pi_i(\omega') = \Pi_i(\omega)$.

*Projections Preserve Ignorance:* If $\omega \in S'$ and $S \preceq S'$ then $\Pi_i^{\uparrow}(\omega) \subseteq \Pi_i^{\uparrow}(\omega_S)$.

*Projections Preserve Knowledge:* If $S \preceq S' \preceq S''$, $\omega \in S''$ and $\Pi_i(\omega) \subseteq S'$ then $(\Pi_i(\omega))_S = \Pi_i(\omega_S)$.

Generalized Reflexivity and Stationarity are the analogues of the partitional properties of the possibility correspondence in partitional Aumann structures or Kripke structures. In particular, Generalized Reflexivity yields the truth property of knowledge. Stationarity will guarantee the introspection properties. Essentially it rules out mistakes of information processing. The properties Projections Preserve Ignorance and Projections Preserve Knowledge guarantee the coherence of knowledge and awareness of individuals down the lattice structure. They compare the possibility sets of an individual in a state $\omega$ and its projection $\omega_S$. The properties guarantee that, first, at the projected state $\omega_S$ the individual knows nothing she does not know at $\omega$, and second, at the projected state $\omega_S$ the individual is not aware of anything she is unaware of at $\omega$ (Projections Preserve Ignorance). Third, at the projected state $\omega_S$ the individual knows every event she knows at $\omega$, provided that this event is based in a space lower than or equal to $S$ (Projections Preserve Knowledge). These properties also imply that at the projected state $\omega_S$ the individual is aware of every event she is aware of at $\omega$, provided that this event is based in a space lower than or equal to $S$.

Heifetz, Meier, and Schipper (2006) show that unawareness structures are capable of modeling non-trivial unawareness and strong notions of knowledge. Unawareness-knowledge structures have been axiomatized by Halpern and Rego (2008) and Heifetz, Meier, and Schipper (2008). Halpern and Rego (2008, 2014), Heifetz, Meier, and Schipper (2008), and Heinsalu (2013) show equivalence results with respect to alternative approaches to modeling knowledge and awareness. See Schipper (2015) for a review. The important point is that these structures are capable of explicitly modeling knowledge and awareness in interactive settings without loss of generality.

So far, our structure provides for qualitative notions of knowledge and awareness only. For optimization, players may want to quantify or weight states that they consider possible. Let $\Delta(S)$ be the set of probability measures on $S$. For a probability measure $\mu^{S'} \in \Delta(S')$, the marginal $\mu^{S'}_{|S}$ of $\mu^{S'}$ on $S \preceq S'$ is defined by

$$\mu^{S'}_{|S}(D) := \mu^{S'}\left(\left(r^{S'}_S\right)^{-1}(D)\right), \quad D \in 2^S.$$

Let $S_{\mu^S}$ be the space on which $\mu^S$ is a probability measure. Whenever for some proposition $E$ we have $S_{\mu^S} \succeq S(E)$ then we abuse notation slightly and write

$$\mu^S(E) = \mu^S(E \cap S_\mu).$$

If $S(E) \not\preceq S_{\mu^S}$, then we say that $\mu^S(E)$ is undefined.

A *prior* $\mu = (\mu^S)_{S \in \mathcal{S}}$ is a projective system of probability measures $\mu^S \in \Delta(S)$, one for each space $S \in \mathcal{S}$, such that for any $S, S' \in \mathcal{S}$ with $S' \succeq S$ we have $\mu^{S'}_{|S} = \mu^S$. A prior is *common* if it is the same for each player. A prior is *positive* if $\mu^{\bar{S}}(\{\omega\}) > 0$ for all $\omega \in \bar{S}$, where $\bar{S}$ is the join of the lattice. This implies positivity also for the probability measures on lower spaces. At $\omega \in \Omega$, player $i$'s belief over states in $\Omega$ is now given by $\mu^{S_{\Pi_i(\omega)}}(\cdot \mid \Pi_i(\omega)) \in \Delta(S_{\Pi_i(\omega)})$.

## 2.2 Networks

We recall some definitions from networks. Let $g = (g_{i,j})_{i,j \in N}$ be a network where $g_{i,j} = 1$ represents an edge between $i$ and $j$. Let $G$ be the set of all networks. To describe the two-way flow of knowledge and awareness, we consider the closure of $g$ denoted by $\bar{g}$ and defined by $\bar{g}_{i,j} = \max\{g_{i,j}, g_{j,i}\}$. Note that $\bar{g}_{i,j} = \bar{g}_{j,i}$ for all $i, j \in N$.

A *path* in a network $g$ between $i$ and $j$, $i \neq j$, is a sequence of links $\bar{g}_{i_1 i_2} = \bar{g}_{i_2,i_3} = ... = \bar{g}_{i_{K-1},i_K} = 1$ for $k \in \{1, ..., K-1\}$, $i_1 = i$ and $i_K = j$, and $i_k \neq i_\ell$ for all $k, \ell = 1, ..., K$, $k \neq \ell$. That is, a path between $i$ and $j$ is a sequence of links starting at $i$ and ending at $j$ such that all nodes in "between" are pairwise distinct. A *cycle* in a network $g$ is a sequence of links $\bar{g}_{i_1,i_2} = \bar{g}_{i_2,i_3} = ... = \bar{g}_{i_{K-1},i_K} = 1$ for some $K > 1$ such that $i_1 = i_K$ and $i_k \neq i_\ell$ for all $k, \ell = 2, ..., K-1$, $k \neq \ell$. That is, a cycle is essentially path that ends where it starts and all other nodes in "between" are pairwise distinct.

A set $C(g) \subseteq N$ is a *component* of the network $g$ if for any $i, j \in C(g)$ there is a path between them and there does not exist a path between any player in $C(g)$ and a player in $N \setminus C(g)$. A component $C(g)$ is *minimal* if it does not contain any cycle and $g_{i,j} = 1$ implies $g_{j,i} = 0$. Let $\mathcal{C}(g)$ be the set of all components of the network $g$. A network $g$ is *connected* if it has a unique component. If that component is minimal, we say it is *minimally connected*. The complete network is denoted by $g^N$. The empty network is denote by $g^\emptyset$.

A *tree* is a connected network without cycles. A *forest* is a network such that each component is a tree. Note that a connected network is a tree if and only if it has $n-1$ links. A tree has at least two leaves, where a leave is a node that has exactly one link. In a tree, there is a unique path between any two nodes. An example of a tree is a star. A *star* is a network for which there exists a player $i$ such that for $j, k \in N$, $g_{j,k} = 1$ implies either $j = i$ or $k = i$. In such a case, $i$ is the *center* of the star. A star is *center-sponsored* if $g_{j,k} = 1$ implies $j = i$.

See Jackson (2008), Goyal (2007) or Vega-Redondo (2007) for further details on networks.

## 2.3 Network Games with Unawareness

The network games defined in this section combine ideas on network formation games Bala and Goyal (2000) (and Jackson and Wolinsky, 1996) with Bayesian games with unawareness by Meier and Schipper (2014) and the solution concept of interim correlated rationalizability introduced in Dekel, Fudenberg, and Morris (2007).

We assume the benefit of the network to player $i$ is represented by a function $v_i : G \times \Omega \longrightarrow \mathbb{R}$. It shall depend on the knowledge and awareness of players that are linked to player $i$. To this extent, we first define for each player $i \in N$ an auxiliary value function $\nu: \bigcup_{S \in \mathcal{S}} \left(2^S \setminus \{\emptyset^S\}\right)^\uparrow \longrightarrow \mathbb{R}$. It is defined on the set of non-contradictory propositions in the unawareness structure (i.e., vacuous events are excluded). We say that value is *monotone* if $E \subseteq F$ implies $\nu(E) \geq \nu(F)$ and $E \subsetneqq F$ implies $\nu(E) > \nu(F)$. Note that this assumption implies that the value increases both in knowledge and awareness. For instance, consider information sets of two players at $\omega$, $\Pi_i(\omega)$ and $\Pi_j(\omega)$. Extend them to events in the lattice structure by taking for each information set the union on inverse images, $\Pi_i^{\uparrow}(\omega)$ and $\Pi_j^{\uparrow}(\omega)$. Now $\Pi_i^{\uparrow}(\omega) \subseteq \Pi_j^{\uparrow}(\omega)$ because $\Pi_i(\omega) \subseteq \Pi_j(\omega)$ (i.e., both players have the same awareness but $i$ has weakly better knowledge than $j$) or $\Pi_i(\omega) \subseteq \Pi_j^{\uparrow}(\omega)$ (i.e., player has weakly higher awareness than player $j$ and does not know less than player $j$).

Second, we let

$$v_i(g, \omega) := \nu \left( \bigwedge_{j \in C : C \in \mathcal{C}(g), i \in C} \Pi_j^{\uparrow}(\omega) \right).$$

This formulation captures the assumption that all players in a component can share their awareness and knowledge. Player $i$'s benefit depends now on the distributed knowledge and awareness of players in his component. Monotonicity of the auxiliary value function $\nu$ implies that the benefit function $v_i$ is monotone in shared knowledge and awareness among players of $i$'s component.

