Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning*

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May 2, 2017

Abstract

We study the informational effectiveness of electoral campaigns. Voters may not think about all political issues and have incomplete information with regard to political positions of candidates. Nevertheless, we show that if candidates are allowed to microtarget voters with messages then election outcomes are as if voters have full awareness of political issues and complete information about candidate's political positions. Political competition is paramount for overcoming the voter's limited awareness of political issues but unnecessary for overcoming just uncertainty about candidates' political positions. Our positive results break down if microtargeting is not allowed or voters lack political reasoning abilities. Yet, in such cases, negative campaigning comes to rescue.

Keywords: Electoral competition, campaign advertising, multidimensional policy space, microtargeting, dog-whistle politics, negative campaigning, persuasion games, unawareness.

JEL-Classifications: C72, D72, D82, P16.

*We thank the editors of QJPS as well as Pierpaolo Battigalli, Oliver Board, Giacomo Bonanno, Jon Eguia, Ignacio Esponda, Boyan Jovanovic, Jean-François Laslier, Alessandro Lizzeri, Tymofiy Mylovanov, Joaquim Silvestre, Walter Stone, Thomas Tenerelli and seminar participants at NYU Stern and participants at WEAI 2012 for helpful comments. Burkhard is grateful for financial support through NSF SES-0647811. An earlier version was circulated 2011 under the title "Political Awareness and Microtargeting of Voters in Electoral Competition".

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"You are not allowed to lie but you don't have to tell everything to the people, must not tell them the entire truth. That's how I held it."

Konrad Adenauer, first Chancellor of the Federal Republic of Germany, last interview

1 Introduction

We study to what extent electoral compaign can effectively promote awareness of political issues and reveal information about political positions of candidates. We assume that voters may not think about all political issues and have incomplete information about political positions of candidates, while candidates are aware of all political issues, know the preferences of voters, and can use modern sophisticated campaign strategies including microtargeting to persuade voters. We show that despite this stark asymmetry between voters and candidates, election outcomes under voters' unawareness of many political issues and incomplete information about candidates' political positions are equivalent to election outcomes under full awareness of political issues and complete information about candidate's political positions. We also show that this positive result depends crucially on the strength of electoral competition, the ability of candidates to microtarget voters, and voters' political reasoning abilities.

Our model reflects a number of what we argue are realistic features. First, while traditionally candidates in electoral competition are portrayed as being completely opportunistic in their choice of political positions (i.e., Downs, 1957), we consider candidates who are constrained by their ideology as their agendas might to some extent reflect personal convictions (Hillygus and Shields, 2008, p. 40). Second, political positions of candidates may not be obvious to all voters and may pertain to many political issues that are shaping the complex political environment of an election. Candidates may raise only some political issues in a campaign but may be completely silent on others. On some of the political issues raised, candidates may be intentionally vague about their political position while on others they may be completely transparent. Third, we allow for microtargeting of voters with specific campaign messages and issues. By narrowly communicating issue messages, candidates reduce the risk of alienating other voters, thereby broadening the range of issues on the campaign agenda. In order to successfully tailor campaign messages to voters, candidates need to have information about political preferences of voters, a fourth feature of our model. In the past, it was impossible for candidates to know political preferences of individual voters. Rather, they had to be content with aggregate information about voters' preferences from opinion polls or similar sources. Yet, modern information technology allows to collect a wealth of individual data on voters, apply sophisticated data mining tools and use this information strategically in campaigns.[1]

[1]Recent campaigns merged voter registration files with consumer data that include names, addresses, address

To provide some intuition for the results, consider for simplicity two candidates and just one voter. The policy space is a multidimensional Euclidean space. Each issue corresponds to a dimension. The political positions of the candidates and the most preferred policy point of the voter are points in this space. The voter evaluates a candidate by how far the candidate's policy point is away from his most preferred policy point using Euclidean distance but only in a subspace spanned by the issues that he is aware of. The voter is unaware of all but one issue and that candidates can campaign by raising issues to him. Being unaware of an issue means that the voter does not consider this issue at all. In other words, he conceives of a policy subspace that misses the dimension corresponding to that issue. Further, we assume for the moment that once an issue has been raised by some candidate, the political positions of both candidates on this issue become completely transparent to the voter. Finally, we assume that both candidates know the voter's most preferred policy point and each other's political positions. We claim that in this extremely simplified model candidates will raise enough issues so as to produce an election result that would emerge also under full awareness of all issues. To see this note that candidates face a zero-sum game. If raising an issue is not beneficial to one candidate, it will be to the other. Thus, either all issues will be raised or raising further issues won't change the voting outcome anymore. Now assume that policy points on issues that have been raised do not become automatically transparent. Each candidate can provide some information on her political position. This information can be vague, that a candidate can be silent on issues, but that she can not bluntly lie in the sense of not including her political position (in the subspace of issues revealed) in the information she provides. This is reminiscent of models of verifiable information à la Grossman and Hart (1980), Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986) except that our model involves unawareness of some dimensions of the information and more than one informed party. We claim that despite unawareness and incomplete information, all "relevant" information is revealed to the voter. To see this, note that if a candidate does not provide more precise information on an issue that he is aware of then a voter should realize that this because the information is not favorable to the candidate.

Our novel model allows us to highlight the role of voters' awareness of political issues. If voters are aware of all political issues (but uncertain about candidates' political positions), the positive result could also be obtained even without political competition among candidates. In fact, even if there were just a single candidate, voters could deduce her political position. In contrast, we show by example that when voters are unaware of some political issues, political competition among candidates becomes crucial for unraveling. If there is just one candidate, then she would simply be silent on issues that are unfavorable to her and voters would have no

histories, driving records, criminal records, and consumer purchases like magazine subscriptions, mortgage information, credit-card purchases, gun ownership etc. and used it "microtarget messages through direct mail, email, telephone calls, and personal visits."(see Hillygus and Shields, 2008, p. 159, 161).

way to discover and learn about these issues. In a sense, the presence of voters' unawareness may explain at least to some extent why electoral competition is believed to be paramount for the well-functioning of democracy.

Extending the result to more than one voter is non-trivial. This is because a voter not receiving precise public information of an issue from a candidate is now uncertain whether it is because the information is not favorable to the candidate or whether releasing such information would diminish the candidate's ability to capture some other voter's vote. This is where mircotargeting comes to rescue. If each candidate can tailor campaign messages to each voter, he has no excuse anymore for not providing precise information. We show by example that our positive results break down if microtargeting is not allowed.

While our positive results paint an optimistic picture of the informational effectiveness of electorial campaigning, we believe an equally important role of our results is that they serve as a benchmark for exploring why fully revealing outcomes may not be observed in reality. In particular, we show by example the importance of sophisticated political reasoning abilities of the electorate. The advantage of our solution concept, an extensive-form version of iterated admissibility, is that we can explicitly identify the rationality and higher order reasoning about candidates that required from voters for a positive result, i.e., the voters' ability to optimize, their reasoning about candidates, their reasoning about candidates' reasoning etc. We show by example how our results break down if voters can only reason up to a few levels. Moreover, we show that negative campaigning can overcome some of the failures due to insufficient political reasoning abilities. In doing so, we discover a notable difference between majoritarian and proportional voting systems: Negative campaigning overcomes lack of political reasoning even without the need of microtargeting in majoritarian voting systems but proportional voting systems still require microtargeting for unraveling of information.[2]

The paper is organized as follows: The next section introduces the model. This is followed by the main results in Section 3. In Section 4 we explore with the help of some examples the voters' limited awareness of political issues and the role of electoral competition that mitigates these limits. Section 5 highlights how lack of microtargeting hampers information revelation in electoral campaigns. The role of voters' political reasoning abilities is explored in Section 6 and Section 7 show how negative campaigning facilitates information revelation even in the face of limited political reasoning abilities. We conclude and discuss related literature in Section 8. Proofs and additional material are collected in an appendix.

[2]There is empirical evidence that negative campaigning is less frequent in parliamentary systems (Walter, 2014) although this is viewed as a result of multiparty competition in parliamentary systems.

2 Model

Allowing for dynamically changing multidimensional policy spaces and voters who are aware of different subspaces only and who may become aware of larger subspaces during the political campaign poses a modeling challenge in terms of tractability. To wit, we make use of extensive-form games with unawareness introduced in Heifetz, Meier, and Schipper (2013).[3] Such a game consists of a collection of game trees partially ordered by a subtree relation. Initially a player may be unaware of some dimension of the problem and may perceive the strategic situation as a subtree. Only during the course of play, he becomes aware of more and more dimensions of which he was initially unaware and perceives increasingly more expressive subtrees as a richer description of the strategic situation.

More formally, let $I = \{1, ..., m\}$ be a finite set of *political issues*. Examples of issues are "War on terrorism", "Immigration", "Health care" etc. Different policies with regard to an issue are associated with different points in the real interval $[0, 1]$, one interval for each issue. Thus, the full-dimensional policy space considered in this model is $[0, 1]^{|I|}$. Since we aim to study electoral competition when some voters may not be aware of all issues, we also need to consider subspaces of the full-dimensional policy space. We let Y denote a finite set of policy points in $[0, 1]^{|I|}$. For every nonempty subset I' of issues, $I' \subseteq I$, let $Y_{|_{I'}}$ be the projections of policy points in Y onto the subspace $[0, 1]^{|I'|}$ spanned by I'.

There are two candidates, a and b. Each candidate $k \in \{a, b\}$ has a fixed ideological policy point $y^k \in Y$. If candidate k's political position in Y is y^k, then for any nonempty $I' \subseteq I$ his political position in $Y_{|_{I'}}$ is $y^k_{|_{I'}}$.

There is a finite set of voters denoted by $N = \{1, ..., n\}$. Each voter $j \in N$ has a unique most preferred policy point x^j in the full-dimensional policy space, $[0, 1]^{|I|}$, with the projection to $[0, 1]^{|I'|}$ denoted by $x^j_{|_{I'}}$, for any nonempty $I' \subseteq I$. We denote by x^j_i the i-component of the vector x^j (similarly for y^k_i).

A voter j's utility from candidate k depends on voter j's awareness of political issues and is given by the Euclidean distance between his most preferred point and the candidate's policy but only in the subspace of issues that he is aware of. I.e., the utility of voter j from voting for candidate k when voter j is aware of issues in I' with $\{1\} \subseteq I' \subseteq I$ and candidate k's political position on these issues is $y^k \in Y_{|_{I'}}$ is

$$u^j(I', y^k, x^j) = - \parallel x^j - y^k \parallel_{I'} := - \sqrt{\sum_{i \in I'} (x^j_i - y^k_i)^2}. \quad (1)$$

We assume that at the beginning of the campaign voters are aware of only one default issue,

[3] See Halpern and Rego (2014), Feinberg (2012) and Grant and Quiggin (2013) for related work.

which is issue 1. That is, even when neither candidate raises any issue, voters are aware of issue 1 and the set of all possible policy points of candidates, $Y_{|\{1\}}$, regarding issue 1. It allows for well-defined preferences of voters even if no issues are raised in the campaign.[4]

We start with describing one game tree denoted by T^I. At the first stage of T^I, nature c (i.e., "chance") moves and selects for each candidate a most preferred policy point in the finite set $Y \subseteq [0,1]^{|I|}$. We assume that candidates have complete information about each other's political positions (and the voters' most preferred policy points). That is, each of their information sets is a singleton. After the move of nature, candidates simultaneously campaign for votes. In this campaign, each candidate reveals to each voter some subset of issues and some information (i.e., a nonempty subset of policy points) about her own political position on these issues. The information provided to a voter is observed by this voter only and not by other voters (i.e., microtargeting). Note that our model encompasses public campaigns as a special case if we require that each candidate provides the same information to all voters.

After the campaign, each voter votes for a candidate and the game ends. Each voter takes into account her awareness of issues and the (inferred) information on the candidates' political positions. Since not all issues may have been raised during the campaign, a voter may not be aware of all issues. Consequently, she is unable to think about these issues and does not realize that they could have been raised in a different campaign. This means that the voter's information set emanating from a node in the tree T^I may be a subset of corresponding nodes in a poorer description of the game. In this less expressive tree, nature chooses policy points of candidates only in a space spanned by a subset of issues and candidates can raise only subsets of those issues during the campaign. That is, our model involves a collection of trees, $(T^{I'})_{\{1\} \subseteq I' \subseteq I}$, one for each subset of issues that includes the default issue. For each $T^{I'}$, $\{1\} \subseteq I' \subseteq I$, nature selects a profile of candidates' (projected) policy points in $Y_{|_{I'}} \times Y_{|_{I'}} \subseteq [0,1]^{|I'|} \times [0,1]^{|I'|}$. In tree $T^{I'}$, candidates are only aware of issues I' and have complete information about all players political positions (i.e., singleton information sets) in $[0,1]^{|I'|}$. Candidates campaign simultaneously for votes by revealing to each voter some (possibly proper) subset of issues in I' and some information on their own political positions on these issues. After the campaign, voters vote on candidates taking only the subset of issues raised during this campaign and information provided in this campaign into account when forming their expectations. That is, a voter's information set emanating from a node in tree $T^{I'}$ may be a subset of corresponding nodes in an even poorer description of game $T^{I''}$ with $\{1\} \subseteq I'' \subset I'$. Note that an information set of a voter may contain several nodes (within one tree) because of the uncertainty over the policy points of candidates on the issues they are aware of.

[4]Our results do not depend on the fact that ex ante all voters are aware of the same default issue. We could allow that ex ante different voters are aware of different subset of issues and our results would remain true.