To define network games, we require actions and utility functions. Let $A_i$ be player $i$'s nonempty finite set of actions. We let $A = \times_{i \in N} A_i$ with generic element $a \in A$. For future reference, we also introduce the notation $A_{-i} := \times_{j \in N \setminus \{i\}} A_j$ with generic element $a_{-i} \in A_{-i}$. A consequence function $\gamma : A \longrightarrow \Delta(G)$ maps action profiles into distributions of networks. The interpretation of actions and the specific form of the consequence function will depend on the particular network model we will study in the next sections. In some models, the action of the player will simply be her links she forms with other players. In some other application it may be wage demands or offers that lead to employment and hence links among players.

Player $i$'s utility function is a mapping $u_i : A \times \Omega \longrightarrow \mathbb{R}$. Although the precise functional form of the utility function of players will depend on the model, we require for any model that it is a mapping of the benefit function defined above. That is $u_i(a, \omega) = f_i \left( \mathbb{E}_{\gamma}[v_i(g, \omega) \mid a] \right)$ for some continuous function $f_i$ that will depend on the network model we will study in the next sections.

Note that our game is a network formation game with *incomplete information* including unawareness. Most of the literature on strategic network formation pertains to complete information. The solution concepts like Nash networks (Bala and Goyal, 2000) or pairwise stability (Jackson and Wolinsky, 1996) are not applicable. Moreover, it is not clear that there is any satisfactory equilibrium notion in our context that would capture "stability". To see this note that once a player becomes aware of some event, she may acquire further knowledge about it by forming yet another link. Such link may lead unexpectedly also to additional awareness and yet desires to form additional links etc. If there is a notion of stability, it would capture the long run after *discovery* and *learning* has taken place. Yet, we are more interested in *what* is discovered and learned based on current knowledge and awareness. That's why we will adapt *interim (correlated) rationalizability*, a solution concept studied in Dekel, Fudenberg, and Morris (2007). It captures common knowledge of rationality in the context of incomplete information.

**Definition 1 (Interim rationalizability)** *Define inductively for $i \in N$:*

$$R_i^0(\omega) := A_i$$

$$R_i^{k+1}(\omega) := \left\{ a_i \in A_i : \begin{array}{c} \text{There exists } \beta_i \in \Delta(A_{-i} \times S_{\Pi_i(\omega)}) \text{ such that} \\ (1)\ \beta_i(a_{-i}, \omega') > 0 \text{ implies } a_{-i} \in \times_{j \in N \setminus \{i\}} R_j^k(\omega') \\ (2)\ a_i \in \arg\max_{a_i' \in A_i} \sum_{(a_{-i}, \omega') \in A_{-i} \times S_{\Pi_i(\omega)}} u_i(a_i', a_{-i}, \omega') \cdot \beta_i(\{(a_{-i}, \omega')\}) \\ (3)\ \sum_{a_{-i} \in A_{-i}} \beta_i(\{a_{-i}, \omega'\}) = \mu\left(\{\omega'\} \mid \Pi_i(\omega)\right) \end{array} \right\}$$

$$R_i(\omega) := \bigcap_{k=0}^{\infty} R_i^k(\omega).$$

*If $a_i \in R_i^k(\omega)$, we say that $a_i$ is $k$-level rationalizable at $\omega$. If $a_i \in R_i(\omega)$, we say that $a_i$ is rationalizable at $\omega$. Similarly, say that a network $g$ is $k$-level rationalizable at $\omega$ if $a \in \times_{i \in N} R_i^k(\omega)$ is $k$-level rationalizable at $\omega$ and $\gamma(a)(g) > 0$.*

While for most observations, we can rely on interim rationalizability, we acknowledge it is a quite weak solution concept. For some observations, we need to strengthen it. We introduce interim prudent rationalizability by incorporating an admissibility criterion. Essentially, while interim rationalizability is the interim analogue to iterative elimination of strictly dominated actions, interim prudent rationalizability is the analogue to iterative elimination of weakly dominated actions.

**Definition 2 (Interim Prudent Rationalizability)** *We define inductively for $i \in N$:*

$$P_i^0(\omega) := A_i$$

$$P_i^{k+1}(\omega) := \left\{ a_i \in A_i : \begin{array}{c} \text{There exists } \beta_i \in \Delta(A_{-i} \times S_{\Pi_i(\omega)}) \text{ such that} \\ (1)\ \beta_i(a_{-i}, \omega') > 0 \text{ iff } a_{-i} \in \times_{j \in N \setminus \{i\}} P_j^k(\omega') \text{ and } \mu\left(\{\omega'\} \mid \Pi_i(\omega)\right) > 0 \\ (2)\ a_i \in \arg\max_{a_i' \in A_i} \sum_{(a_{-i}, \omega') \in A_{-i} \times S_{\Pi_i(\omega)}} u_i(a_i', a_{-i}, \omega') \cdot \beta_i(\{(a_{-i}, \omega')\}) \\ (3)\ \sum_{a_{-i} \in A_{-i}} \beta_i(\{a_{-i}, \omega'\}) = \mu\left(\{\omega'\} \mid \Pi_i(\omega)\right) \end{array} \right\}$$

$$P_i(\omega) := \bigcap_{k=0}^{\infty} P_i^k(\omega).$$

*If $a_i \in P_i^k(\omega)$, we say that $a_i$ is k-level prudent rationalizable at $\omega$. If $a_i \in P_i(\omega)$, we say that $a_i$ is prudent rationalizable at $\omega$. Similarly, say that a network $g \in G$ is k-level prudent rationalizable at $\omega$ if there exists a k-level rationalizable action profile $a \in\in \times_{i \in N} P_i^k(\omega)$ with $\gamma(a)(g) > 0$.*

In comparison with the definition of interim rationalizability, interim prudent rationalizability requires now that if $a_{-i} \in \times_{j \in N \setminus \{i\}} P_j^k(\omega')$ and $\mu(\{\omega'\} \mid \Pi_i(\omega)) > 0$ then $\beta_i(a_{-i}, \omega') > 0$. That is any prudent rationalizable action of opponent $j$ at a state that receives strict positive probability with player $i$'s belief must get also strict positive probability.

## 2.4 Specialists versus Generalists

In this section, we limit us to the discussion to unawareness structures with two types of players, specialists and generalists. Roughly, a specialist is a player who has "precise" knowledge about an event but is not be aware of many events. In contrast, a generalist is a player who is aware of many events but may not know most of them. As such, the specialists and generalists are prototypes of extreme forms of knowledge and awareness which will help us to illustrate some main ideas in the following sections.

**Definition 3** *An unawareness structure* diverse specialists *if there subset of players $N^s \subseteq N$ such that for all $\omega \in \bar{S}$*

*(i) Diversity: For any $i, j \in N^s$ with $i \neq j$ the awareness levels $S_{\Pi_i(\omega)}$ and $S_{\Pi_j(\omega)}$ are pairwise incomparable (with respect to the lattice order).*

*(ii) Specialist: For any $i \in N^s$, $\Pi_i(\omega)$ is a singleton.*

*(iii) Nontriviality: For any $S \succ \underline{S}$, where $\underline{S}$ is the meet of $\mathcal{S}$, we have $|S| \geq 2$.*

That is, each specialist has the "deepest" knowledge about his "special topic" (Property (ii)) and any two specialists have different "specialities" (Property (i)). Clearly, it would be beneficial to society if they combine their "deep but diverse knowledge". Unfortunately, each of them with his own "tunnel" view sees no point of doing so.[3] The point

[3]In German, the term "Fachidiot" may capture the nature of the specialist.

of (iii) is that together with (ii) it implies that specialists really know something since the cannot just know tautologies (i.e., universal events). Point (iii) is stated stronger (but neater) than needed. It would be enough to require that $|S| \geq 2$ for all $S \in \mathcal{S} \setminus \{\underline{S}\}$ for which there exists $i \in N^s$ and $\omega \in \Omega$ with $\Pi_i(\omega) \subseteq S$. Note also that since $\mathcal{S}$ is a lattice, the meet $\underline{S}$ exists.

**Definition 4** *An unawareness structure has* generalists *if there exists a subset of players $N^g \subseteq N$ such that for all $\omega \in \bar{S}$, $\Pi_i(\omega) = \bar{S}$ for all $i \in N^g$, where $\bar{S}$ is the join of the lattice of spaces.*

Roughly generalists "knows nothing" (beyond tautologies) but are "aware of everything". They have "extremely broad but superficial knowledge".

In the following subsections, we will often focus on the case in which the population consists of diverse specialists and just one generalist. In such a case, we label the generalist with "1".

Because of their "tunnel view", for any state a specialist considers, there is no benefit to combining his knowledge with the knowledge of other specialists or the generalist This is stated more formally in the following lemma.

**Lemma 1** *Consider an unawareness structure in which the set of players is partitioned into two subsets, diverse specialists, $N^s$, and generalists, $N^g$. Then for any $\omega \in \Omega$, $g \in G$, and $i \in N^s$, we have*

1. $\Pi_i^{\uparrow}(\omega) = \bigwedge_{j \in C: C \in \mathcal{C}(g), i \in C} \Pi_j^{\uparrow}(\omega)$,
2. $\nu\left(\Pi_i^{\uparrow}(\omega)\right) = \nu\left(\bigwedge_{j \in C: C \in \mathcal{C}(g), i \in C} \Pi_j^{\uparrow}(\omega)\right)$,
3. $v_i(g^{\emptyset}, \omega) = v_i(g, \omega)$.

*That is, each specialist does not consider an increase (and hence strict positive benefit) of knowledge from combining her knowledge with that of any other specialists or generalists.*

Proof. We start by proving 1. Let $i \in N^s$ denote a specialist. For any generalist, $k \in N^g$, since for any $\omega \in \bar{S}$, $\Pi_k(\omega) = \bar{S}$, we have $\Pi_k(\omega_{S_{\Pi_i(\omega)}}) = S_{\Pi_i(\omega)}$ by Projections Preserve Knowledge. Again, by Projections Preserve Knowledge, we have $\Pi_k(\omega_{S_{\Pi_i(\omega)}}) = S_{\Pi_i(\omega)}$ for any $\omega \in \Omega$.

For all other specialists $j \in N^s \setminus \{i\}$, $\omega \in \Omega$, $\Pi_j(\omega) = \{\omega_{\Pi_j(\omega)}\}$ by Definition 3 (ii). By Definition 3 (i), for any player $i \in N^s$ and any other player $j \in N^s \setminus \{i\}$, and $\omega \in \bar{S}$, $\Pi_j(\omega_{\Pi_i(\omega)}) \subseteq S$ for some space $S \prec S_{\Pi_i(\omega)}$, $S \in \mathcal{S}$. Such a space $S$ exists in $\mathcal{S}$ since $\mathcal{S}$ is a lattice. Note that $\Pi_j(\omega_{\Pi_i(\omega)})$ represents $j$'s knowledge and awareness in the mind of player $i$. For $\omega \in S'$ with $S' \preceq \bar{S}$, $\Pi_j(\omega_{\Pi_i(\omega)}) \subseteq S$ with $S \preceq S'$. Thus, we must have by Generalized Reflexivity, $\Pi_i^{\uparrow}(\omega) \wedge \left(\bigwedge_{j \in N^s \setminus \{i\}} \Pi_j^{\uparrow}(\omega_{\Pi_i(\omega)})\right) \wedge \left(\bigwedge_{k \in N^g} \Pi_k^{\uparrow}(\omega_{\Pi_i(\omega)})\right) = \Pi_i^{\uparrow}(\omega)$. This implies 1.

2. follows now directly from the definition of $\nu$ and 3. follows from the definition of $v_i$. □

In contrast to specialists, the generalist realizes that their are benefits of combining knowledge of specialists. This is stated more formally in the following lemma.

**Lemma 2** *Consider an unawareness structure that consists of a subset of diverse specialists, $N^s$, and exactly one generalist (labeled by "1"). Then for any $\omega \in \Omega$, $g \in G$, we have*

1. $\Pi_1^{\uparrow}(\omega) \supsetneqq \bigwedge_{j \in C: C \in \mathcal{C}(g), 1 \in C} \Pi_j^{\uparrow}(\omega)$,
2. $\nu\left(\Pi_1^{\uparrow}(\omega)\right) < \nu\left(\bigwedge_{j \in C: C \in \mathcal{C}(g), 1 \in C} \Pi_j^{\uparrow}(\omega)\right)$,
3. $v_1(g^{\emptyset}, \omega) < v_1(g, \omega)$.

*That is, the generalist considers the increase (and hence strict positive benefit) from combining her knowledge with that of specialists.*

Proof. By (ii) and (iii) of Definition 3 as well as Generalized Reflexivity, we have for any $\omega \in \bar{S}$ and $i \in N^s$, $\left(r^{\bar{S}}_{S_{\Pi_i(\omega)}}\right)^{-1}(\Pi_i(\omega)) \subsetneqq \bar{S}$. Moreover, by Definition 4, for any $\omega \in \bar{S}$, $\Pi_1(\omega) = \bar{S}$. Thus, $\Pi_1(\omega) \wedge \left(\bigwedge_{j \in N^s} \Pi_j^{\uparrow}(\omega)\right) \subsetneqq \Pi_1(\omega)$. This implies 1.

2. follows now directly from the definition of $\nu$ and 3. follows from the definition of $v_i$. □

# 3 Unilateral Cost of Initiating Communication

We now study a model where the cost of communicating are just born by the player who initiates the communication.

An *action* of player $i \in N$ is a profile $g_i = (g_{i,1}, ..., g_{i,i-1}, g_{i,i+1}, ..., g_{i,n})$. We denote by $G_i$ player $i$'s set of actions and by $G = \times_{i \in N} G_i$ the set of action profiles, one for each player. Thus, $\gamma = id_G$. I.e., the consequence function is simply the identity on the set of networks.

The *utility function* of player $i$ is a function $u_i : G \times \Omega \longrightarrow \mathbb{R}$ defined by

$$u_i(g, \omega) := v_i(g, \omega) - c \cdot \sum_{j \in N} g_{i,j}$$

for some *cost of initiating a connection* $c > 0$. Note that the cost of initiating a connection is borne just to the player who initiates a link, not to the players to which he links to. We call this the *unilateral communication cost model.*

**Assumption 1 (Small costs)** *We say* costs are small *in the unilateral communication cost model if $c > 0$ is such that for all $i \in N$, $\omega \in \Omega$, $g_{-i} \in G_{-i}$, and all $g_i, g'_i \in G_i$ with $g'_{i,k} \neq g_{i,k}$ for exactly one $k \in N \setminus \{i\}$ and $g'_{i,\ell} = g_{i,\ell}$ for all $\ell \in N \setminus \{i, k\}$, if*

$$\sum_{\omega' \in S_{\Pi_i(\omega)}} \left(v_i(g_i, g_{-i}, \omega') - v_i(g'_i, g_{-i}, \omega')\right) \mu^{S_{\Pi_i(\omega)}} \left(\{\omega'\} \mid \Pi_i(\omega)\right) > 0$$

*then*[4]

$$\sum_{\omega' \in S_{\Pi_i(\omega)}} \left(v_i(g_i, g_{-i}, \omega') - v_i(g'_i, g_{-i}, \omega')\right) \mu^{S_{\Pi_i(\omega)}} \left(\{\omega'\} \mid \Pi_i(\omega)\right) > 2c.$$

That is, as long as the expected benefit from forming an additional link is strict positive, it always pays to bear the cost of forming that link. So costs of initiating a link are small in the sense that they are never an impediment of to forming a link as long as the link is expected to be valuable.

**Proposition 1** *Consider the unilateral communication cost model and an unawareness structure with diverse specialists only. If $g$ is a rationalizable network at $\omega \in \Omega$ then $g$ is the empty network and thus ex post inefficient. In fact, no communication is the strict dominant action for each player at every $\omega \in \Omega$.*

Proof. Let $\beta_i \in \Delta(G_{-i} \times S_{\Pi_i(\omega)})$ be a conjecture of player $i$ such that $\sum_{g_{-i} \in G_{-i}} \beta_i(\{g_{-i}, \omega_{S_{\Pi_i(\omega)}}\}) = \mu^{S_{\Pi_i(\omega)}} \left(\{\omega_{S_{\Pi_i(\omega)}}\} \mid \Pi_i(\omega)\right)$. By Lemma 1, for any $g_{-i} \in G_{-i}$, $v_i(g_i, g_{-i}, \omega_{S_{\Pi_i(\omega)}}) = v_i(g'_i, g_{-i}, \omega_{S_{\Pi_i(\omega)}})$

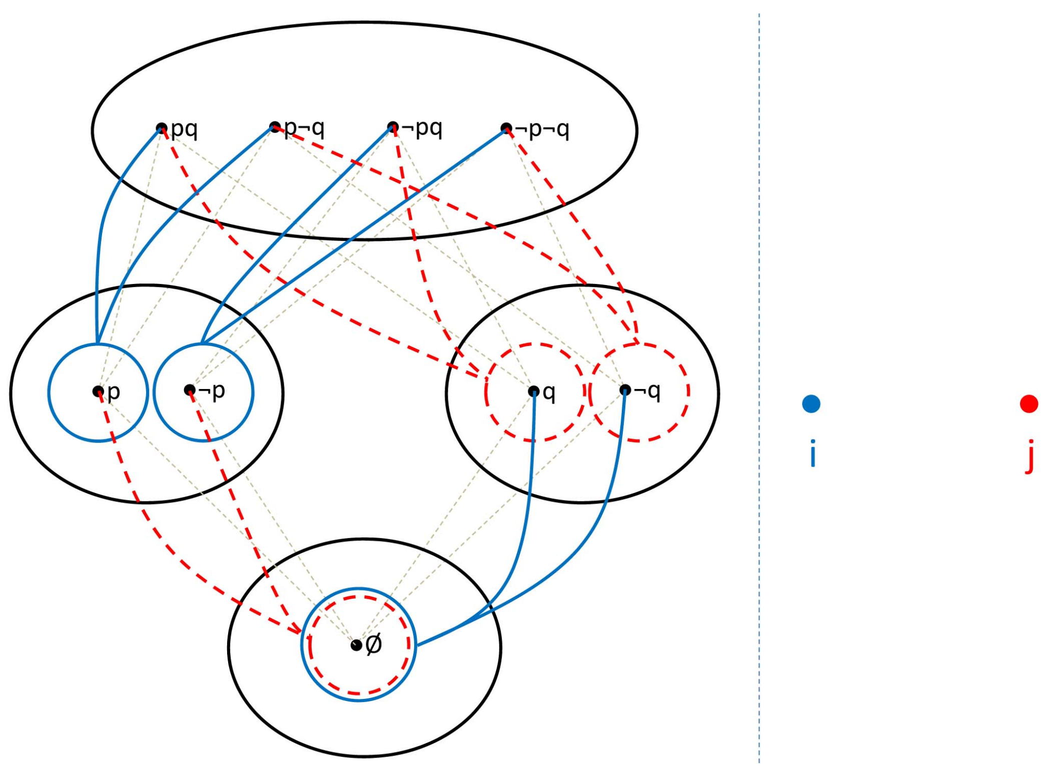
[4] We could have written $c$ without factor 2 in front of $c$. This factor is only required for the bilateral communication cost model in the next section.