To complete the description of the game, we need to specify preferences. We assume that each voter prefers the candidate that is "closest" to her given her awareness and information. For candidates we consider two types of preferences. For any terminal node z, let $\sigma(z)(a)$ be the share of voters voting for candidate a. Candidates *care only about winning the election* if the utility function of candidate a is defined by

$$u^a(z) = \begin{cases} 1 & \text{if } \sigma(z)(a) \geq \frac{1}{2} \\ -1 & \text{otherwise} \end{cases} \tag{2}$$

and candidate b's utility function is given by

$$u^b(z) = -u^a(z). \tag{3}$$

For simplicity we don't allow for ties in payoffs but confer to candidate a a slight advantage in case both candidates obtain the same number of votes. We call the game in which candidates care only about winning the election the *majoritarian election model*.

Candidates *care only about the share of voters* if the utility function of candidate a is defined by

$$u^a(z) = \sigma(z)(a) \tag{4}$$

and candidate b's utility function is given by

$$u^b(z) = 1 - u^a(z). \tag{5}$$

We call the game in which candidates care about the share of voters the *proportional election model*. Note that under both specifications of utility functions the game is a strictly competitive for candidates.

The collection of game trees partially ordered by set inclusion on the set of issues and the information sets outlined above shall satisfy the properties of generalized extensive-form games introduced in Heifetz, Meier, and Schipper (2013).

For any nonempty subset I', $\{1\} \subseteq I' \subseteq I$, the I'-partial game is the collection of trees $(T^{I''})_{\{1\} \subseteq I'' \subseteq I'}$ such that all information sets emanating at nodes in the trees of this collection are contained within trees of this collection.

For each player $k \in N \cup \{a, b\}$ we denote by H_k the set of all information sets of player k (across all trees) and by $I(h_k)$ the set of issues such that information set h_k belongs to the tree $T^{I(h_k)}$.

2.1 Strategies

Let $I^{k,j} \subseteq I$ be the set of issues raised by candidate $k \in \{a, b\}$ to voter $j \in N$. To ease notation we assume that each candidate raises at least the default issue 1 to each voter, i.e., $\{1\} \subseteq I^{k,j}$,

$k \in \{a, b\}$, $j \in N$.

When deciding to vote for one candidate or another, each voter j takes into account only issues that are raised to him during the campaign by either candidate (apart from the default issue). For instance, if candidate a campaigned to voter j on issues in the set $I^{a,j}$ and candidate b campaigned to voter j on issues in $I^{b,j}$, then voter j takes into account issues in $I^{a,j} \cup I^{b,j} \subseteq I$. That is, the policy space perceived by voter j is restricted to the domain $[0, 1]^{|I^{a,j} \cup I^{b,j}|}$.

A strategy of voter j is a function that assigns to each information set of voter j a candidate she votes for. That is,

$$s_j : H_j \longrightarrow \{a, b\}. \tag{6}$$

Note that a voter's strategy assigns to each of the voter's information sets in *each tree* the candidate for which he votes. Since a voter may be unaware of many policy issues and consequently may not perceive all trees, he can not "choose" such a strategy ex ante before the game starts. Rather, at each of his information sets the voter chooses an action. Strategies of voters will be used here just as objects of candidate's beliefs. As we will see below, candidates form beliefs about the behavior of voters.

For each candidate $k \in \{a, b\}$, let $y^k(h_k)$ be the policy point selected by nature in $Y_{|_{I(h_k)}}$ after which information set h_k of candidate k occurs. A strategy for candidate $k \in \{a, b\}$, specifies for each information set $h_k \in H_k$ of candidate k which issues and which information on those issues she provides to each voter. We assume that each candidate cannot bluntly lie about her policy point but she can be vague. That is, if $I^{k,j} \subseteq I(h_k)$ is the nonempty set of issues provided by candidate k to voter j at information set h_k, then her (projected) policy point $y^k(h_k)_{|_{I^{k,j}}}$ must be in the set of policy points provided by candidate k to voter j at the information set h_k. Note that at the information set h_k candidate k's message to voter j can be silent on some issues in $I(h_k)$. Finally, we do not require that the same set of issues and information is provided to each voter, i.e., we allow for microtargeting of voters. Formally, a strategy for candidate $k \in \{a, b\}$ is

$$s_k : H_k \longrightarrow \prod_{j \in N} \left[\bigcup_{\{1\} \subseteq I^j \subseteq I} 2^{Y_{|_{I^j}}} \right] \tag{7}$$

such that for every voter $j \in N$, there exists a nonempty subset of issues I^j with $\{1\} \subseteq I^j \subseteq I(h_k)$ such that $y^k(h_k)_{|_{I^j}} \in (s_k(h_k))_j \in 2^{Y_{|_{I^j}}}$, where $(s_k(h_k))_j$ is the jth component in the profile $s_k(h_k)$. With this notation, $(s_k(h_k))_j$ is the information provided by candidate k to voter j. Note that candidate k's "true" policy point $y^k(h_k)_{|_{I^j}}$ at the information set h_k (subject to possibly being silent on some issues) is required to be in the set of possible policy points $(s_k(h_k))_j$ provided to voter j. Note that although we assume each candidate to be aware of all issues, we require his strategy to assigns actions to his information sets even in lower trees where his "unaware incarnations live." This is because candidates' strategies are objects of beliefs of voters.

For any player $k \in N \cup \{a, b\}$ we denote by S_k player k's set of strategies. Moreover, for any strategy $s_k \in S_k$ and any subset I' of issues with $\{1\} \subseteq I' \subseteq I$, we denote by $s_k^{I'}$ the I'-partial strategy in the I'-partial game induced by s_k. This is the strategy s_k restricted to k's information sets in the I'-partial game. $S_k^{I'}$ denotes the set of I'-partial strategies of player k.

2.2 Belief Systems

Each voter forms beliefs about candidates' policy points and (partial) strategies. For every information set of the voter, his belief is restricted to issues that the voter is aware of. Voter j's belief system is a tuple

$$(\beta_j(h_j))_{h_j \in H_j} \in \prod_{h_j \in H_j} \Delta \left(Y_{|_{I(h_j)}} \times Y_{|_{I(h_j)}} \times S_a^{I(h_j)} \times S_b^{I(h_j)} \right) \tag{8}$$

such that for all h_j, $\beta_j(h_j)$ assigns probability 1 to the subset of candidates' policy points selected by nature in $Y_{|_{I(h_j)}} \times Y_{|_{I(h_j)}}$ and candidates' strategy profiles in $S_a^{I(h_j)} \times S_b^{I(h_j)}$ that reach h_j in the $T^{I(h_j)}$-partial game. That is, at every one of his information sets h_j, voter j is certain to have reached his information set h_j.[5]

For two information sets h and h' in a given tree $T^{I'}$, we say h precedes h' (or h' succeeds h) if for node $n' \in h'$, there is a path $n, ..., n'$ in $T^{I'}$ such that $n \in h$.

At every information set of candidate $k \in \{a, b\}$, we assume that she knows her policy point and the policy point of the opponent candidate $-k$ selected by nature. She forms beliefs about the other candidate's strategy and the strategies of voters. For $k \in \{a, b\}$, candidate k's belief system is a tuple

$$(\beta_k(h_k))_{h_k \in H_k} \in \prod_{h_k \in H_k} \Delta \left(S_{-k}^{I(h_k)} \times \prod_{j \in N} S_j^{I(h_k)} \right). \tag{9}$$

For $k \in N \cup \{a, b\}$, we denote by B_k the collection of player k's belief systems.

[5]We could require a belief system to satisfy Bayesian updating whenever possible. Yet, Bayesian updating whenever possible will be implied by our solution concept. See Meier and Schipper (2012) and (for standard games and standard extensive-form rationalizability) Shimoji and Watson (1998).

2.3 Prudent Rationalizability

We make use of a solution concept that has a long tradition in formal models of politics and voting, namely iterated admissibility or iterative elimination of weakly dominated strategies (see Farquharson, 1969, Brams, 1975, Moulin, 1979, and Gretlein, 1982). Since we deal with dynamic extensive-form games rather than with just static normal-form games, we make use of an extensive-form version, called prudent rationalizability. It has been introduced in Heifetz, Meier, and Schipper (2011) for extensive-form games with unawareness. It is a version of extensive-form rationalizability (see Pearce, 1984, and Battigalli, 1997) featuring *cautious* behavior. Moreover, it is equivalent to iterated admissibility applied to the normal-form games associated with the extensive-form game (Shimoji and Watson, 1998, Meier and Schipper, 2012).

The use of an extensive-form version of iterated admissibility has several advantages over the use of equilibrium concept. First, standard solution concepts like perfect Bayesian equilibrium or sequential equilibrium are not even defined in our more general framework that allows for unawareness. Second, even if the definitions of such equilibrium concepts are extended to our framework, the interpretation would be highly problematic while our solution concepts offers a clear interpretation. In our model, voters may not think about all political issues before the election and consequently they may be surprised about the issues arising in the campaign. They simply could not have learned an equilibrium convention that is guiding their behavior in such a setting. In contrast, our solution concept can be interpreted as possible behavior arising from political reasoning of players. Voters may ask themselves about the strategic intentions of candidates. E.g., *why* did a candidate provide this information and *why* did the candidate raise those political issues. Third, cautious reasoning about the rationality of candidates and their cautious reasoning about voters' cautious reasoning etc. lead to behavior captured by our solution concept. This cautiousness embodied in our solution concept introduces automatically some degree of sceptism akin to Milgrom and Roberts (1986)'s refinement of sequential equilibrium as cautious beliefs of voters put (at least initially) some weight on unfavorable policy points. Another nice side benefit is that the levels of reasoning captured by our solution concept provide us with a "measure" of political sophistication of voters, a feature we will exploit in later sections when we study the robustness of our positive results. Finally, standard equilibrium solution concepts also require auxiliary assumptions on probability distributions over moves of nature like a common prior and independence that we do not need to impose, thus adding robustness to our positive results.

For any player $k \in N \cup \{a, b\}$, with a belief system β_k, a strategy s_k of player k is *rational* at information set $h_k \in H_k$, if there exists no other action $s'_k(h_k)$ at h_k such that by only replacing the action $s_k(h_k)$ with action $s'_k(h_k)$ (which results in some new strategy) yields k a strictly higher expected utility.

Prudent rationalizability adapted to our context takes the following form: For $k \in N \cup \{a, b\}$, let

$$S_k^0 = S_k.$$

For $\ell \geq 1$, define inductively for $k \in \{a, b\}$,

$$B_k^\ell \quad = \quad \left\{ \beta_k \in B_k : \begin{array}{l} \text{For every information set } h_k, \text{the support of } \beta_k(h_k) \text{ is} \\ S_{-k}^{I(h_k),\ell-1} \times \prod_{j \in N} S_j^{I(h_k),\ell-1}. \end{array} \right\},$$

for $j \in N$

$$B_j^\ell \quad = \quad \left\{ \beta_j \in B_j : \begin{array}{l} \text{For every information set } h_j, \text{ if there exists some profile} \\ \text{of policy points } (y^a, y^b) \in Y_{|_{I(h_j)}} \times Y_{|_{I(h_j)}} \text{ and some profile} \\ \text{of candidates' strategies } (s_a, s_b) \in S_a^{\ell-1} \times S_b^{\ell-1} \text{ such that} \\ (y^a, y^b, s_a, s_b) \text{ reaches } h_j \text{ in the tree } T^{I(h_j)}, \text{ then the support} \\ \text{of } \beta_j\left(h_j\right) \text{ is the set of policy profiles and strategy profiles} \\ (y^a, y^b, s_a, s_b) \in Y_{|_{I(h_j)}} \times Y_{|_{I(h_j)}} \times S_a^{I(h_j),\ell-1} \times S_b^{I(h_j),\ell-1} \\ \text{such that } (y^a, y^b, s_a, s_b) \text{ reaches } h_j. \end{array} \right\},$$

and for any player $k \in N \cup \{a, b\}$,

$$S_k^\ell \quad = \quad \left\{ s_k \in S_k^{\ell-1} : \begin{array}{l} \text{There exists } \beta_k \in B_k^\ell \text{ such that for every information set } h_k \\ \text{player } k \text{ is rational at } h_k. \end{array} \right\}.$$

The set of prudent rationalizable strategies of player $k \in N \cup \{a, b\}$ is

$$S_k^\infty = \bigcap_{\ell=1}^{\infty} S_k^\ell.$$

At each round of elimination, a strategy is kept if there exists a full support belief on the remaining strategies of other players and possible moves of nature for which the strategy is rational at every information set of the player. The prudence or cautiousness of players enters through the full support beliefs about the remaining strategies and possible moves of nature. It means that at each level, a player does not completely exclude any of the opponents' remaining strategies. This feature will be essential for our result. See Heifetz, Meier, and Schipper (2011) and Meier and Schipper (2012) for further discussions of the solution concept.

Since the game is finite, existence of a nonempty set of prudent rationalizable strategy profiles follows directly from a result in Heifetz, Meier, and Schipper (2011). Moreover, since the space of policy points Y is finite, at most finite number of eliminations of strategies suffice.

3 Positive Results

Despite voters' unawareness of issues and uncertainty about the candidates' policy points, we claim that electoral competition is sufficient for the emergence of election outcomes that are equivalent to outcomes with full awareness of issues and full information of policy points. Note that we do not claim that all issues or all information are revealed during the campaign. All we claim is that the revelation of further issues and information won't change the election outcome.

Proposition 1 (Proportional Election Model) *At every prudent rationalizable outcome of the proportional election model with unawareness of political issues and incomplete information about candidates' policy points, if a voter votes for a candidate, then he prefers to vote for the same candidate when having full awareness of all political issues and complete information about the candidates' policy points. Conversely, if a voter strictly prefers to vote for a candidate under full awareness of political issues and complete information about candidates' policy points, then in any prudent rationalizable outcome of the proportional election model with unawareness of political issues and incomplete information about candidates' policy points, he votes for the same candidate.*

While the proof is naturally somewhat tedious due to the multiplicity of multidimensional policy spaces and the change of dimensions during the play, the basic idea is to show unraveling of sufficient issues and information such that election outcomes are the same as under full awareness and complete information.

Roughly we show that with any first-level prudent rationalizable strategy, the voter who receives from exactly one candidate information that he has her most preferred policy point (in the policy space that she is aware of at that information set) must vote for that candidate. Any second-level prudent rationalizable strategy of candidates must be such that if the candidate has the best policy point for the voter in some subspace and all higher-dimensional spaces, then he must reveal it to the voter. For any $\ell \geq 1$, at level-$(2\ell + 1)$ prudent rationalizable strategies voters vote for the candidate who reveals unambiguously the ℓ-closest or any closer policy point to the voter, while at level-$(2\ell + 2)$ prudent rationalizable strategies candidates reveal if possible to voters the ℓ-closest or closer policy point in an appropriate policy space or any higher-dimensional policy space.

We prove an analogous result for the majoritarian election model. The positive results for both the majoritarian and proportional model serve as important benchmarks for our later analysis. We will see that the proportional model differs from the majoritarian model when later we discuss the lack of voters' political reasoning abilities and negative campaigning.

Proposition 2 (Majoritarian Election Model) *At every prudent rationalizable outcome of the majoritarian election model with unawareness of political issues and incomplete information about candidates' policy points, if a candidate obtains the majority of votes then he also obtains the majority of votes under full awareness of all political issues and complete information about the candidates' policy points. Conversely, if a majority of voters strictly prefer to vote for a particular candidate under full awareness of political issues and complete information about candidates' policy points, then in any prudent rationalizable outcome of the majoritarian election model with unawareness of political issues and incomplete information about candidates' policy points, this candidate obtains a majority of votes.*

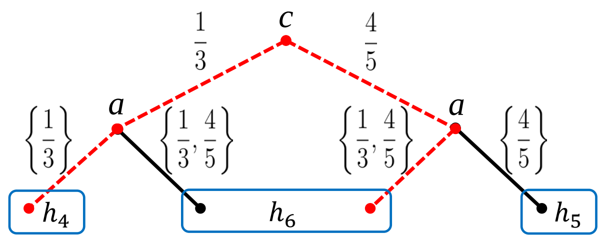
The proofs of both results are contained in the appendix. In fact, for convenience we state the proofs in reverse order. The proof for the majoritarian election model applies with minor modifications also to the proportional model. The proofs differ mainly in the set of "relevant" voters that candidates care about. In the proportional model, candidates care about every voter while in the majoritarian election model candidates do not necessarily care about subsets of voters larger than a majority.

4 The Role of Political (Un)Awareness and Electorial Competition

4.1 Political Competition is Not Necessary under Full Awareness

Consider just one issue and a single voter who is aware of this (default) issue. Assume that the voter's most preferred point is $\frac{1}{2}$. Further, he is uncertain about the policy point of the single candidate (say, a) which is in the set $Y = \left\{\frac{1}{3}, \frac{4}{5}\right\}$. The voter decides whether or not to elect the candidate. If the candidate is not elected, a fixed "status quo" policy, $y^* = \frac{3}{4}$ is implemented instead. Note that the voter strictly prefers policy point $\frac{1}{3}$ to both the "status quo" policy and policy point $\frac{4}{5}$ and the "status quo" policy is strictly preferred to policy point $\frac{4}{5}$. The simplified game form is depicted in Figure 1. Nature, c, moves first and selects the political position of the candidate, which is either $\frac{1}{3}$ or $\frac{4}{5}$. Then candidate a provides information about her political position to the voter. She can either be precise about her political position (i.e., sending either message $\{\frac{1}{3}\}$ or $\{\frac{4}{5}\}$ depending on the move of nature) or be ambiguous by sending message $\{\frac{1}{3}, \frac{4}{5}\}$. Consequently the voter's information set depends on the message sent by the candidate. The information sets are indicated by blue rectangles and denoted by h_1, h_2, and h_3, respectively. At these information sets, the voter can either vote for the candidate or do not vote for the candidate. In order to save space, we omitted the voter's action from the game form in Figure 1.

Figure 1: Unraveling without unawareness of issues and competition

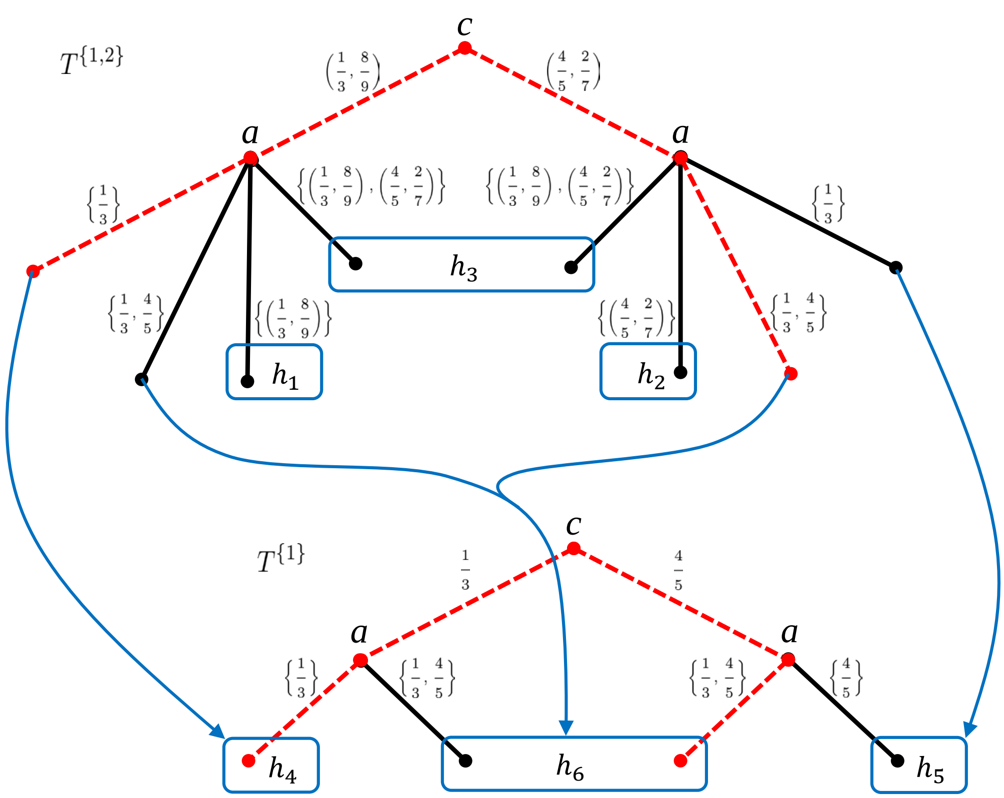


The unique prudent rationalizable strategy of the candidate we illustrate with the red dashed lines in Figure 1. It is fully informative to the voter. In order to sketch briefly the intuition, note that at the first level of the procedure, if the voter receives the message $\left\{\frac{1}{3}\right\}$, he knows that candidate a has policy point $\frac{1}{3}$. Consequently, he votes for a. Anticipating this at at the second level of the procedure, if candidate a has indeed policy point $\frac{1}{3}$ then she is happy to tell the voter. At level 3 of the procedure, the voter now knows that if she didn't receive message $\left\{\frac{1}{3}\right\}$, then candidate a must have policy point $\frac{4}{5}$. Consequently, the voter does not vote for candidate a in such cases. Hence, information unravels. For a detailed statement of the prudent rationalizable strategies in this example, see Appendix B.

4.2 Unraveling Fails under Unawareness without Political Competition

Consider two issues, and a single voter who is initially aware only of the default issue, issue 1. Only if the voter becomes aware of the other issue, issue 2, during the campaign, he perceives the full-dimensional policy space. In the full-dimensional policy space, the voter's most preferred point is $(\frac{1}{2}, \frac{1}{2})$. He is uncertain about the policy point of the single candidate (say, a), which is in the set $Y = \left\{\left(\frac{1}{3}, \frac{8}{9}\right), \left(\frac{4}{5}, \frac{2}{7}\right)\right\}$. The voter decides whether or not to elect the candidate. If the candidate is not elected, a fixed "status quo" policy, $y^* = \left\{\left(\frac{3}{4}, \frac{1}{2}\right)\right\}$ is implemented instead. Note that in this two-dimensional example the voter strictly prefers "status quo" policy $\left(\frac{3}{4}, \frac{1}{2}\right)$ to any of policy points $\left(\frac{1}{3}, \frac{8}{9}\right)$ and $\left(\frac{4}{5}, \frac{2}{7}\right)$, meaning that under full awareness and complete information, the candidate should not be elected. The simplified game form is depicted in Figure 2. The figure shows two game trees, denoted by $T^{\{1,2\}}$ and $T^{\{1\}}$, respectively. Note first that the lower game tree, $T^{\{1\}}$, is identical to Figure 1. It depicts precisely the perception of the voter when he remains unaware of issue 2 during the campaign. In the upper game three, $T^{\{1,2\}}$, nature draws first the policy point of the candidate, either $\left(\frac{1}{3}, \frac{8}{9}\right)$ or $\left(\frac{4}{5}, \frac{2}{7}\right)$, in the full-dimensional policy space. This is followed by candidate a choosing her campaign message. She has essentially four choices: She could precisely disclose her policy point to the voter, leading to the voter's information sets h_1 and h_2, respectively, depending on the move of nature. She could remain vague about her policy point but raise the voter's awareness of issue 2 leading to the voter's information set h_3. Note that in both cases, the voter's information set is located in the upper tree $T^{\{1,2\}}$, indicating that he is aware of both issues. Alternatively, the candidate can precisely disclose her policy on issue 1 only and remain completely silent on issue 2. In such a case, the voter remains unaware of issue 2 and his information set is now located in the lower tree $T^{\{1\}}$. This is indicated by the blue arrows to the information sets h_4 and h_5, respectively. Finally, the candidate may not disclose any information about her policy and remain also silent on issue 2 leading to information set h_6 in the lower tree $T^{\{1\}}$. Again, we truncate the game form at the information set of the voter and do not explicitly draw the voting actions of the voter in order to save space.

Figure 2: Failure of Unraveling without competition under unawareness



As in the previous example, we indicate the unique prudent rationalizable strategy of the candidate by the red intermitted path in both game trees. We observe that in any prudent rationalizable outcome, the voter must remain unaware issue 2 as any information set of the voter reached along any prudent rationalizable path lies in the lower game tree, $T^{\{1\}}$. Unawareness of issue 2 prevents unraveling of information in this example. If nature chooses $(\frac{1}{3}, \frac{8}{9})$ then candidate a is elected even though he would not be elected under full awareness and information.

To gain some intuition for the failure of unraveling, note that regardless of what information on the candidate's policy point is disclosed to the voter, as long as the candidate makes the voter aware of issue 2, the voter would not vote for the candidate as the "status quo" is strictly preferred to any possible policy point of the candidate. Thus, the candidate has no interest in making the voter aware of issue 2. Consequently, the game is perceived by the voter as identical to the one described in Example 1. Thus, when nature draws the candidate's policy point $\left(\frac{1}{3}, \frac{8}{9}\right)$, the candidate only discloses $\left\{\frac{1}{3}\right\}$ and keeps silent on issue 2. When nature draws the candidate's policy point $\left(\frac{4}{5}, \frac{2}{7}\right)$, the candidate remains vague on the first issue and silent on the second. All the voter can deduce is that the candidate's policy point is $\frac{4}{5}$ (on the first issue) but he cannot even contemplate about the second issue. Hence, the candidate is elected even though he would not be elected under full awareness. In Appendix B we state in detail the prudent rationalizable strategies in this example.

5 The Role of Microtargeting Voters

Our model reflects a rather novel feature of political campaigns as we allow candidates to microtarget of voters with specific messages. This has also been called *dog-whistle politics* – targeting a message so that it can be heard only by those it is intended to reach, like the high–pitched dog whistle that can be heard by dogs but is not audible to the human ear. In this section, we analyze the role of microtargeting for unraveling of information in electoral campaigns.

An early instance of microtargeting is the 2000 presidential campaign in which Bush sent a letter to the U.S. Conference of Catholic Bishops, who have influence over a traditional Democratic constituency, in which he pledged that taxes should not be used to fund research that involves the destruction of human embryos. Yet, stem cell research was not mentioned in nominated speeches of either candidate, not covered on their television advertising, not raised in presidential debates nor displayed on campaign web sites (see Hillygus and Shields, 2008, p. 2).

The normative appropriateness of such strategies has been questioned both in political science and political economy. For instance, Hillygus and Shields (2008, pp. 13) write "The fragmentation of campaign dialogue also has potential implications beyond the electoral contest itself. Elections have always been a blunt instrument for expressing the policy preferences of the public but the multiplicity of campaign messages makes it even more difficult to evaluate whether elected representatives are following the will of the people. Microtargeting enables candidates to focus attention on the issues that will help them win, irrespective of whether they are of concern to the broader electorate. ... How does a winning candidate interpret the policy directive of the electorate if different individuals intended their vote to send different policy messages? Can politicians claim a policy mandate if citizens are voting on the basis of different policy promises?" In the political economy, the received view is that "(p)olitical communication is mass communication. If a politician was able to design a different talk for each elector, maybe each of these talks would be very clear. Actually, politicians can easily give way to the temptation of making different promises to different people" (Laslier, 2006). Moreover, until recently it was felt that "for practical reasons, it is impossible to perfectly realize this targeting." Yet, modern information technology allows to collect a wealth of individual data on voters, apply sophisticated data mining tools[6], and use this information strategically in campaigns. Recent campaigns merged voter registration files with consumer data that include names, addresses, address histories, driving records, criminal records, and consumer purchases like magazine subscriptions, mortgage information, credit-card purchases, gun ownership etc. (see Hillygus and Shields, 2008, p. 159, 161).[7] This information was then used to "microtarget messages through direct mail, email, telephone calls, and personal visits." As Sara Taylor, a strategist for Bush's 2004 presidential campaign summed up "We could identify exactly who should be mailed, on what issues, and who should be ignored completely." (quoted from Hillgus and Shields, 2008, p. 161). Our main results cautions against an entirely negative normative assessment of microtargeting voters since information unravels despite microtargeting. In fact, in this section we like to demonstrate that microtargeting facilitates unraveling and may be necessary for unraveling.