for any $g_i, g_i' \in G_i$. Since $c > 0$, for any $g_{-i} \in G_{-i}$, $u_i(g_i, g_{-i}, \omega_{S_{\Pi_i(\omega)}}) > u_i(g_i', g_{-i}, \omega_{S_{\Pi_i(\omega)}})$ for any $g_i$ with $g_{i,j} = 0$ for all players $j \in N$ and $g_i'$ with $g_{i,j}' = 1$ for some player $j \in N$. Thus, for every player $i \in N$, proposing no link is strict dominant at $\omega$.

To prove that the empty network $g^{\emptyset}$ is ex post inefficient, consider a minimally connected network $g$. $\bigwedge_{i \in N} \Pi_i^{\uparrow}(\omega) = \bigwedge_{i \in N} \{\omega_{S_{\Pi_i(\omega)}}\}^{\uparrow}$. By diversity of specialists, $\bigwedge_{i \in N} \{\omega_{S_{\Pi_i(\omega)}}\}^{\uparrow} \subsetneqq \{\omega_{S_{\Pi_j(\omega)}}\}^{\uparrow}$ for any $j \in N$. Hence $v_i(g, \omega) > v_i(g^{\emptyset}, \omega)$ by monotonicity of $\nu$ for all $i \in N$ and thus $\sum_{i \in N} v_i(g, \omega) > \sum_{i \in N} v_i(g^{\emptyset}, \omega)$. A minimal connected network as $n - 1$ links. Since cost are assumed to be small, $\sum_{i \in N} u_i(g, \omega) = \sum_{i \in N} v_i(g, \omega) - (n-1) \cdot c > \sum_{i \in N} v_i(g^{\emptyset}, \omega) = \sum_{i \in N} u_i(g^{\emptyset}, \omega)$. □

Figure 1 illustrates Proposition 1. The unawareness structure is depicted at the left. There is a lattice of four spaces. For illustration, we labeled each state with primitive propositions that are true at that state. The upmost space is rich enough to describe both the event $p$ and $q$. The left space is confined to $p$ whereas the right space to $q$. The lowest space allows for the empty description only. The symbol "¬" stands for negation. I.e., "$\neg p$" means that the event $p$ does not obtain.

Figure 1: Two Diverse Specialists



There are two specialists. The possibility correspondences of both players are depicted with the solid blue and intermitted red lines and ovals, respectively. The blue player specializes in $p$ whereas the red player in $q$. Neither is aware of what the other is aware of (except for tautologies and contradictions of lowest expressive power). Each player believes that the other is unaware of what he is aware of. The only rationalizable network, the empty network, is depicted at the right of Figure 1.

The following proposition emphasizes the important role of generalists in society. Although their knowledge is extremely superficial, due to their broad knowledge they are able to see connections that specialists do not see. They realize what others know and want to connect to them. In the resulting social network, generalists are in the center. They "talk a lot" although they "know nothing". This allows society to make efficient use of the distributed knowledge of specialists.

**Proposition 2** *Consider the unilateral communication cost model and an unawareness structure that consists of diverse specialists and one generalist. If $g$ is a rationalizable network at $\omega \in \Omega$ then $g$ is minimally connected and thus ex post efficient. In particular, $g$ is a center-sponsored star with the generalist in the center at every $\omega$. Not proposing any links is the strict dominant action for all specialists at $\omega$ and communicating with all specialists is the unique second-level rationalizable action for the generalist at $\omega$.*

PROOF. For all specialists, $i \in N^s$, the action $g_i$ with $g_{i,j} = 0$ for all $j \in N$ is the strict dominant action at any $\omega \in \bar{S}$ and thus the unique rationalizable action by similar arguments as in the proof of Proposition 1. For all $i \in N^s$, $g_i \in R_i^1(\omega)$ implies $g_{i,j} = 0$ for all $j \in N$. In fact, $\{g_i\} = R_i^1(\omega)$.

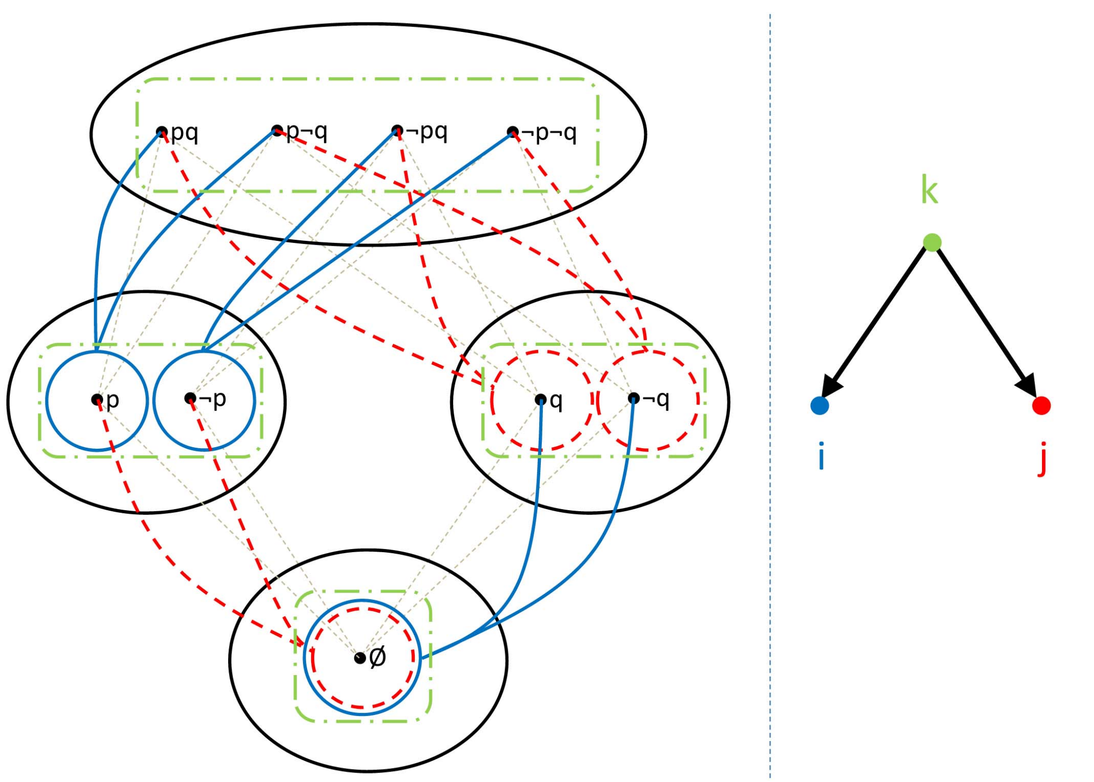
Denote by 1 the generalist. We need to show that for the generalist, the action $g_1$ with $g_{1,j} = 1$ for all $j \in N^s$ is the unique rationalizable action at $\omega$. Since the second-level belief of the generalist 1 is certain of first-level rationalizable actions of all other players, we have $\beta_1(g'_{-1}, \omega') = 0$ for all $g'_{-1} \notin \times_{i \in N^s} R_i^1(\omega')$ and $\omega' \in \bar{S}$. Moreover, $marg_{S^{\Pi_1(\omega')}} \beta_1(g_{-1}, \omega') = \mu^{\bar{S}}(\{\omega'\} \mid \Pi_1(\omega)) = \mu^{\bar{S}}(\{\omega'\} \mid \bar{S}) = \mu^{\bar{S}}(\{\omega'\})$. That is, the generalist's second order belief must be unique. Since $\mu$ is strict positive and the unawareness structure consists of diverse specialists and a generalist, at any $\omega \in \bar{S}$ the generalist is certain that the other players are specialists. Thus, by Lemma 2, for all $\omega \in \bar{S}$, $v_1(g_1, g_{-1}, \omega) > v_1(g'_1, g_{-1}, \omega)$ for all $g'_1 \in G_1$ such that $C \in \mathcal{C}(g'_1, g_{-1})$ with $1 \in C$, $C \subsetneqq N$. Since costs are small, for all $\omega \in \bar{S}$, $u_1(g_1, g_{-1}, \omega) = v_1(g_1, g_{-1}, \omega) - (n-1) \cdot c > v_1(g'_1, g_{-1}, \omega) - c \cdot \sum_{i \neq 1, i \in C, C \in \mathcal{C}(g'_1, g_{-1}), 1 \in C} g'_{1,k} = u_1(g'_1, g_{-1}, \omega)$ for all $g'_1 \in G_1$ such that $C \in \mathcal{C}(g'_1, g_{-1})$ with $1 \in C$, $C \subsetneqq N$. Hence $\sum_{\omega' \in \bar{S}} u_1(g_1, g_{-1}, \omega') \mu^{\bar{S}}(\{\omega'\}) > \sum_{\omega' \in \bar{S}} u_1(g'_1, g_{-1}, \omega') \mu^{\bar{S}}(\{\omega'\})$ for all $g'_1 \in G_1$ such that