In the following example, we assume that each candidate cannot send different messages to different voters but must send the same message to all voters. We can interpret this as public campaign messages sent to all voters. For simplicity, we ignore the issue of unawareness and show that microtargeting is already crucial under full awareness. There are three possible

[6]In 2011, the Obama campaign posted a job ad for a "Predictive Modeling and Data Mining Scientists/Analysts" on KDnuggets, a website who bills itself as "Data Mining Community's Top Resource for Data Mining and Analytics Software, Jobs, Consulting, Courses, and more". See http://www.kdnuggets.com/jobs/11/07-13-obama2012-predictive-modeling-data-mining-scientists-analysts.html.

[7]There are several commercial companies in the US like Aristotle, Camelot, and Catalist who collect individual voter records, merge them with other public and commercial data, and provide them for a fee to campaigns. For instance, Catalist claims to maintain a "database of over 265 million persons (more than 180 million registered voters and 85 million unregistered adults)". The data include "Registered Voters and Non-Registered persons (with contact information)" but also "Commercial and Census Data ..." (see http://catalist.us). The company Aristotle claims that "(i)n addition to the wealth of demographics Aristotle already provides for high level micro-targeting, you can now identify your voters based on their interests and hobbies. Aristotle maintains a list of over 5.4 million voters who hold hunting and fishing licenses, as well as individuals who subscribe to a wide array of magazine subscriptions including family, religious, financial, health, culinary and Do-It Yourself publications." Premium data are priced at $0.06 per record for over 50,000 records (see http://www.aristotle.com or http://www.voterlistsonline.com).

policy points of candidates, y_1, y_2, and y_3. Moreover, there are three voters. For simplicity we consider just three information sets of voters in Table 1. The preferences of voters are such that their first-level prudent rationalizable strategies at those information sets are given in Table 1.[8] Voter 3 strictly prefers candidate a with policy point y_1 no matter whether candidate b's policy

Table 1: First-Level Prudent Rationalizable Strategies of Voters

Information Set		First-level Prud. Rat. Strategies					Winning
Message by a	Message by b	Voter 1		Voter 2		Voter 3	Candidate
$\{y_1\}$	$\{y_2\}$	a	a	b	b	a	a
$\{y_1\}$	$\{y_3\}$	b	b	a	a	a	a
$\{y_1\}$	$\{y_2, y_3\}$	a	b	a	b	a	a or b

point is y_2 or y_3. Voter 1 prefers candidate a over candidate b if the latter's policy point is y_2 while he prefers candidate b over candidate a if candidate b's policy point is y_3. Voter 2 has preferences dual to voter 1. Consequently, when candidate b reveals $\{y_2, y_3\}$, voters 1 and 2 have full-support beliefs that would make voting for candidate a rational as well as full-support beliefs that would make voting for candidate b rational.

There are second-level prudent rationalizable strategies of candidate b ascribing to send message $\{y_2, y_3\}$ to all voters after nature moves (y_1, y_2) or (y_1, y_3). At those information sets this message is rational with the following belief: b believes with sufficiently high probability that candidate a sends message $\{y_1\}$ and the voter votes for b after receiving the profile of message $(\{y_1\}, \{y_2, y_3\})$, otherwise he votes for a. Finally, with any third-level prudent rational belief, voters 1 and 2 cannot deduce anymore candidate b's policy point after receiving message $\{y_1\}$ from candidate a and $\{y_2, y_3\}$ from candidate b.

In other words, voter 1 cannot identify candidate b's strategic intention of not providing the more favorable message, $\{y_3\}$. Candidate b might not be able to send this message because her policy point is in fact y_2. Alternatively, candidate b may actually have policy point y_3 but is afraid that saying so would alienate voter 2. By sending message $\{y_2, y_3\}$ instead, candidate b keeps the chance of obtaining both votes.

Thus, together with our main results this example highlights a positive role of microtargeting voters with possibly different messages. Microtargeting facilitates information revelation as a voter can now deduce that a candidate's political position is unfavorable to her from the fact that the candidate has not provided precise information. With public information only, a voter not receiving favorable information from a candidate does not know whether the candidate does

[8] That is, voter 1 strictly prefers y_1 to y_3 to y_2, while voter 2 strictly prefers $y_2 > y_3 > y_1$. Voter 3 strictly prefers y_3 to any other policy points.

not have this information or whether the candidate just tries to avoid alienate other voters.[9]

Interestingly this example turns out to be a special case of a more general preference aggregation paradox according to which each voter may have von Neumann-Morgenstern preferences but society does not. We elaborate on this theoretical point further in Appendix C.

6 The Role of Political Reasoning Abilities

In Section 3, we presented strong positive results on the informational effectiveness of electoral competition with microtargeting of voters due to unraveling of awareness and information. Our solution concept assumes that voters are able to infer information from messages of candidates. E.g., a voter who receives the message that the candidate's policy point is ℓth-closest or closer to his is able to infer that the candidate has exactly the ℓth-closest policy point. This is implied by level-$(2\ell-1)$ mutual cautious belief in rationality among the voter and candidate. This raises at least two questions: First, do our results really rely on the voters' ability of (higher-order) reasoning about political campaigns? Second, if yes, what can be said about election outcomes in a context in which voters' rationality is limited in the sense that they still try to do what is best for them but their reasoning about rationality is limited to some finite level k? Answers to these questions are empirically relevant for two reasons: First, casual evidence suggests that elections in real world may not achieve full unraveling. Second, experimental evidence with other games suggests that human reasoning is largely limited to two levels of reasoning only (e.g., Nagel, 1995).

Fortunately, our model and solution concept allow us to investigate this question. In particular, we consider it as a strength that we can derive implications of finite-level reasoning directly from our iterative solution concept. To prove the point that two levels of reasoning won't be always sufficient for our unraveling results, consider now a policy space with three possible policy points of candidates, $Y = \{y_1, y_2, y_3\}$, no unawareness, two candidates, and just a single voter who strictly prefers y_1 to y_2, and y_2 to y_3. Focus on the move of nature $(y^a, y^b) = (y_2, y_3)$. In Table 2, the second column denotes all possible actions of candidate a after this move of nature and the second row lists all possible actions of candidate b after this move of nature. Thus, each cell corresponds to an information set of the voter. In each cell, we indicate the actions that first-level prudent rationalizable strategies of the voter can ascribe to this information set. Every second-level prudent rationalizable strategy of a candidate prescribes to fully reveal its policy point if the policy point is y_1 and it never prescribes to fully reveal its policy point if it

[9] This is reminiscent of Koessler (2008) who shows in a particular two-states, two-actions, two-receivers persuasion game that private communication may lead to full disclosure in sequential equilibrium while public communication may not.

Table 2: First-Level Prudent Rationalizable Actions for the Voter

		Information provided by candidate b			
		$\{y_3\}$	$\{y_1, y_3\}$	$\{y_2, y_3\}$	Y
Information	$\{y_2\}$	a	a, b	a	a, b
provided by	$\{y_1, y_2\}$	a	a, b	a	a, b
candidate	$\{y_2, y_3\}$	a	a, b	a, b	a, b
a	Y	a	a, b	a, b	a, b

is y_3. Any second-level prudent rationalizable strategy of candidate a may prescribe to reveal $\{y_2\}$, $\{y_1, y_2\}$, $\{y_2, y_3\}$, or Y while any second-level prudent rationalizable strategy of candidate b may prescribe to reveal $\{y_1, y_3\}$, $\{y_2, y_3\}$, or Y. Assume that the voter has limited political reasoning capabilities in the sense that he does not form beliefs about the candidates' belief in the voter's rationality. That is, the voter won't necessarily believe in the second-level prudent rationalizable strategies of candidates. In this case, the process of eliminating strategies stops after the second level. Note that there are many second-level prudent rationalizable outcomes where the voter votes for candidate b even though he would prefer candidate a over candidate b under complete information. This is because not enough information is revealed after two levels of elimination of imprudent strategies.

It is possible albeit tedious to prove a more general result. For every finite level ℓ of eliminating imprudent strategies, there is a generic policy space (in the sense that the policy points of candidates could be perturbed slightly without affecting the result) with a sufficiently large but finite number of possible policy points for candidates and a ℓ-level prudent rationalizable election outcome that differs from the outcome under full awareness and complete information. Thus, the "richer" the policy space, the higher are the demands on the political reasoning capabilities of voters in order for the unraveling results to obtain. We conclude that the political reasoning abilities of voters are very crucial for our positive results.

7 A Positive Role of Negative Campaigning

In the two previous sections, we observed that limited political reasoning capabilities of voters or the inability of candidates to microtarget voters may prevent unraveling of information in electoral campaigns. In this section, we explore to what extent positive results can be regained when allowing for negative campaigns of candidates. A crucial feature of negative campaigns is that a candidate can now reveal information not only about his own policy point but also about the policy point of its opponent.

We consider first the case in which information revealed is now public to all voters instead targeted to particular voters. This is because we want to claim that negative campaigning facilitates unraveling of information even when microtargeting may not be feasible at least in a majoritarian election. Formally, a strategy for candidate $k \in \{a, b\}$ is now redefined

$$s_k : H_k \longrightarrow \bigcup_{\{1\} \subseteq I' \subseteq I} 2^{Y_{|I'}} \times 2^{Y_{|I'}} \tag{10}$$

such that $(s_k(h_k))_k, (s_k(h_k))_{-k}) \in 2^{Y_{|I'}} \times 2^{Y_{|I'}}$ for some I' with $\{1\} \subseteq I' \subseteq I(h_k)$ satisfying $\left(y^k(h_k)_{|_{I'}}, y^{-k}(h_k)_{|_{I'}}\right) \in (s_k(h_k))_k, (s_k(h_k))_{-k})$, where $(s_k(h_k))_k$ denotes k's "message" about her own policy points, and $(s_k(h_k))_{-k}$ denotes k's "message" about her opponent's policy points.

As in the previous model, we assume that while she can be vague, each candidate can not bluntly lie about its own policy points nor about the opponent's. In the definition above, this is reflected in the restriction $\left(y^k(h_k)_{|_{I'}}, y^{-k}(h_k)_{|_{I'}}\right) \in (s_k(h_k))_k, (s_k(h_k))_{-k})$. That is, if I' is the nonempty set of issues[10] raised by candidate k at information set h_k, then the profiles of (projected) policy points $\left(y^k(h_k)_{|_{I'}}, y^{-k}(h_k)_{|_{I'}}\right)$ must be in the set of policy points revealed by candidate k to voters at the information set h_k.

At the first glance, the assumption of verifiable information may appear to be stronger than in the previous model because a candidate might be tempted to lie about the opponent's policy point rather than about his own. Yet, as Geer (2006, p. 6) points out "(f)or a negative appeal to be effective, the sponsor of that appeal must marshal more evidence, on average, than for positive appeals. The public, like our legal system, operates on the assumption of 'innocent until proven guilty.' A candidate cannot ... simply assert that their opposition favor a tax increase. They must provide some evidence for this claim ..." This is echoed by some political consultants and campaign managers (see Geer, 2006, pp. 53). Geer (2006, pp. 54) supports the claim with empirical evidence from a content analysis of some television ads of presidential campaigns from the 1964 to 2000. Thus, assuming verifiability of information about opponents may be weaker than assuming verifiability of the candidate's own information. In any case, we like to point that in this modified model information about the "entire" policy profiles becomes verifiable, while only "partial" information about policy profiles is assumed to be verifiable in the previous model.

Belief systems and the solution concept are redefined with modified strategies of candidates accordingly. We call this the model with negative campaigning because candidates can now reveal unfavorable information about the opponent.

[10] In the definition of candidates' strategies above, we require that candidate k reveals the same set of issues for herself and for the opponent. This is without loss of generality since a voter is assumed to reason about any issues that were raised during the campaign.

We are now able to show for the majoritarian election model that even when voters lack political reasoning capabilities beyond two rounds of prudent rationalizability (as was assumed in the example of Section 6) or candidates lack the ability to microtarget voters, prudent rationalizable outcomes are equivalent to outcomes under full awareness and complete information.

Proposition 3 (Majoritarian Election Model with Negative Campaigning) *At every second-level prudent rationalizable outcome of the majoritarian election model with negative campaigning under unawareness of political issues and incomplete information about candidates' policy points, if a candidate obtains the majority of votes then he could also obtain the majority of votes under full awareness of all political issues and complete information about the candidates' policy points.*

The proof in the appendix goes roughly as follows: With any first-level prudent rationalizable strategies of a voter, she votes for the candidate whose policy point she is certain to weakly prefer and may even strictly prefer to the other candidate's policy point. Any second-level prudent rationalizable strategy of candidates must be such that if a candidate has a strictly preferred policy point for a voter in some subspace and all higher-dimensional spaces, then he reveals his own policy point and the other's.

To gain intuition for why less political reasoning capabilities are required in the model with negative campaigning, recall that candidates face a zero-sum game. If providing certain information is not beneficial to one candidate, it will be to the other. In the baseline model, a candidate with the strictly less preferred policy point for a voter would hide precise information on its policy points since revealing it would benefit the opponent. Hence, sophisticated political reasoning capabilities are required for the voter to reason why this candidate does not provide more precise information. However, in the extended model, even though the candidate may provide vague information to the voter, the opponent candidate can now reveal the information on the candidate so that the voter has all relevant information for a vote equivalent to one under full awareness and complete information.