$C \in \mathcal{C}(g_1', g_{-1})$ with $i \in C$, $C \subsetneq N$. Thus, $g_1$ is the unique best response to the unique second level belief $\beta_1(g_{-1}, \cdot)$. Hence there is no other second-level rationalizable action for the generalist but $g_1$.

Ex post efficiency follows by similar arguments as in the proof of Proposition 1 from monotonicity and the assumption that cost are small. □

Figure 2 illustrates Proposition 2. The unawareness structure is as in Figure 1. Yet, we added a generalist whose possibility correspondence is given by the green dash-dotted ovals. The generalist knows nothing (beyond tautologies) but is aware of everything. This follows from his possibility set on the highest space that encompasses all states. Consequently, the properties of the possibility correspondence force the information sets on the lower spaces. The only reationalizable network is depicted at the right side of the Figure 2.

Figure 2: Two Diverse Specialists and a Generalist




What can be said beyond specialists and generalists?

**Definition 5** *At $\omega \in \Omega$ player $i$ knows that she has better knowledge than player $j$ if for $\omega' \in \Pi_i(\omega)$, $\Pi_i(\omega') \subsetneqq \Pi_j^{\uparrow}(\omega') \cap S_{\Pi_i(\omega')}$. Similarly, at $\omega \in \Omega$ player $i$ knows that player $j$ has better knowledge than her if for $\omega' \in \Pi_i(\omega)$, $\Pi_j^{\uparrow}(\omega') \cap S_{\Pi_i(\omega')} \subsetneqq \Pi_i(\omega')$. At $\omega \in \Omega$, player $i$ knows that player $j$ knows that he has better knowledge than her if for $\omega' \in \Pi_i(\omega)$, $\omega'' \in \Pi_j(\omega')$, $\Pi_j(\omega'') \subsetneqq \Pi_i^{\uparrow}(\omega'') \cap S_{\Pi_j(\omega'')}$.*

**Proposition 3** *Let $g$ is a rationalizable network in the unilateral communication cost model. For any $\omega \in \Omega$ and $i \in N$,*

*(i) if player $i$ knows that she has better knowledge than any other player $j \in N \setminus \{i\}$, then $g_{i,j} = 0$ for all $j \in N \setminus \{i\}$. In fact, this is a strict dominant action for player $i$ at $\omega$.*

*(ii) if player $i$ knows that all other players $j \in N \setminus \{i\}$ know that they have better knowledge than $i$, then $g_{i,j} = 1$ for some $j \in N \setminus \{i\}$.*

Proof. (i) Let $g$ be a rationalizable network and $\omega$ and $i$ be such that for any $\omega' \in \Pi_i(\omega)$, $\Pi_i(\omega') \subsetneqq \Pi_j^{\uparrow}(\omega') \cap S_{\Pi_i(\omega')}$ for any $j \neq i$. Then for any $(g_{-i}, \omega') \in G_{-i} \times \Pi_i(\omega)$, $v_i(g, \omega') = v_i(g_i', g_{-i}, \omega')$ for any $g_i' \in G_i$. Since links that $i$ initiates are costly to $i$, $g_i$ with $g_{i,j} = 0$ for all $j \in N \setminus \{i\}$ is strict dominant for $i$ at $\omega$.

(ii) Before we prove (ii), note that at $\omega \in \Omega$, if $i$ knows that $j$ knows that $j$ has better knowledge than $i$, then $j$ and $i$ know that $j$ has better knowledge than $i$. Hence, by (i) for all players $j \in N \setminus \{i\}$, $g_{j,i} = 0$ is first-level rationalizable. In fact, it is strict dominant. Any second-level belief of player $i$ is certain of that. Any best response to such a second-level belief implies $g_{i,j} = 1$ for some $j \in N \setminus \{i\}$. □

# 4 Bilateral Communication Costs and Forcing Others to Listen

In the previous section, a player can unilaterally decide to communicate with another player. Consent to listen is granted automatically by the other player. This is realistic in some but not all contexts. For instance, we often listen to someone contacting us just out of politeness. That is, consent to listen is granted automatically quasi by social convention or by civil manners. Yet, in some contexts we may apologize and decline a request of appointment when we think it is not beneficial to us. In other cases, people with power such as managers can compel their subordinates to listen to them. People in power are granted the right by society or hierarchy to impose on others the cost of listening to them.

When considering both specialists and generalists, there is naturally the question of whom should be put into such a position of power? This is a question about the efficient design of social networks. To analyze this, we introduce a strategic network formation game that bears features both of the simultaneous link announcement game of Myerson (1991, p. 448)[5] and non-cooperative network formation games of Bala and Goyal (2000).

As before, an *action* of player $i \in N$ is a profile $g_i = (g_{i,1}, ..., g_{i,i-1}, g_{i,i+1}, ..., g_{i,n})$, where we interpret $g_{i,j} = 1$ as player $i$'s intention to link to player $j$. Whether or not such a link is established will now depend on the power of player $i$ as well as the action of player $j$. Partition the set of players $N = N^{**} \cup N^{*}$ with $N^{**} \cap N^{*} = \emptyset$. Given the action profile $g = (g_1, ..., g_n)$, the network $\hat{g}$ is defined by

$$\hat{g}_{i,j} = \begin{cases} \min\{g_{i,j}, g_{j,i}\} & \text{if } i, j \in N^{**} \\ g_{i,j} & \text{if } i \in N^{**}, j \in N^{*} \\ g_{j,i} & \text{if } i \in N^{*}, j \in N^{**} \\ \min\{g_{i,j}, g_{j,i}\} & \text{if } i, j \in N^{*} \end{cases}$$

That is, players in $N^{**}$ have power over players in $N^{*}$ but not vice versa. Players have no powers over players at the same level. That is, players in $N^{**}$ do not have power over other players in $N^{**}$. Moreover, players in $N^{*}$ have no power of players in $N^{*}$. So far, this describes a deterministic consequence function $\gamma(g) = \hat{g}$ (more precisely, $\gamma(g)(\hat{g}) = 1$).

**Definition 6** *We say that player $i$ is put in the* position of power *if $i \in N^{**}$. We say that a subset of players $N' \subseteq N$ is in the* same position of power *if $N' \subseteq N^{**}$ or $N' \subseteq N^{*}$.*

There is still two-way flow of knowledge and awareness. Moreover, the benefits from forming links are as in the previous section.

The utility function is now redefined for each player $i \in N$ with respect to network $\hat{g}$ by

$$u_i(\bar{\hat{g}}, \omega) := v_i(\bar{\hat{g}}, \omega) - c \cdot \sum_{j \in N} \bar{\hat{g}}_{i,j}$$

for some *cost* $c > 0$. Note that a player $j \in N^{**}$ has now the power to force the cost of communication $c$ upon player $i \in N^{*}$ even if player $i$'s action involves $g_{i,j} = 0$ because in

[5]See Jackson (2008), Section 11.1.2.

such a case $\hat{g}_{i,j} = g_{j,i} = \bar{\hat{g}}_{i,j}$. As before, we denote by $\bar{\hat{g}}$ the closure of the network $\hat{g}$. We call the model outlined so far in the bilateral communication cost model. We still assume that the costs of communicating are small (Assumption 1). That is, for any player and state, as long as the expected benefit from forming an additional link is strict positive, it socially efficient to forming that link and given that both sides to a link bear the cost of communication.

**Proposition 4** *Consider the bilateral communication cost model and an unawareness structure that may have any mix of diverse specialists and generalists in which all or none are in a position of power. If $\hat{g}$ is a rationalizable network, then $\hat{g}$ is empty and this is ex post inefficient. In particular, no communication is a strict dominant action for any specialist and any action is second-level rationalizable for any generalist.*

PROOF. Since all players are in the same position of power, a link between $i$ and $j$ gets formed if and only if $g_{i,j} = g_{j,i} = 1$. We will show that for any specialist $i$ the strict dominant action is $g_i$ with $g_{i,j} = 0$ for all $j \in N \setminus \{i\}$ and for any generalist any action is rationalizable.