To understand why microtargeting is not required anymore, recall from Section 5 that without microtargeting a voter may not be able to infer from a vague message the candidate's policy point. She does not know whether the candidate sends a vague message because he does not have a favorable policy point or because he has a favorable policy point but does not want to say so in public in order to not alienate other voters. This uncertainty kicks in at the third and higher level of reasoning. With negative campaigning, just two levels of reasoning suffice to reveal sufficient information such that election outcomes do not differ from the case of full awareness and complete information. Thus, negative campaigning is not just a substitute for limited political reasoning capabilities of voters but also for sophistication of candidates' campaign strategies.

What about the proportional election model? Surprisingly, the election system matters for the positive result on negative campaigning. In fact, an analogous result to Proposition 3 *does not hold* for the proportional election model. The following counterexample demonstrates that in the proportional election model, negative campaigning is insufficient for unraveling sufficient information such that election outcomes are equivalent to outcomes under full awareness and complete information of voters if campaign messages are public and no microtargeting of voters is allowed.

Example. Consider the case of a just one-dimensional policy space $Y = \{y^I, y^{II}\}$. There are two voters, $N = \{1, 2\}$, with most preferred policy points $x^1 = y^I$ and $x^2 = y^{II}$. Note that unless the policy points of candidates coincide, the share of voters voting for either one candidate is exactly $\frac{1}{2}$ when voters have complete information about candidates' policy points.

It will suffice to focus on two moves of nature, the case of identical policy points among candidates $(y^a, y^b) = (y^I, y^I)$ as well as the case of different policy points $(y^a, y^b) = (y^I, y^{II})$. The arguments for the other moves of nature are analogous by symmetry. Table 3 shows the first-level prudent rationalizable strategies of voters. The upper table pertains to the case $(y^a, y^b) = (y^I, y^I)$ while the lower is for case $(y^a, y^b) = (y^I, y^{II})$. Each cell refers to a common information set of voters that is reached by a particular combination of actions of candidates a (rows) and b (columns).

Unfortunately, we will show that *any* strategy of each candidate is second-level prudent rationalizable. Consider Case 1: $(y^a, y^b) = (y^I, y^I)$ (upper table). Noteworthy, the strategy of the voter may prescribe to vote for different candidates at different information sets among those 9 information sets at which voters are certain of $(y^a, y^b) = (y^I, y^I)$ because voting for any candidate is rational. Candidate k's action $\left\{(y^I, y^I)\right\}$ is rational at the information set reached by the move of nature, (y^I, y^I) if k believes with sufficiently high probability that the same action is taken by candidate $-k$ and that voters vote for candidate k only at their information set reached by the move of nature (y^I, y^I) and the profile of candidates' actions $\left(\left\{(y^I, y^I)\right\}, \left\{(y^I, y^I)\right\}\right)$ and otherwise vote for the opponent. By analogous arguments, any other action of candidate k is second-level prudent rationalizable after the move of nature (y^I, y^I).

Consider now Case 2: $(y^a, y^b) = (y^I, y^{II})$ (bottom table). All actions of candidate k are payoff-equivalent no matter what his opponent does except action $Y \times Y$. Consider for instance actions $\left\{(y^I, y^{II})\right\}$ and $Y \times Y$. At any information set of voters reached with candidate k's action $\left\{(y^I, y^{II})\right\}$, exactly one voter votes for each candidate. Action $\left\{(y^I, y^{II})\right\}$ is second-level prudent rationalizable for candidate k with a belief system that assigns sufficiently high

Table 3: First-level prudent rationalizable strategies of voters

Case 1 : $(y^a, y^b) = (y^I, y^I)$		Actions of candidate b			
	(y^I, y^I)	$\{(y^I, y^I)\}$	$\{y^I\} \times Y$	$Y \times \{y^I\}$	$Y \times Y$
Actions of candidate a	$\{(y^I, y^I)\}$	any, any	any, any	any, any	any, any
	$\{y^I\} \times Y$	any, any	(a, b)	any, any	(a, b)
	$Y \times \{y^I\}$	any, any	any, any	(b, a)	(b, a)
	$Y \times Y$	any, any	(a, b)	(b, a)	any, any

Case 2 : $(y^a, y^b) = (y^I, y^{II})$		Actions of candidate b			
	(y^I, y^{II})	$\{(y^I, y^{II})\}$	$\{y^I\} \times Y$	$Y \times \{y^{II}\}$	$Y \times Y$
Actions of candidate a	$\{(y^I, y^{II})\}$	(a, b)	(a, b)	(a, b)	(a, b)
	$\{y^I\} \times Y$	(a, b)	(a, b)	(a, b)	(a, b)
	$Y \times \{y^{II}\}$	(a, b)	(a, b)	(a, b)	(a, b)
	$Y \times Y$	(a, b)	(a, b)	(a, b)	any, any

probability to action $Y \times Y$ of candidate $-k$ and exactly one voter voting for each candidate at their information set reached by those candidates' actions while both vote for $-k$ at the information set reached by the move of nature (y^I, y^{II}) and the candidates' action profile $(Y \times Y, Y \times Y)$. Also action $Y \times Y$ is second-level prudent rationalizable for candidate k with a belief system that assigns sufficiently high probability to action $Y \times Y$ of candidate $-k$ and that both voters vote for k at the information set reached by the move of nature (y^I, y^{II}) and the candidates' action profile $(Y \times Y, Y \times Y)$.

Thus, we have shown that for any strategy of candidate k, there is a full-support belief of candidate k with which the strategy is second-level prudently rational at every of his information sets. Moreover, since no strategies of candidates have been eliminated at the first-level of the prudent rationalizability procedure, no strategies of voters are eliminated at the second-level of prudent rationalizablity. Hence, prudent rationalizability does not eliminate any candidate's strategies at a higher level and information fails to unravel. $\square$

Note that the arguments of the previous counterexample do not apply to the majoritarian election model. The crucial difference is that in the majoritarian election model candidates care only about winning the election. Candidate a can now secure his election by choosing the fully revealing action $\{(y^I, y^{II})\}$ in Case 2 above, while due to prudent beliefs there is some uncertainty that candidate a is elected by choosing $Y \times Y$. (Recall that we excluded ties in our model, see p. 7.) This is different from the proportional election model where candidate a just obtains $\frac{1}{2}$ of all votes when choosing the fully revealing action.[11]

Microtargeting would help to overcome the unraveling problem of the example in the proportional election model. To see this, note that the only reason why candidate k may use action $Y \times Y$ in Case 2 in the example above is that he can have hopes that candidate $-k$ uses $Y \times Y$ as well and that both voters vote for k. But since he entertains prudent beliefs, there remains some uncertainty whether voters will actually vote like that. Instead, with microtargeting candidate a can ensure that voter 1 votes for him by sending her privately message $\{(y^I, y^{II}\}$ while still remain hopeful on voter 2 by sending her the message $Y \times Y$. Since candidate b acts analogously at the second level of the prudent rationalizability procedure, just two levels of prudent rationalizability are enough to produce election outcomes equivalent to outcomes under full awareness and complete information. It turns out that this holds generally in the proportional election model.

We now allow again for microtargeting of voters just we did in the baseline model. Formally, a strategy for candidate $k \in \{a, b\}$ is now redefined

$$s_k : H_k \longrightarrow \prod_{j \in N} \left[\bigcup_{\{1\} \subseteq I^j \subseteq I} 2^{Y_{|_{I^j}}} \times 2^{Y_{|_{I^j}}} \right] \tag{11}$$

such that for every voter $j \in N$, there exists I^j with $\{1\} \subseteq I^j \subseteq I(h_k)$ with $(s_k(h_k))_{j,k}, (s_k(h_k))_{j,-k}) \in 2^{Y_{|_{I^j}}} \times 2^{Y_{|_{I^j}}}$ satisfying $\left(y^k(h_k)_{|_{I^j}}, y^{-k}(h_k)_{|_{I^j}}\right) \in (s_k(h_k))_{j,k}, (s_k(h_k))_{j,-k})$, where $(s_k(h_k))_{j,k}$ denotes k's "message" to voter j on k's policy points, and $(s_k(h_k))_{j,-k}$ denotes k's "message" to voter j on $-k$'s policy points.

Belief systems and the solution concept are redefined with modified strategies of candidates accordingly. We call this the model with negative campaigning *and microtargeting.*

Proposition 4 (Proportional Election Model with Negative Campaigning and Microtargeting) *At every second-level prudent rationalizable outcome of the proportional election model with negative campaigning and microtargeting under unawareness of political issues and incomplete information about candidates' policy points, every voter prefers to vote for the same candidate as when he has full awareness of all political issues and complete information about the candidates' policy points.*

[11] Note that the difference between the proportional and majoritarian election models with respect to negative campaigning is not due to the particular tie-breaking rule in the latter model. Consider for instance an alternative tie-breaking rule according to which each candidate wins with probability 1/2 in case of a tie. This tie-breaking rule mimics the shares of voters in the proportional election model in the counterexample. In this case, Proposition 3 holds trivially because any candidate can win under full awareness and complete information.

The proof is contained in the appendix. We conclude that with negative campaigns just "two levels" of political reasoning are required for sufficient unraveling of awareness and information. That is, with negative campaigning it is enough that each voter naively votes for the candidate that she believes is closest to her based on the information that emerges in the campaign and candidates take this into account when choosing their campaign messages. Yet, there is a crucial difference between the majoritarian and proportional election models with respect to the sophistication of candidates' campaign strategies. The majoritarian election model achieves unraveling with less sophisticated campaign strategies of candidates that do not require microtargeting of voters while in the proportional election model unraveling requires also microtargeting of voters.

In any case, we conclude that negative campaigning improves the informational efficiency of electoral campaigns. This is in contrast to the public opinion on negative campaigning, who views it as being detrimental to the political process, a view that is echoed widely in the political science literature (for an overview, see Geer, 2006, pp. 2-3, 15-18). An exception is Geer (2006) who argues that negative campaigning improves the "information environment" of elections. Based on a content analysis of presidential campaigns from 1964 to 2000 he shows that negative TV campaign advertisements bring up more issues per ad to voters than "positive" campaign ads. He also argues that for "a negative appeal to be effective, the sponsor of that appeal must marshal more evidence, on average, than for positive appeals."

8 Conclusions and Related Literature

We study to what extent a well-functioning democracy requires a knowledgable electorate. Whereas we conventionally think that an informed electorate is necessary for democracy to function well, our results suggest that the voters can be miserably uninformed in general and still make the same choices at the ballot box as if they were fully informed about political positions of candidates and aware of all political issues. This is because modern electoral campaigns reveal sufficient information to voters. Our model allows us to show that our results go through even when voters are entirely unaware of some political issues. In such a case, political competition is paramount for unraveling of information. Moreover, campaign strategies like micro-targeting and negative campaigning, which are frequently deplored, enhance the effectiveness of the campaign in providing the requisite information to identify the better candidate. This is especially important when voters lack sophisticated political reasoning abilities.

Our study is most closely related to Demange and Van der Straeten (2013), Hafer and Landa (2013), and Janssen and Teteryatnikova (2015b), who also model electorial campaigns with verifiable information. Demange and Van der Straeten (2013) also model electoral campaigning with a persuasion game à la Milgrom (1981). Candidates can send a signal on each issue about their fixed ideological policy point on that issue. Signals are unbiased, normally distributed with the variance controlled by the candidate. There is just a single voter who is aware of all issues, has a prior belief that is independent across candidates and across issues, and normally distributed. This voter observes both the signals and the variances chosen by candidates. Candidates do not interact strategically as each just considers the effect of her own strategy on the voter. They show unraveling in the sequential equilibrium of the game. Moreover, they show that when voters are naive and take messages at face-value, unraveling breaks down. Their model is very similar to ours in that both use verifiable information. Yet, while in their model candidates chose unbiased signals distributed normally with chosen variance, in our model candidates chose arbitrary but finite sets of policy points that contain their true policy point. We make no distributional assumptions. Their conclusions are very similar to ours as well, which shows that the results in either paper cannot be an artifact of differing modeling assumptions. Their result on naive voters is akin to our observation that unraveling breaks down when voters have limited political reasoning capabilities. Our analysis suggests that their result could be extended to strategic interaction among candidates. Moreover, our analysis suggests that their results could be extended to unraveling even in the presence of unawareness if strategic interaction among candidates is added. We also show that allowing for more than one voter introduces additional difficulties because unraveling may break down in the absence of microtargeting. Since they consider just one voter, the issue of microtargeting does not even arise since there is no difference between public and private campaign messages. Finally, we study negative campaigning, a feature that cannot be studied Demange and Van der Straeten (2013) because they do not consider strategic interaction among candidates.

Hafer and Landa (2013) consider a game between two senders and a continuum of voters. Each player has an ideal political position but voters initially do not know their own position. Senders can send a message public to all voters that consists either of their own position or no message. If their position corresponds to a voter's position, the voters realizes it. Otherwise, if the sender's communicated political position is not equal to the voter's position, then the voter learns her own position only with a probability which is interpreted as the voter's sophistication to decode political messages. Nothing is inferred by the voter if no message is sent. Voting outcomes are determined by majority. Their model is similar in spirit to ours, but there are obvious differences. While both models use verifiable information, we allow for a richer message space. Moreover, while in their model voters decode the same message of the receiver differently depending on their own type, we consider microtargeting (i.e., different messages to different voters) and allow for voters to have different beliefs about the political positions coarsely communicated by candidates. Although they show that in undominated sequentially rational equilibrium some type always wants to communicate there may be realizations of sender's types such that in equilibrium no sender communicates. We obtain unraveling for any realization of senders' types. Finally, we model political sophistication of voters as their ability to interactively reasoning about the candidate's strategies (i.e., roughly how many iterations of elimination of weakly dominated strategies the voters is able to do) while Hafer and Landa (2013) consider a fixed probability of voters learning their own type from receiving a message not corresponding to their own type. Most interestingly, in their equilibrium more might be communicated to voters if they have a lower probability of learning their own type while in our setting limits on the political sophistication of voters are detrimental to unraveling unless negative campaigning is considered as well. This suggests that the impact of voters sophistication on campaigns depends crucially on how sophistication is modeled. Their model is silent on negative campaigning.

Janssen and Teteryatnikova (2015b) study negative campaigning among two candidates with a model of verifiable information. They also show that negative campaigning helps unraveling. Their analysis differs from ours as they do not allow for unawareness, confine the analysis to a one-dimensional policy space, use a version of perfect Bayesian equilibrium with identical off-equilibrium path beliefs of voters, do not consider microtargeting, and focus on the case in which candidates care only about their share of voters. Their model is closely related to their earlier paper on disclosure of horizontal attributes by firms, Janssen and Teteryatnikova (2015a) (see also Celik, 2014, Koessler and Renault, 2012, and Sun, 2011). This suggests that our model and results can be extended to a setting in which competing firms disclose vertical and horizontal attributes to potential buyers.

A related paper on negative campaigning is Polborn and Yi (2006). There are two candidates each "owning" an issue. Candidates are allowed to either remain silent or provide correct precise information on their own issue or the opponent's issue. Their result most closely related to ours is a version of our Proposition 3 without unawareness. Since they consider just a representative voter and Perfect Bayesian equilibrium, they cannot explore microtargeting or limited political reasoning capabilities of voters. On the other hand, they consider "budget constraints" such that candidates face the choice between either a positive or a negative campaign, something that we don't consider here.

There have been earlier work who conceive of electorial campaigns as advertising. Coate (2004a, b) and Ashworth (2006) who consider beside a policy dimension also the quality of candidates. Lobbies finance campaigns in order to enhance prospects of like-minded parties. Parties can provide verifiable information in public advertising campaigns about their own policies that they committed. Negative campaigns are ruled out. The focus of their work is on the effect of campaign financing.

Our model can be viewed as answering early critique in political science on the uni-dimensional

Downs model. Stokes (1963) postulated that "the space in which political parties compete can be of highly variable structure. Just as the parties may be perceived and evaluated on several dimensions, so the dimensions that are salient to the electorate may change widely over time." He also criticized the assumption of a commonly perceived policy space by stating "(b)ut with the space formed out of perceptions, there is no logically necessary reason why the space of voters and of parties should be identical, and there is good empirical reason to suppose that it often is not." Note that our model captures a "highly variable" policy space, differing "salience of dimensions" over time, and perceptions of the policy space that differ between voters and candidates. There is a large literature in political science on salience of issues in electorial campaigns mostly under the header of issue ownership theory starting with Budge and Farlie (1983) and Petrocik (1996). According to issue ownership theory, "candidates emphasize issues on which they are advantaged". Yet, as Green and Hobolt (2008) emphasize with data from British elections, as parties converge ideologically, their relative competence on an issue becomes more important than ideological considerations. At a first glance, a candidate's relative competence on an issue as reason for the candidate to campaign on that issue seems different from our model. Yet, if political positions in our model are reinterpreted as degrees of competence on the issues, our model becomes a formal model of issue ownership theory. Presumably in such a setting voters prefer uniformly more competence to less, and hence preferences of voters are homogeneous. Our model predicts then that enough relevant issues and information on the candidates' competence is revealed in electoral competition such that election outcomes are identical to the ones under full awareness of all issues and complete information about the candidates' competence. Note that such a result would not hinge anymore on the candidates' abilities to microtarget voters as all voters have homogeneous preferences over competence. Yet, a lack of electoral competition may still prevent unraveling under unawareness of issues because the candidate could be silent on issues he does not "own".

Berliant and Konishi (2005) study whether candidates in a multi-dimensional Downs model with linear utility of voters like to announce policies on all issues. They assume that voters know the state of the world and candidates just have a prior distribution over voters' types. They show that if a Nash equilibrium exists, candidates like to announce policies on all issues. Moreover, they show by example that non-salience may emerge when candidates face Knightian uncertainty and maximize minimal expected utility. Our models differ substantially from each other. Besides differing assumptions about ideological versus opportunistic candidates and differing informational assumptions, it is impossible in their model to be salient on an issue without announcing a policy on this issue. There is also no role for microtargeting of voters. Moreover, our focus on a rationalizablity procedure rather than Nash equilibrium allows us to shed more light on the importance of political reasoning capabilities of voters. Colomer and Llavador (2012) study electoral competition with salience of issues. Ex ante salience of an issue is proportional to the disagreement of the electorate about the status quo policy of the incumbent on this issue. The ex post salience of an issue is highest when both candidates campaign on it by choosing a policy point. Candidates are allowed to campaign only on one issue each. They show that in subgame perfect equilibrium candidates may not campaign on ex ante most salient issues if there is no policy on this issue that would attract broad agreement among the electorate. Again, our model differs substantially from theirs. Most importantly, in our model candidates can choose to campaign on as many issues as they like and can target different issues to different voters. Recently Aragones et al. (2015) presented a formal model of issue selection in electoral competition in which parties must decide on the proposal quality and campaign time spent on each of three issues. The voter's salience of an issue depends on the campaign time of both parties and the voter's type. Their notion of salience is quite different from the notion of awareness in our work since it is a continuous weight whereas awareness is binary.

Our work is related to the large literature on information aggregation of elections. McKelvey and Ordeshook (1985) present an uni-dimensional Downs model with two candidates. Candidates do not know the preferences of voters and uninformed voters do not know the political positions adopted by candidates but there is also a share of informed voters who know the political positions. All participants can learn from "polls" and "interest group endorsements". They show that in a version of self-confirming equilibrium, in which strategies of participants are optimal with respect to the public information available and the public information is consistent with the strategies of participants, election outcomes correspond to full information election outcomes. McKelvey and Ordeshook (1987) present sufficient conditions on the number and distribution of informed and uninformed voters for an analogous result in a multi-dimensional setting, in which poll-data must be broken down by subgroups of voters. Feddersen and Pesendorfer (1997) show that large elections with strategic voters and two fixed policy alternatives can aggregate information about a uni-dimensional state variable. Voters are differently informed by some "information services" instead of by strategically campaigning candidates. They also show that with higher-dimensional uncertainty, elections may not effectively aggregate information and suggest that future research should "focus on the events that precede elections – nominating procedures, campaigns, polls, etc. – as such events determine the information environment." Although our model is very different from the aforementioned models, it can viewed as focusing exactly on "events that precede elections". Gratton (2014) studies electoral competition between two perfectly informed candidates who are faced by voters who have a common value over the policies but he requires them to have some information about what is best for them. Candidates can be of two types, either strategic or truthful. He studies a sequential equilibrium that also entails some forward-induction in that voters can revise beliefs when candidates propose different policies and that leads to an election outcome identical to the full information outcome. Heidhues and Lagerlöf (2003) study a model in which voters possess no information. But candidates receive imperfectly correlated private signals about the state of nature. In equilibrium candidates bias their information transmission through the choice of platforms towards the voters' prior, letting information revelation fail. Laslier and Van der Straeten (2004) show that this conclusion is not robust as soon as the voters have a tiny bit of relevant information. In this case, all equilibria are dismissed by standard refinements except the one in which information revelation occurs.

We are not aware of models of electoral campaigning with microtargeting of voters. Most closely related is Glaezer, Ponzetto, and Shapiro (2005) who study electoral competition à la Downs (1957) but with endogenous voter turnout in which some party members can observe secretly the platform of the candidate on "their side" of the political spectrum while less members of the other parties can observe it. Thus deviating from the median voter does not necessarily alienate voters on the other side while mobilizing support on its own side, producing divergent platforms. Unlike our model, there is no role for parties to microtarget "swing voters" that may be traditionally associated with the other party.

Finally, our work contributes to the growing literature on unawareness. It constitutes the first application of unawareness to political science. We have shown that unraveling of information in electoral competition is robust to voters' unawareness of political issues. It does not imply that unawareness "does not matter" since we show that unraveling under unawareness hinges crucially on the assumption of competition. That is, we effectively show that the voters' limited awareness of political issues is one explanation for adversarial debates in electoral campaigns. Our results are related to Heifetz, Meier, and Schipper (2011) who show that unraveling of information about product quality may break down under unawareness in a model with a monopolist seller and a buyer.[12] Filiz-Ozbay (2012) shows in a different framework that a monopolist insurer may propose incomplete insurance contracts to an insuree who faces unawareness of some relevant contingencies but that competition among insurers leads to completeness. Li, Peitz, and Zhao (2013) study disclosure of product information to consumers under vertical competition in a duopoly when consumers may be unaware of one dimension of the product but otherwise have complete information. Our model and solution concept differs from theirs substantially.

We close with a comment on empirical testing of our theory. The difficulty is that preferences, beliefs, and levels of reasoning are not directly observable in the field. Yet, it should be

[12] Our persuasion game bears also some similarity with Dzuida (2011). She considers an expert who can credibly disclose a number of favorable and unfavorable arguments to a receiver but cannot prove that he disclosed all arguments. In her setting, the receiver knows that she might not know all arguments while in our setting voters do not know that they may not know all issues. This difference is crucial for what voters/receiver can infer from disclosures.

possible to carefully design a laboratory experiment that induces preferences. This is left for further research.

A Proofs

A.1 Proof of Proposition 1

The proof is analogous to the proof of Proposition 2 with the following modifications:

- Erase (N3).
- Replace (N4) by

(N4') if $|N_k^{\ell}(h_k)| < |N|$ then there is no $N' \subseteq N$ satisfying N1 and N2 in place of $N_k^{(\ell)}(h_k)$ and for which $|N'| > |N_k^{(\ell)}(h_k)|$.

- Redefine $\bar{N} := N_k^{(\ell)}(h_k)$ and erase equations (12) and (13).
- Essentially replace "majority of voters" by "every voter" or "some voter", whatever appropriate, throughout.
- Essentially replace "k wins the election" by "k obtains more votes" and "to $-k$ wins the election" by "that $-k$ obtains votes that k could obtain".

$\square$

A.2 Proof of Proposition 2

Consider first the case in which there is a majority of voters who are indifferent between candidates when all voters are fully aware and have complete information about the candidates' profile of policy points $(y^a, y^b) \in Y \times Y$ in the full-dimensional policy space. For those voters, it is rational to vote for any candidate. Hence, for those voters it is trivially true that if any of them prefers to vote for a particular candidate in a prudent rationalizable outcome of the majoritarian election model with unawareness of political issues and incomplete information about candidates' policy points, then he prefers to vote for the same candidate when having full awareness of all political issues and complete information about the candidates' policy points. Thus, from now on we consider only the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points.[13]

We also assume $|N| \geq 2$ since otherwise the majoritarian election model is a special case of the proportional model.

The proof proceeds by induction. Note that the base-case involves the first two levels of prudent rationalizabiliy, while the induction step assumes properties of level $2\ell - 1$ and 2ℓ prudence rationalizable strategies and proves properties of level $2\ell + 1$ and $2\ell + 2$ prudent rationalizable strategies, for $\ell > 1$. This is due to the nature of our two-stage game.

First level: For any candidate $k \in \{a, b\}$ we have $S_k^1 = S_k$. To see this, note that for candidate k, every $s_k \in S_k$ is first-level rationalizable with a belief system β_k such that for every information set h_k the full-support belief $\beta_k(h_k)$ puts sufficiently high probability to strategies of voter j that ascribe voting for k at every information set reached by $s_k(h_k)$ and voting for $-k$ at all other information sets of j, for all $j \in N$.

Before we turn to first-level prudent rationalizable strategies of voters, it will be helpful to introduce the following notation. We say that y^k reaches the information set h_j of voter j if there is a move of nature (y^k, y^{-k}) there is a path (i.e., a sequence of nodes) from (y^k, y^{-k}) to some node in h_j. Let $Y^k(h_j) := \{y^k \in Y_{|_{I(h_j)}} : y^k \text{ reaches } h_j\}$. That is, $Y^k(h_j)$ is the set of candidate's policy points in $Y_{|_{I(h_j)}}$ that voter j considers possible at his information set h_j.

For any voter j and any of his information sets $h_j \in H_j$, define inductively,

$$Y_j^{(1),I(h_j)} \quad := \quad \left\{ y \in Y_{|_{I(h_j)}} : \| x^j - y \|_{I(h_j)} \leq \| x^j - y' \|_{I(h_j)} \text{ for all } y' \in Y_{|_{I(h_j)}} \right\},$$

and for $\ell > 1$,

$$Y_j^{(\ell),I(h_j)} := \left\{ y \in Y_{|_{I(h_j)}} \setminus \left(\bigcup_{\ell' \leq \ell - 1} Y_j^{(\ell'),I(h_j)} \right) : \begin{array}{l} \| x^j - y \|_{I(h_j)} \leq \| x^j - y' \|_{I(h_j)} \\ \text{for all } y' \in Y_{|_{I(h_j)}} \setminus \left(\bigcup_{\ell' \leq \ell - 1} Y_j^{(\ell'),I(h_j)} \right) \end{array} \right\}.$$

That is, $Y_j^{(1),I(h_j)}$ is the set of voter j's most preferred policy points of candidates in the policy space that he is aware of at the information set h_j. Similarly, $Y_j^{(\ell),I(h_j)}$ is the set of voter j's ℓ-most preferred policy points of candidates in the policy space that he is aware of at his information set h_j. Since Y is finite, there is a well-defined set of voter j's least preferred candidates' policy points that he is aware of at his information set h_j, i.e., a finite largest ℓ.