The fact that for any specialist $i$ the strict dominant action is $g_i$ with $g_{i,j} = 0$ for all $j \in N \setminus \{i\}$ follows from analogous arguments as in the proof of Proposition 1. Thus $\{g_i\} = R_i^1(\omega)$ for any specialist $i$ and $\omega \in \Omega$. Hence no link get formed no matter what generalists do. Hence any action is second-level rationalizable for any generalist. (The cost of a link is paid only if the link is formed.)

Ex post inefficiency of the empty network follows from monotonicity and the assumption that cost are small by analogous arguments as in the proof of Proposition 1. □

Bilateral costs have a dramatic effect on social network formation among specialists and generalist. This becomes most obvious when considering a corollary: If $\hat{g}$ is a rationalizable network of of diverse specialists and *one* generalists and that all or none are in a position of power, then $\hat{g}$ is empty and this is ex post inefficient. In particular, no communication is a strict dominant action for any specialist and any action is second-level rationalizable for any generalist. This is in sharp contrast to Proposition 2 for earlier unilateral cost model.

The assumption in Proposition 4 is that both parties to a potential communication need to consent to communication when on the same level of power. Next we show that putting specialists only in a position of power does not help to achieve efficient sharing of knowledge and awareness.

**Proposition 5** *Consider the bilateral communication cost model and an unawareness structure that consists of diverse specialists and generalists and some, all, or none specialists are put in a position of power while none of the generalists are put in a position of power. If $\hat{g}$ is a rationalizable network, then $\hat{g}$ is empty and this is ex post inefficient. In particular, no communication is a strict dominant action for any specialist and any action is second-level rationalizable for any generalist.*

PROOF. This follows by analogous arguments as in the proof of Proposition 4. □

In contrast to the prior result on putting specialists into power, having a generalist in a position of power allows for efficient communication of knowledge and awareness.

**Proposition 6** *Consider the bilaterial communication cost model and an unawareness structure that consists of diverse specialists and one generalist. None of the specialists is put in a position of power while the generalist only is put in a position of power. If $\hat{g}$ is a rationalizable network, then $\hat{g}$ is minimally connected and this is ex post efficient. In particular, $\hat{g}$ is a center-sponsored star with the generalist in the center. No communication is a strict dominant action for any specialist but communication is nevertheless imposed by the generalist in a position of power, for which communicating with everyone is the unique second-level rationalizable action.*

PROOF. This follows by analogous arguments as in the proof of Proposition 2. □

What can be said beyond specialists and generalists? Analogous to the unilateral communication cost model we observe the following:

**Proposition 7** *Let $g$ is a rationalizable network in the bilateral communication cost model. For any $\omega \in \Omega$ and $i \in N$, if player $i$ knows that she has better knowledge than any other player $j \in N \setminus \{i\}$, then $g_{i,j} = 0$ for all $j \in N \setminus \{i\}$. In fact, this is a strict dominant action for player $i$ at $\omega$.*

PROOF. The proof is similar to the proof of Proposition 3 (i). □

Note that a result analogous to Proposition 3 (ii) does not hold in the bilateral communication cost model since anybody who knows that he has better knowledge than others will not accept a link unless forced upon him by a player in a position of power.

# 5 Who Becomes an Entrepreneur?

We now study a simple model of the formation of a firm. Players can be employed by other players. The employer gains the knowledge and awareness of the employee in return for paying a wage. For simplicity, a player who is employed by another player cannot employ other players. That is, a player is either an employee or an employer or "self-employed". Also for simplicity, we assume that players are not allowed to communicate with other players outside the firm.

An action of player $i \in N$ is a profile $w_i = (w_{i,1}, ..., w_{i,i-1}, ..., w_{i,i+1}, ..., w_{i,n}) \in W^{n-1} \subset \mathbb{R}^{n-1}$ such that

(i) if $w_{i,j} > 0$ for some $j \in N$ then $w_{i,k} \geq 0$ for all $k \neq j$,

(ii) if $w_{i,j} < 0$ for some $j \in N$ then $w_{i,k} \leq 0$ for all $k \neq j$.

We interpret $w_{i,j} \geq 0$ as player $i$'s offer to employ player $j$ for the wage $w_{i,j}$. When $w_{i,j} \leq 0$ we interpret it as player $i$'s offer of expertise to player $j$ in return for the wage $-w_{i,j}$ paid by player $j$ to player $i$. With this interpretation the restrictions (i) and (ii) mean that a player cannot be an employer as well as an employee.

In order to study interim rationalizability in the employment game, it will be useful to assume that the game is finite. That is, we assume that there is a finite but sufficiently fine grid of real numbers $W := \{-k\delta, -(k-1)\delta, ..., -2\delta, -\delta, 0, \delta, 2\delta, ..., (k-1)\delta, k\delta\}$ such that $w_i \in W^{n-1}$ for all $i \in N$.

The employment network $g$ is defined by for all $i, j \in N$, $w = (w_i)_{i \in N} \in (W^{n-1})^n$,

$$prob(\{g_{i,j} = 1\} \mid w) = \begin{cases} \frac{1}{|K_j|} & \text{if } i \in K_j := \left\{k \in N \setminus \{j\} \mid w_{k,j} = w_j^{(1)}, w_{k,j} \geq -w_{j,k}\right\} \\ 0 & \text{otherwise} \end{cases}$$

where $w_j^{(1)}$ is the first-order statistic of $(w_{k,j})_{k \in N \setminus \{j\}}$. That is, $w_j^{(1)}$ is the highest wage offered to player $j$ by any other player. The set $K_j$ is defined as the set of players who offer the highest wage to $j$ and whose wage offer exceeds player $j$'s wage demand. When there is more than one player who offers $j$ the highest wage and exceeds player $j$'s wage demand, then any of the highest "bidders" in $K_j$ can employ player $j$ with equal probability. Note that this is essentially a first-price auction among potential employers for player $j$ in which reservation values of $j$ are specific to bidders (i.e., "bidder specific handicaps"). Note that this probabilistic network formation rule defines $\gamma$ in the employment model.

We assume that if player $i$ employs $j$ then there is a two-way flow of knowledge and awareness between those players. Further we assume that if player $i$ employs player $j$ the actual wage received by player $j$ is the wage offered by $i$, $w_{i,j}$. Note that by the definition of the employment network, if player $j$ is an employee of player $i$ then player $i$'s wage offer, $w_{i,j}$ must weakly exceed player $j$'s wage demand, $-w_{j,i}$ (see the definition of $K_j$ above). The utility function of player $i$ is defined by

$$u_i(g, \omega) := v_i(g, \omega) \cdot \max\left\{\max_{j \in N \setminus \{i\}}\{g_{i,j}\}, (1 - \max_{j \in N \setminus \{i\}}\{g_{j,i}\})\right\} - \sum_{j \in N \setminus \{i\}} g_{i,j} \cdot w_{i,j} + \sum_{j \in N \setminus \{i\}} g_{j,i} w_{j,i}.$$

Although this function may look complicated, it has a straightforward interpretation. Note first that if $\max_{j \in N \setminus \{i\}} g_{i,j} = 1$ then player $i$ is an employer. If $1 - \max_{j \in N \setminus \{i\}}\{g_{j,i}\} = 1$ then nobody employs player $i$. Thus, if $\max\left\{\max_{j \in N \setminus \{i\}}\{g_{i,j}\}, (1 - \max_{j \in N \setminus \{i\}}\{g_{j,i}\})\right\} = 1$ then player $i$ is either an employer or is not employed by anyone (which includes the case that player $i$ is "self-employed", i.e., working "alone"). In those cases, player $i$ reaps the benefits of knowledge and awareness of her connected component. Note that if she is "self-employed", her connected component consists just of herself. The employment relations do not only affect the direct benefits from knowledge and awareness but also wages. The term $-\sum_{j \in N \setminus \{i\}} g_{i,j} \cdot w_{i,j}$ are the cost of wages paid by player $i$ to other players. This term is non-zero only if player $i$ is an employer. The last term, $\sum_{j \in N \setminus \{i\}} g_{j,i} w_{j,i}$, is the wage received by player $i$. For this term to be positive, it is necessary that player $i$ is employed by some player. Note that player $j$ is employed by at most one other player. Further note that if $j$ is employed by some other player, then the term $\max\left\{\max_{j \in N \setminus \{i\}}\{g_{i,j}\}, (1 - \max_{j \in N \setminus \{i\}}\{g_{j,i}\})\right\} = 0$. Hence, she does not directly reap the benefits of her own knowledge and awareness but instead receives a wage. If a player is an employer, then she does not receive wages but here utility consists of the expected benefit from the knowledge and awareness of the players she employs and the wage bill she pays. Thus, she essentially receives the "residual" and we may call her an entrepreneur.

The following assumption states that combining knowledge of all players is strictly efficient. Note that by definition $v_i(g^N, \omega) = v_j(g^N, \omega)$ for any $i, j \in N$ since $g^N$ is the complete network.

**Assumption 2** *We assume that* $v_i(g^N, \omega) \geq \sum_{j \in N} v_j(g^\emptyset, \omega) + (n-1)\delta$ *for any* $\omega \in \Omega$.