With these definitions in place, we turn to the first-level prudent rationalizable strategies of voters. For any voter j consider an information set h_j with $Y^k(h_j) \subseteq Y_j^{(1),I(h_j)}$ and $Y^{-k}(h_j) \nsubseteq Y_j^{(1),I(h_j)}$. Note that for any of voter j's belief system β_j, the support of $\beta_j(h_j)$ is the set

[13] Note that it does not imply that there is no majority of voters who are indifferent between candidates under unawareness of some issues but complete information about the policy points in the subspace of issues that they are aware of.

of policy points and strategy profiles of candidates that reach the information set h_j. With any such a belief system, any first-level prudent rationalizable strategy of voter j must ascribe voting for candidate k at h_j. This is because with any such a belief system, voter j is certain that candidate k has his most preferred policy point in $Y_{|_{I(h_j)}}$ while he must assign some strict positive probability to policy points of candidate $-k$ that are strictly less preferred.[14]

Second level: Before we turn to second-level rationalizable strategies, the following definition will be helpful:

For any nonempty subset of issues I' and I'' with $\emptyset \neq I' \subseteq I'' \subseteq I$, denote the projection by $r_{I'}^{I''} : [0,1]^{|I''|} \longrightarrow [0,1]^{|I'|}$. Given an information set h_k of candidate k reached by the move of nature $(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k}))$, let for any $\ell \geq 1$, $N_k^{(\ell)}(h_k) \subseteq N$ be a (possibly empty) subset of voters such that

(N1) every voter $j \in N_k^{(\ell)}(h_k)$ strictly prefers $r^I_{I(h_k)}(y^k)$ over $r^I_{I(h_k)}(y^{-k})$,

(N2) for every voter $j \in N_k^{(\ell)}(h_k)$, $r^I_{I(h_k)}(y^k) \in Y_j^{(\ell'),I(h_k)}$ for some ℓ' with $\ell \geq \ell' \geq 1$,

(N3) the cardinality of $N_k^{(\ell)}(h_k)$ is such that[15]

$$|N_k^{(\ell)}(h_k)| \leq \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or } (k = b \text{ and } |N| \text{ is odd}) \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even),} \end{cases}$$

(N4) if $|N_k^{(\ell)}(h_k)| \leq \frac{1}{2}|N|$ then there is no $N' \subseteq N$ that satisfies properties N1 to N3 in place of $N_k^{(\ell)}(h_k)$ and for which $|N'| > |N_k^{(\ell)}(h_k)|$.

We claim that if s_k is a second-level prudent rationalizable strategy of candidate $k \in \{a, b\}$ then for any information set h_k of candidate k reached by the move of nature $(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k}))$ we have that for any voter $j \in N$, $(s_k(h_k))_j \subseteq Y_{|_{I^j}}$ for some $I^j \subseteq I(h_k)$. Moreover, there is (a possibly empty) subset of voters $N_k^{(1)}(h_k) \subseteq N$ such that for all $j \in N_k^{(1)}(h_k)$ and any I' with $I^j \subseteq I' \subseteq I(h_k)$,

(i) $(r^{I'}_{I^j})^{-1}((s_k(h_k))_j) \subseteq Y_j^{(1),I'}$, and

(ii) voter j strictly prefers $r^I_{I'}(y^k)$ over $r^I_{I'}(y^{-k})$.

We show that these conditions are necessary for second-level prudent rationalizable strategies of candidates. Consider any information set h_k of candidate k such that k knows $(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k}))$.

[14] This condition is just a necessary condition for first-level prudent rationalizable strategies of voters.

[15] Recall from Section 2 that a wins if it obtains weakly more than half of the votes, whereas b wins with strictly more than half of the votes. $\lceil x \rceil$ denotes the smallest integer not less than x.

Let s_k be a second-level prudent rationalizable strategy of candidate $k \in \{a, b\}$. By the definition of strategy, we must have that for any $j \in N$, $(s_k(h_k))_j \subseteq Y_{|_{I^j}}$ for some $I^j \subseteq I(h_k)$. Suppose to the contrary that for every $N_k^{(1)}(h_k)$ there is a nonempty subset of voters $N' \subseteq N_k^{(1)}(h_k)$ such that for any $j \in N'$ properties (i) or (ii) are violated. (If $N_k^{(1)}(h_k)$ is empty, there is nothing to prove.) That is, for any $j \in N'$, there exists $\tilde{I}^j$ with $I^j \subseteq \tilde{I}^j \subseteq I(h_k)$ such that $(r_{I^j}^{\tilde{I}^j})^{-1}((s_k(h_k))_j) \nsubseteq Y_j^{(1),\tilde{I}^j}$ or j does not strictly prefer $r_{\tilde{I}^j}^I(y^k)$ over $r_{\tilde{I}^j}^I(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to any voter $j \in N'$ such that an information set h_j of voter $j \in N'$ in $T^{\tilde{I}^j}$ is reached with $Y^{-k}(h_j) \cap Y_j^{(1),\tilde{I}^j} \neq \emptyset$.

Let $\bar{N} \subset N$ be such that $N_k^{(1)}(h_k) \subseteq \bar{N}$ and

$$|\bar{N}| = \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases} \tag{12}$$

We can partition the set of voters N into $\left\{N_k^{(1)}(h_k) \setminus N', N', \bar{N} \setminus N_k^{(1)}(h_k), N \setminus \bar{N}\right\}$. With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j such that

$$\text{if } j \in \begin{cases} N_k^{(1)}(h_k) \setminus N' & \text{then } j \text{ votes for } k \\ N' & \text{then } j \text{ votes for } -k \\ \bar{N} \setminus N_k^{(1)}(h_k) & \text{then } j \text{ votes for } k \\ N \setminus \bar{N} & \text{then } j \text{ votes for } -k \end{cases},$$

which implies that candidate k must assign strictly positive probability to $-k$ winning the election. Yet, candidate k can strictly improve his expected payoff at h_k given $\beta_k(h_k)$ by replacing $(s_k(h_k))_j$ for $j \in N'$ with $\{r_{I(h_k)}^I(y^k)\}$, because at any information set h_j of voter $j \in N'$ reached by this modified strategy of candidate k, any first-level prudent rationalizable strategy of voters $j \in N'$ must ascribe voting for k implying that k wins the election, a contradiction.

For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$.

Induction step: The following definitions are helpful. For any $\ell > 1$, we say that *strategy s_j of voter j satisfies condition ℓ* if for every h_j such that for some $k \in \{a, b\}$,

1. for some ℓ^k with $\ell \geq \ell^k \geq 1$, $Y^k(h_j) \cap Y_j^{(\ell^k),I(h_j)} \neq \emptyset$ and $Y^k(h_j) \cap Y_j^{(\ell'),I(h_j)} = \emptyset$ for all $\ell' > \ell^k$, and
2. if for some ℓ^{-k} with $\ell \geq \ell^{-k} \geq 1$, $Y^{-k}(h_j) \cap Y_j^{(\ell^{-k}),I(h_j)} \neq \emptyset$ and $Y^{-k}(h_j) \cap Y_j^{(\ell''),I(h_j)} = \emptyset$ for all $\ell'' > \ell^{-k}$, then $\ell^{-k} > \ell^k$,

then voter j votes for k at h_j. Intuitively, this condition states that if candidate k reveals information to voter j that is weakly better than her ℓ-most preferred policy points and $-k$

reveals information to voter j that is not unambiguously better than k's information, then voter j votes for candidate k.

For any $\ell > 1$, we say that *strategy s_k of candidate k satisfies condition ℓ* if for every $y^k \in Y$ and every information set h_k of candidate k reached by the move of nature $(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k}))$ we have that for all voters $j \in N$, $(s_k(h_k))_j \subseteq Y_{|_{I^j}}$ for some $I^j \subseteq I(h_k)$. Moreover, there is (a possibly empty) subset of voters $N^{(\ell)}_k(h_k) \subseteq N$ such that for all $j \in N^{(\ell)}_k(h_k)$ and any I' with $I^j \subseteq I' \subseteq I(h_k)$,

(I) there is $\ell^{I'}$ with $\ell \geq \ell^{I'} \geq 1$ such that $r^I_{I'}(y^k) \in Y_j^{(\ell^{I'}),I'}$ (and hence $(r^{I'}_{I^j})^{-1}((s_k(h_k))_j) \cap Y_j^{(\ell^{I'}),I'} \neq \emptyset$) and $(r^{I'}_{I^j})^{-1}((s_k(h_k)_j) \cap Y_j^{(\ell'),I'} = \emptyset$ for all ℓ' such that $\ell' > \ell^{I'}$, and

(II) voter j strictly prefers $r^I_{I'}(y^k)$ over $r^I_{I'}(y^{-k})$.

Assume now that we have proved that for every voter $j \in N$ the $(2\ell - 1)$-level prudent rationalizable strategies of voter j satisfy condition ℓ and that for every candidate $k \in \{a, b\}$ the 2ℓ-level prudent rationalizable strategies of candidate k satisfy condition ℓ. We claim that for any voter $j \in N$, the $(2\ell+1)$-level prudent rationalizable strategies satisfy condition $\ell+1$ and for every candidate $k \in \{a, b\}$, the $(2\ell + 2)$-level prudent rationalizable strategies of candidate k satisfy condition $\ell + 1$.

Consider a voter $j \in N$ with information set h_j. Suppose that for some ℓ^k with $\ell+1 \geq \ell^k \geq 1$, $Y^k(h_j) \cap Y_j^{(\ell^k),I(h_j)} \neq \emptyset$ and $Y^k(h_j) \cap Y_j^{(\ell'),I(h_j)} = \emptyset$, for all ℓ' with $\ell' > \ell^k$. Unless we also have that for some ℓ^{-k} with $\ell^k \geq \ell^{-k} \geq 1$, $Y^{-k}(h_j) \cap Y_j^{(\ell^{-k}),I(h_j)} \neq \emptyset$ and $Y^{-k}(h_j) \cap Y_j^{(\ell''),I(h_j)} = \emptyset$, for all ℓ'' with $\ell'' > \ell^{-k}$, then with any $2\ell + 1$ prudent rationalizable strategy, voter j must vote for k at h_j. To see this note that for any belief system of voter j, $\beta_j \in B_j^{2\ell+1}$, the support of the belief $\beta_j(h_j)$ at h_j is the set of 2ℓ-prudent rationalizable strategies of candidates who by assumption satisfy condition ℓ. Thus, the voter is certain at h_j of $y^k_{|_{I(h_j)}} \in Y_j^{(\ell^k),I(h_j)}$. Moreover, since candidates' strategies satisfy condition ℓ, voter j with belief $\beta_j(h_j)$ cannot assign strict positive probability to policy points $y^{-k}_{|_{I(h_j)}}$ of candidate $-k$ that are strictly preferred to k's policy point since otherwise $-k$ would have revealed it. It follows that voter j's $(2\ell + 1)$-level prudent rationalizable strategies satisfy condition $\ell + 1$.

Consider any information set h_k of candidate k such that k knows $(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k}))$. Let s_k be a $(2\ell + 2)$-level prudent rationalizable strategy of candidate k. By the definition of strategy, we must have that for any $j \in N$, $(s_k(h_k))_j \subseteq Y_{|_{I^j}}$ for some $I^j \subseteq I(h_k)$. Suppose to the contrary that for every $N^{(\ell+1)}_k(h_k)$ there is a nonempty subset of voters, $N' \subseteq N^{(\ell+1)}_k(h_k)$, such that for any $j \in N'$ there exists $\tilde{I}^j$ with $I^j \subseteq \tilde{I}^j \subseteq I(h_k)$ for which properties (I) or (II) are violated. (If $N^{(\ell+1)}_k(h_k)$ is empty, there is nothing to prove.) That is, we have for all $\tilde{\ell}$ with $\ell + 1 \geq \tilde{\ell} \geq 1$, $r^I_{\tilde{I}^j}(y^k) \notin Y_j^{(\tilde{\ell}),\tilde{I}^j}$ or $(r^{\tilde{I}^j}_{I^j})^{-1}((s_k(h_k))_j) \cap Y_j^{(\ell'),\tilde{I}^j} \neq \emptyset$ for some $\ell' > \tilde{\ell}$, or j does not strictly prefer $r^I_{\tilde{I}^j}(y^k)$ over $r^I_{\tilde{I}^j}(y^{-k})$.

Note that since $j \in N_k^{(\ell+1)}(h_k)$ we have by N1 that j strictly prefers $r^I_{I(h_k)}(y^k)$ over $r^I_{I(h_k)}(y^{-k})$. Moreover, by N2 we must have $r^I_{I(h_k)}(y^k) \in Y_j^{(\ell''),I(h_k)}$ for some ℓ'' with $\ell + 1 \geq \ell'' \geq 1$.

With any belief system $\beta_k \in B_k^{2\ell+2}$, candidate k must assign strict positive probability to $(2\ell+1)$-level prudent rationalizable strategies of candidate $-k$ that reveal information to voter $j \in N'$ such that an information set h_j of voter j in $T^{\tilde{I}^j}$ is reached with $Y^{-k}(h_j) \cap Y_j^{(\tilde{\ell}),\tilde{I}^j} \neq \emptyset$.

Redefine $\bar{N} \subset N$ such that $N_k^{(\ell+1)}(h_k) \subseteq \bar{N}$ and

$$|\bar{N}| = \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases} \tag{13}$$

Partition the set of voters N into $\left\{N_k^{(\ell+1)}(h_k) \setminus N', N', \bar{N} \setminus N_k^{(\ell+1)}(h_k), N \setminus \bar{N}\right\}$. With any belief system $\beta_k \in B_k^{2\ell+2}$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j such that

$$\text{if } j \in \begin{cases} N_k^{(\ell+1)}(h_k) \setminus N' & \text{then } j \text{ votes for } k \\ N' & \text{then } j \text{ votes for } -k \\ \bar{N} \setminus N_k^{(\ell+1)}(h_k) & \text{then } j \text{ votes for } k \\ N \setminus \bar{N} & \text{then } j \text{ votes for } -k \end{cases},$$

which implies that candidate k must assign strictly positive probability to $-k$ winning the election. Yet, candidate k can strictly improve his expected payoff at h_k given $\beta_k(h_k)$ by replacing $(s_k(h_k))_j$ for $j \in N'$ with $\{r^I_{I(h_k)}(y^k)\}$, because at any information set h_j of voter $j \in N'$ reached by this modified strategy of candidate k, any $(2\ell+1)$-level prudent rationalizable strategy of voters $j \in N'$ must ascribe voting for k implying that k wins the election, a contradiction.