We say that the employment network $g$ is rationalizable if there is a rationalizable profile of actions $w = (w_1, ..., w_n)$ for which there is strict positive probability that $g$ is formed.

The model outlined so far in this section we call the entrepreneurship model.

**Proposition 8** *Consider the entrepreneurship model and an unawareness structure that consists of diverse specialists. If $g$ is a first-level rationalizable employment network, then $g$ is the empty network and thus ex post inefficient. In any first-level rationalizable action profile, if a wage is offered to another player, it must be zero. Moreover, for any first-level rationalizable action profile, the wage demanded by a player must exceed her value of knowledge from being self-employed.*

Proof. By Lemma 1, for all $i \in N$, $\omega \in \Omega$, $v_i(\omega_{S_{\Pi_i(\omega)}}, g^{\emptyset}) = v_i(\omega_{S_{\Pi_i(\omega)}}, g)$ for all $g \in G$. Thus, for any $i \in N$, $\omega \in \Omega$, if $w_i \in R_i^1(\omega)$ with $w_{i,j} \geq 0$ for some $j \neq i$, $j \in N$, then we must have $w_{i,j} = 0$. That is, any first-level rationalizable wage offered by a player $i$ to another player $j$ must be zero.

For any $i \in N$ and $\omega \in \Omega$, if $w_i \in R_i^1(\omega)$ with $w_{i,j} < 0$ for some $j \neq i$, $j \in N$, then $w_{i,j}$ is such that $v_i(\omega_{S_{\Pi_i(\omega)}}, g^{\emptyset}) < -w_{i,j}$. That is, any first-level rationalizable wage demand of player $i$ from player $j$ must be such that it exceeds player $i$'s value from staying self-employed, her outside option. This is essentially player $i$'s participation constraint for being an employee.

It follows now that any first-level rationalizable network must be the empty network $g^{\emptyset}$. By Assumption 2 this is ex post inefficient. □

For developing a result on specialists and a generalist, we need the stronger solution concept of interim prudent rationalizability. The reason is that a large wage demand may be rationalizable with some rather unreasonably confident belief that somebody would pay such a wage. Note that when somebody gets a job, the wage offered to him (and not the wage demanded) will determine the actual wage paid to him. The wage demanded just determines the likelihood of getting the job. Thus, a player may prudently chose a wage demand that exceeds his reservation value (i.e., the benefit of enjoying his knowledge and awareness from self-employment) just by the minimum increment $\delta$.

We also make the following simplifying assumption. For each player, the benefit from self-employment shall be constant across states. Clearly, that's an unrealistic assumption. It allows us to rule out inefficiencies that may arise from the generalist not knowing the value of self-employment of specialists. Note that the following assumption implies that the generalist knows the value of self-employment of any specialist.

**Assumption 3** *For all* $i \in N$, $v_i(\omega, g^{\emptyset}) = v_i(\omega', g^{\emptyset})$ *for all* $\omega, \omega' \in \bar{S}$.

We are ready to state our main result for the entrepreneurship model.

**Proposition 9** *Consider the entrepreneurship model and an unawareness structure with specialists and one generalist. If* $g$ *is a second-level prudent rationalizable network then* $g$ *is a center-sponsored star with the generalist in the center. It is ex post efficient. Moreover, any prudent rationalizable wage offer by the generalist to another player equals the value of self-employment by this player plus* $\delta$*. Any prudent rationalizable wage demand by a specialist equals the same amount. No strict positive wage offer by a specialist is prudent rationalizable. No strict positive wage demand by the generalist is prudent rationalizable. Every specialist earns a wage equal to his value of self-employment plus* $\delta$ *whereas the generalist retains the residual, i.e., the difference between the benefit of joining the knowledge of all specialists and wages paid to specialists.*

Proof. The first part of the proof bears similarity to the proof of Proposition 8. Let's index the generalist with "1". By Lemma 1, for all specialists $i \in N \setminus \{1\}$, $\omega \in \Omega$, $v_i(\omega_{S_{\Pi_i(\omega)}}, g^{\emptyset}) = v_i(\omega_{S_{\Pi_i(\omega)}}, g)$ for all $g \in G$. Thus, for any $i \in N$, $\omega \in \Omega$, if $w_i \in P_i^1(\omega)$ with $w_{i,j} \geq 0$ for some $j \neq i$, $j \in N$, then we must have $w_{i,j} = 0$. That is, any first-level prudent rationalizable wage offered by specialist $i$ to another player $j$ must be zero.

For any specialist $i \in N \setminus \{1\}$ and $\omega \in \Omega$, if $w_i \in P_i^1(\omega)$ with $w_{i,j} < 0$ for some $j \neq i$, $j \in N$, then $w_{i,j}$ is such that

$$-w_{i,j} = v_i(\omega_{S_{\Pi_i(\omega)}}, g^{\emptyset}) + \delta.$$

That is, any first-level prudent rationalizable wage demand of specialist $i$ from player $j$ must be such that it exceeds specialist $i$'s value from staying self-employed, her outside option, by exactly $\delta$. This is essentially specialist $i$'s participation constraint for being an employee. A higher wage demand may result in non-employment when employment at $v_i(\omega_{S_{\Pi_i(\omega)}}, g^{\emptyset}) + \delta$ could have been feasible. (A higher wage demand does not increase the wage paid. It just decreases the likelihood of the event being matched. ) Since beliefs are full support, the specialist puts strict positive probability to wages offers such that this is the case.

For the generalist (indexed by "1"), for any $\omega \in \Omega$, if $w_1 \in P_1^1(\omega)$ then either (a)

$w_{1,i} \geq 0$ for all $i \in N \setminus \{1\}$ with

$$\sum_{(\omega', w_{-1}) \in \bar{S} \times W_{-1}} \left( v_1(g, \omega') - \sum_{i \in N \setminus \{1\}} g_{1,i} w_{1,i} \right) \cdot \gamma(w)(g) \cdot \beta_1 \left( \{\omega', w_{-i}\} \right) \geq \sum_{\omega' \in \bar{S}} v_1(g^{\emptyset}, \omega') \cdot \mu(\{\omega'\} \mid \Pi_1(\omega)), \quad (1)$$

or (b) number (b) $w_{1,i} < 0$ for some $i \in N \setminus \{1\}$ with

$$-w_{1,i} = \delta + \sum_{\omega' \in \bar{S}} v_1(g^{\emptyset}, \omega') \cdot \mu(\{\omega'\} \mid \Pi_1(\omega)).$$

Condition (a) means that the expected benefit joining knowledge and awareness shall be greater than the expected benefit from being self-employed. Condition (b) just means that the wage demand of the generalist should equal to his expected benefit from being self-employed plus the minimum effort. These conditions follow from the fact that the generalist can ensure himself at the last the expected benefit of his own awareness in the empty network by staying "self-employed". Thus, when employing specialists, he will pay wages as high as long as his expected utility stay above the expected benefit from his own awareness when being self-employed (condition (a)). Moreover, any such wage offer is prudently rationalizable with a belief to puts sufficient weight on the case that specialists do not accept a lower offer. (At level 1, any wage demand by specialists is still "believable" since any first level belief is completely unrestricted.) When being employed, he will demand a wage as high as the expected benefit from his own awareness when being self-employed plus $\delta$ (condition (b)) by the similar arguments as for the specialists.

At the second level, for any $\omega \in \Omega$, if $w_1 \in P_1^2(\omega)$ then $w_{1,i} \geq 0$ for some $i \in N \setminus \{1\}$ implies that $w_{1,i} = v_i(\omega, g^{\emptyset}) + \delta$ for all $i \in N \setminus \{1\}$. Note that by Assumption 3, $v_i(\cdot, g^{\emptyset})$ is constant in $\omega \in \bar{S}$. This is prudent rationalizable because the generalist is certain at the second level that specialist accept such a wage offer and offering more would only lowers his profit. Moreover, the generalist is also certain at the second level that no specialist offers him any positive wage. What is left to show that such wage offers by the generalist yield a larger profit to the generalist than her outside option of self-employment. To this end, note for any $\omega'' \in \bar{S}$,

$$\sum_{\omega' \in \bar{S}} v_1(g^N, \omega') \cdot \mu\left(\{\omega'\} \mid \Pi_1(\omega)\right) - \sum_{i \in N \setminus \{1\}} v_i(g^{\emptyset}, \omega'') - (n-1)\delta \\ \geq \sum_{\omega' \in \bar{S}} v_1(g^{\emptyset}, \omega') \cdot \mu(\{\omega'\} \mid \Pi_1(\omega)) \quad (2)$$

The left hand side is the expected utility of the generalist from the second-level prudent rationalizable network. Since it is a star with the generalist at the center, it has one connected component and hence its benefit to the generalist is the same as the complete network in every state $\omega' \in \bar{S}$. The second term of the left hand side is constant in the state $\omega'' \in \bar{S}$ by Assumption 3. By Assumption 2 $v_i(g^N, \omega') - \sum_{j \in N \setminus \{i\}} v_j(g^{\emptyset}, \omega') - (n-1)\delta \geq v_i(g^{\emptyset}, \omega')$ for all $\omega' \in \bar{S}$. Taking expectations and using Assumption 3, we obtain inequality (2). Note that the second-level prudent rationalizable network is ex post efficient. □