Hence, since Y is finite, there is a finite $2\bar{\ell}$ such that for any $\ell > 2\bar{\ell}$ no strategy is eliminated anymore. Moreover, conditions $\bar{\ell}$ imply that neither candidate can change the election outcome by revealing his policy point in the full-dimensional space at any of his information sets in T^I. Conversely, since we assumed that there is a majority of voters who strictly prefer to vote for one particular candidate when all voters have full awareness of political issues and complete information about the policy points, this candidate must also win in any prudent rationalizable outcome under unawareness and incomplete information. $\square$

A.3 Proof of Proposition 3

By the same arguments as in the proof of Proposition 2, we just consider the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points.

First level: For the same reason as in the proof of the Proposition 2, we have $S_k^1 = S_k$ for any candidate $k \in \{a, b\}$.

For any voter j, consider an information set h_j such that for any $y^k \in Y^k(h_j)$ and $y^{-k} \in Y^{-k}(h_j)$, $\| x^j - y^k \|_{I(h_j)} \leq \| x^j - y^{-k} \|_{I(h_j)}$ and for some $y^k \in Y^k(h_j)$ and $y^{-k} \in Y^{-k}(h_j)$, $\| x^j - y^k \|_{I(h_j)} < \| x^j - y^{-k} \|_{I(h_j)}$. Note that for any of voter j's belief system β_j, the support of $\beta_j(h_j)$ is the set of profiles of policy points and strategy profiles of candidates that reach the information set h_j. With any such a belief system, any first-level prudent rationalizable strategy of voter j must ascribe voting for candidate k at h_j. This is because with any such a belief system, voter j at h_j is certain that candidate k is (weakly) preferred to candidate $-k$ for any profile of candidates policy points in $Y_{|_{I(h_j)}}$, while he must assign some strictly positive probability to profiles of policy points (y^k, y^{-k}) for which he strictly prefers y^k to y^{-k}.

Second level: For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$. Given an information set h_k of candidate k reached by the move of nature $(r_{I(h_k)}^I(y^k), r_{I(h_k)}^I(y^{-k}))$, let

$$\bar{N}_k(h_k) = \left\{ j \in N : \text{voter j strictly prefers } r_{I(h_k)}^I(y^k) \text{ to } r_{I(h_k)}^I(y^{-k}) \right\}. \tag{14}$$

Denote by m_k the majority of number of voters required for candidate k to win, which is defined by

$$m_k := \begin{cases} \lceil \frac{1}{2}|N| \rceil & \text{if } k = a \text{ or (if } k = b \text{ and } |N| \text{ is odd)} \\ \frac{1}{2}|N| + 1 & \text{else (i.e., if } k = b \text{ and } |N| \text{ is even).} \end{cases}$$

Since we just need to consider the case in which there is no majority of voters who are indifferent between the candidates when all voters have full awareness of political issues and complete information about candidates' policy points, for every move of nature in the upmost tree there is a candidate k and an information set h_k of candidate k reached by this move of nature such that $|\bar{N}_k(h_k)| \geq m_k$. Fix the move of nature that reaches h_k with $|\bar{N}_k(h_k)| \geq m_k$.

We claim that if s_k is a second-level prudent rationalizable strategy of candidate k, then there is a maximal subset of voters $N_k(h_k) \subseteq \bar{N}_k(h_k)$ such that $|N_k(h_k)| \geq m_k$ and for all $j \in N_k(h_k)$ and any I' with $I^k \subseteq I' \subseteq I(h_k)$ (where I^k is the subset of issues revealed to voters by strategy s_k at information set h_k)

(i) $\| x^j - y' \|_{I'} \leq \| x^j - y'' \|_{I'}$ for any $y' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_k)$ and $y'' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{-k})$,

(ii) voter j strictly prefers $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

Suppose to the contrary that there is no such a $N_k(h_k)$. Then there is subset of voters $N' \subseteq N$ with $|N'| \geq m_{-k}$ and I' such that for any $j \in N'$, $\| x^j - y' \|_{I'} > \| x^j - y'' \|_{I'}$ for some $y' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_k)$ and $y'' \in (r_{I^k}^{I'})^{-1}((s_k(h_k))_{-k})$, or j does not strictly prefer $r_{I'}^I(y^k)$ over $r_{I'}^I(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to voters such that an information set h_j of voter $j \in N'$ in $T^{I'}$ is reached at which for some $y' \in Y^k(h_j)$ and $y'' \in Y^{-k}(h_j)$, $\| x^j - y' \|_{I'} > \| x^j - y'' \|_{I'}$ or $\| x^j - r^I_{I'}(y^k) \|_{I'} \geq \| x^j - r^I_{I'}(y^{-k}) \|_{I'}$. Thus, with any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j and strategies of candidate $-k$ such that all voters in N' vote for candidate $-k$, in which case candidate k loses the election. Yet, candidate k can strictly improve her expected payoff at h_k given β_k by replacing $(s_k(h_k))$ with $\left\{\left(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k})\right)\right\}$, because at any information set h_j of voter $j \in \bar{N}_k(h_k)$ reached by this modified strategy of candidate k, any first-level prudent rationalizable strategy of the voter must ascribe voting for k. Since $|\bar{N}_k(h_k)| \geq m_k$, it implies that k wins the election, a contradiction. $\square$

A.4 Proof of Proposition 4

First level: The arguments about first-level rationalizable strategies for both candidates and voters are analogous to the proof of Proposition 3.

Second level: For all voters $j \in N$, $S_j^2 = S_j^1$ since $S_k^1 = S_k$ for $k \in \{a, b\}$. We claim that if s_k is a second-level prudent rationalizable strategy of candidate k, then for all $j \in \bar{N}_k(h_k)$ (where $\bar{N}_k(h_k)$ is defined as in (14) in the proof of Proposition 3) and I' with $I^k \subseteq I' \subseteq I(h_k)$ (where I^j is the subset of issues revealed to voter j by $s_k(h_k)$)

(i) $\| x^j - y' \|_{I'} \leq \| x^j - y'' \|_{I'}$ for any $y' \in (r^{I'}_{I^k})^{-1}((s_k(h_k))_{j,k})$ and $y'' \in (r^{I'}_{I^k})^{-1}((s_k(h_k))_{j,-k})$,

(ii) voter j strictly prefers $r^I_{I'}(y^k)$ over $r^I_{I'}(y^{-k})$.

Suppose to the contrary that there is a voter $j \in \bar{N}_k(h_k)$ and I' with $I^k \subseteq I' \subseteq I(h_k)$ such that $\| x^j - y' \|_{I'} > \| x^j - y'' \|_{I'}$ for some $y' \in (r^{I'}_{I^k})^{-1}((s_k(h_k))_{j,k})$ and $y'' \in (r^{I'}_{I^k})^{-1}((s_k(h_k))_{j,-k})$, or j does not strictly prefer $r^I_{I'}(y^k)$ over $r^I_{I'}(y^{-k})$.

With any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to strategies of candidate $-k$ that reveal information to voters in $\bar{N}_k(h_k)$ such that an information set h_j of voter j in $T^{I'}$ is reached at which for some $y' \in Y^k(h_j)$ and $y'' \in Y^{-k}(h_j)$, $\| x^j - y' \|_{I'} > \| x^j - y'' \|_{I'}$ or $\| x^j - r^I_{I'}(y^k) \|_{I'} \geq \| x^j - r^I_{I'}(y^{-k}) \|_{I'}$. Thus, with any belief system $\beta_k \in B_k^2$, candidate k at h_k must assign strict positive probability to first-level prudent rationalizable strategies of voter j and strategies of candidate $-k$ such that voter j votes for candidate $-k$. Yet, candidate k can capture the vote of voter j for sure and thus strictly improve her expected payoff at h_k given β_k by replacing $(s_k(h_k)_{j,k}, s_k(h_k)_{j,-k})$ with $\left\{\left(r^I_{I(h_k)}(y^k), r^I_{I(h_k)}(y^{-k})\right)\right\}$, because at any information set h_j of voter j reached by this modified strategy of candidate k, any first-level prudent rationalizable strategy of the voter must ascribe voting for k (this follows from the fact that $j \in \bar{N}_k(h_k)$), a contradiction. $\square$

B Further Details on Examples of Section 4

Below tables summarize the prudent rationalizable strategies for every player and every level for the example of Section 4.1.

Table 4: Prudent Rationalizable Strategies in Section 4.1

	voter		
info. set	$S_v^1 = S_v^2$		S_v^3
h_4	a	a	a
h_5	*not a*	*not a*	*not a*
h_6	a	*not a*	*not a*

	candidate				
info. set	S_a^1				$S_a^2 = S_a^3$
$\frac{1}{3}$	$\{\frac{1}{3}\}$	$\{\frac{1}{3}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}\}$
$\frac{4}{5}$	$\{\frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$

When nature selects $\frac{1}{3}$ as the policy point of the one candidate (i.e., candidate a), then with any second-level prudent rationalizable strategy of the candidate, the candidate discloses her policy point. When nature selects $\frac{4}{5}$, then with any second-level prudent rationalizable strategy of the candidate, the candidate's message contains both policy points $\{\frac{1}{3}, \frac{4}{5}\}$. Although in this case the message appears to be uninformative, the voter can correctly deduce the candidate's policy point as being $\frac{4}{5}$ because if it had be $\frac{1}{3}$, the candidate would have disclosed it. In this sense, information complete unravels (even without competition among candidates).

Similarly, below tables summarize the prudent rationalizable strategies for every player and every level for the example of Section 4.2.

Table 5: Prudent Rationalizable Strategies in Section 4.2

	voter		
info. set	$S_v^1 = S_v^2$		S_v^3
h_1	*not a*	*not a*	*not a*
h_2	*not a*	*not a*	*not a*
h_3	*not a*	*not a*	*not a*
h_4	a	a	a
h_5	*not a*	*not a*	*not a*
h_6	a	*not a*	*not a*

	candidate	
info. set	$S_a^1 = S_a^0$	S_a^2
$\left(\frac{1}{3}, \frac{8}{9}\right)$	$\{(\frac{1}{3}, \frac{8}{9})\}$, $\{(\frac{1}{3}, \frac{8}{9}), (\frac{4}{5}, \frac{2}{7})\}$, $\{\frac{1}{3}\}$, $\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}\}$
$\left(\frac{4}{5}, \frac{2}{7}\right)$	$\{(\frac{4}{5}, \frac{2}{7})\}$, $\{(\frac{1}{3}, \frac{8}{9}), (\frac{4}{5}, \frac{2}{7})\}$, $\{\frac{4}{5}\}$, $\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$
$\frac{1}{3}$	$\{\frac{1}{3}\}$, $\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}\}$
$\frac{4}{5}$	$\{\frac{4}{5}\}$, $\{\frac{1}{3}, \frac{4}{5}\}$	$\{\frac{1}{3}, \frac{4}{5}\}$

In any second-level prudent rationalizable strategy of the candidate, she is silent on the second dimension. Consequently, the voter is kept unaware of the second dimension and information does not unravel.

C Microtargeting and Preference Aggregation

Microtargeting circumvents well-known limits to aggregation of voters' preference under uncertainty. Voters have probabilistic beliefs about candidates' positions. A candidate may actually find it useful to keep a voter uncertain about his political position.[16] In such cases, the "preference" of society may not correspond to a von Neumann-Morgenstern utility function even though each voter's preference is captured by a von Neumann-Morgenstern utility. Our example inSection 5 can be recast in an example with three voters, in which one and the same candidate is elected if each of the possible political positions of the candidates were commonly known but the other candidate is elected under uncertainty over the candidate's political positions. In this sense, there is a role for "ambiguity" in electoral competition.[17]

To be precise, analogous to our example in Section 5, consider a set of three outcomes $\{x, y, z\}$ and lotteries over those outcomes. There are three voters, each having a preference relation on lotteries over outcomes as follows:

Voter	Preferences		
1	$(1,0,0) \succ_1 (0,1,0)$	$(1,0,0) \prec_1 (0,0,1)$	$(1,0,0) \prec_1 (0,\frac{1}{2},\frac{1}{2})$
2	$(1,0,0) \prec_2 (0,1,0)$	$(1,0,0) \succ_2 (0,0,1)$	$(1,0,0) \prec_2 (0,\frac{1}{2},\frac{1}{2})$
3	$(1,0,0) \succ_3 (0,1,0)$	$(1,0,0) \succ_3 (0,0,1)$	$(1,0,0) \succ_3 (0,\frac{1}{2},\frac{1}{2})$
Majority	$(1,0,0) \succ_M (0,1,0)$	$(1,0,0) \succ_M (0,0,1)$	$(1,0,0) \prec_M (0,\frac{1}{2},\frac{1}{2})$

Observe that each voter's preference is consistent with von Neumann-Morgenstern utility. The last line of the table shows the "social choice" using simple majority over pairwise comparisons. Clearly, this social choice is inconsistent with a von Neumann-Morgenstern utility of society.

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[16] Related problems have been noted in Zeckhauser (1969) and Shepsle (1970, 1972).

[17] This topic lead to an extensive literature with different approaches. See Downs (1957), Shepsle (1972), Alesina and Cukierman (1990), Aragonès and Neeman (2000), Aragonès and Postlewaite (2007), Glazer (1990), Jensen (2009), Laslier (2006), McKelvey (1980), Meirowitz (2005), and Page (1976).

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