What can be said beyond specialists and generalists? Analogous to the unilateral and bilateral communication cost model we observe the following:

**Proposition 10** *Consider the entrepreneurship model. For any $\omega \in \Omega$ and $i \in N$, if player $i$ knows that she has better knowledge than any other player $j \in N \setminus \{i\}$, then $w_{i,j} = 0$ for all $j \in N \setminus \{i\}$ in any rationalizable outcome. That is, player $i$ will not be an entrepreneur. In fact, this is a strict dominant action for player $i$ at $\omega$.*

PROOF. Let $\omega \in \Omega$ such that player $i \in N \setminus \{j\}$ knows that she has better knowledge than any player $j$. For any $w_i \in P_i^1(\omega)$ it must be that $w_{i,j} = 0$. To see this, note that for any two $g, g' \in G$, $v_i(g, \omega') = v_i(g', \omega')$ for all $\omega' \in \Pi_i(\omega)$. For any $w_{i,j} > 0$, there is strict positive probability that $i$ employs $j$ for the wage $w_{i,j}$, which makes $i$ worse off. This is strictly dominated by $w_{i,j} = 0$ at $\omega$. □

So someone with superior knowledge compared to others is never an employer since she doesn't need anybody. How about someone with inferior knowledge compared to all others?

**Proposition 11** *Consider the entrepreneurship model. For $\omega \in \Omega$, if every player $i \in N \setminus \{j\}$ knows that she has better knowledge than player $j$, then $w_{i,j} = 0$ in any rationalizable outcome.*

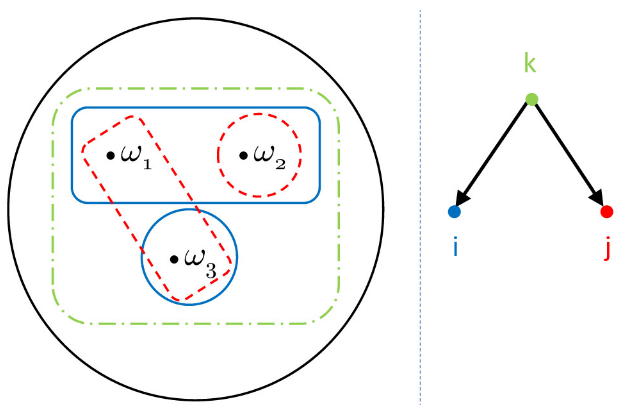
Proof. Let $\omega \in \Omega$ such that any every player $i \in N \setminus \{j\}$ knows that she has better knowledge than player $j$.

For any $w_i \in P_i^1(\omega)$ it must be that $w_{i,j} = 0$. To see this, let $g, g' \in G$ be such that $g_{i,k} = g'_{i,k}$ for all $k \in N \setminus \{i, j\}$ and $g_{i,j} \neq g'_{i,j}$. Note that $v_i(g, \omega') = v_i(g', \omega')$ for all $\omega' \in \Pi_i(\omega)$. For any $w_{i,j} > 0$, there is strict positive probability that $i$ employs $j$ for the wage $w_{i,j}$, which makes $i$ worse off. This is strictly dominated by $w_{i,j} = 0$ at $\omega$. Note that this holds for all $i \in N \setminus \{j\}$ at $\omega$. Thus, we conclude that at $\omega$ no player $i \in N \setminus \{j\}$ offers employment to player $j$. □

If someone has inferior knowledge compared to others and others know that, then they will never employ him. The reason is that a player can only be employed by at most another player. So there is no reason to employ a player with inferior knowledge (since also indirectly one cannot gain from him via connections to other players).

Players with knowingly inferior knowledge may emerge as an entrepreneur even without superior awareness. This is illustrated in the following example:

Figure 3: Entrepreneur with inferior knowledge




**Example 1** *Consider the information structure in Figure 3 (left side). There are three players, player i (solid blue), player j (dashed red), and player k (dashed-dotted green). There is no unawareness. All players are aware of everything. Player k knows nothing. Player i knows $\omega_3$ if $\omega_3$ obtains. He knows the event $\{\omega_1, \omega_2\}$ if either state occurs. Player j has different knowledge. She knows $\omega_2$ if $\omega_2$ obtains. Moreover, she knows the event $\{\omega_1, \omega_3\}$ in either of those states. If both players i and j share their knowledge, then they would have perfect information in any state.*

*Let the benefit function be such that for* $\omega \in \{\omega_1, \omega_2, \omega_3\}$,

$$\begin{aligned} v_i(g,\omega) &= v_j(g,\omega) = v_k(g^N,\omega) = 100 \text{ for all } g \in G \text{ with } \bar{g_{i,j}} = 1 \\ v_i(g,\omega) &= v_j(g,\omega) = 49 \text{ for all } g \in G \text{ with } \bar{g}_{i,j} = 0 \\ v_k(g^\emptyset,\omega) &= 0 \\ v_k(g,\omega) &= 49 \text{ for all } g \in G \text{ with } \bar{g}_{k,i} \neq \bar{g}_{k,j} \end{aligned}$$

*We also let* $\delta = 1$.

*With this specification, the employment network at the right of Figure 3 is an interim prudent rationalizable network. In fact, it is even an ex post equilibrium. Essentially both players* $i$ *and* $j$ *may expect to employ each other. Each demands* $v_i(g^\emptyset,\omega) + \delta = 50$. *But in fact, neither offers the other employment. The role of the entrepreneur is taken up by player* $k$ *who makes wage offers* $v_i(g^\emptyset,\omega) + \delta = 50$ *to both, players* $i$ *and* $j$.

*To see that nobody has an incentive to deviate, first note that neither* $i$ *nor* $j$ *would outbid player* $k$. *This is because* $50 = v_i(g^N,\omega) - v_j(g^\emptyset,\omega) - \delta = v_i(g^\emptyset,\omega) + \delta = 50$. *Player* $i$ *would have to pay player* $j$ *strictly more than 50 in order to entice him to surely take his wage offer instead* $k$*'s offer. In such a case, player* $i$ *would make less profit (i.e., follows from left hand side) than what he would obtain from staying employed by* $k$ *(right hand side). Player* $i$ *also doesn't want to just bid the same amount as* $k$ *for player* $j$. *In such a case, either players* $i$ *and* $k$ *would win the employment contract for* $j$ *with equal probability. We claim that* $\frac{1}{2}(v_i(g^N,\omega) - v_i(g^\emptyset,\omega) - \delta) + \frac{1}{2}v_i(g^\emptyset,\omega) < v_i(g^\emptyset,\omega) + \delta$. *This is clear since this inequality can be rewritten as* $50 - \frac{3}{2} = \frac{1}{2}v_i(g^N,\omega) - \frac{3}{2}\delta < v_i(g^\emptyset,\omega) = 49$. *Finally, we need to check that player* $k$ *does the optimal decision. Note that player* $k$ *would not want to deviate to self-employment. I.e., we need to verify that* $0 = v_k(g^N,\omega) - 2(v_i(g^\emptyset,\omega) + \delta) \geq v_k(g^\emptyset,\omega) = 0$. *This inequality says that the profit from employing* $i$ *and* $j$ *(left-hand side) is larger than the profit from staying self-employed/unemployed (right-hand side). Note further that player* $k$ *would not want to deviate to employ only of the players* $i$ *or* $j$. *I.e.,* $v_k(g,\omega) - v_i(g^\emptyset,\omega) - \delta = -1 < 0$ *for any employment network* $g \in G$ *with* $\bar{g}_{k,i} \neq \bar{g}_{k,j}$. *Finally, note that the example satisfies our assumptions imposed on the benefit function in this section.* □

The example bears both features of a coordination failure and Bertrand competition. First, note that players need to coordinate who becomes an entrepreneur and who becomes an employee. Clearly, there are two other equilibria in which player $i$ employs $j$ or $j$ employs $i$. Second, note that undercutting player $k$ is not profitable because of the fixed grid size $\delta$. This is reminiscent of Bertrand competition on a finite price grid.

Anyway, the example demonstrates that players who know less than others may not be "unemployed" but may become entrepreneurs.

## 6 Discussion

Our analysis remained mostly in the context of specialists and a generalist although we also consider context in which some player has better or worse knowledge than any other. It would be desirable to derive interesting characterizations (or at least some interesting conditions) of rationalizable networks for any distribution of knowledge and awareness among players. So far we were unable to come up with interesting conditions. This is not too surprising. First, most known models of strategic network formation depend on details of the model. Second, our solution concept (even the in form of the stronger version of interim prudent rationalizability) is rather weak.

It may be possible to complement our approach with existing approaches to explain interesting multi-level hierarchies. In particular, we conjecture that we could derive multi-level hierarchies when considering limited capacity to information processing (in the spirit of Radner, 1993, for instance) of generalists. That is, although generalists have superior awareness, they may not be able to absorb quickly the specialists' knowledge and thus may delegate "putting together" subsets of specialists to lower-level managers.

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