CHAPTER 7

International Transportation in the Heckscher-Ohlin Model

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Trade models with local transportation firms carrying the country’s imports have appeared in the literature, but standard Heckscher-Ohlin (HO) models containing transport firms that compete in international transport markets do not seem to have been advanced. I present a model in this direction. The analysis follows the insight that world prices $p$ cannot clear world markets without international freight rates $f$, one rate for each good, simultaneously clearing the markets for international transportation. From the single-country, partial perspective, both $p$ and $f$ are determined in a worldwide market system and the comparative static system of a country’s activity equilibrium is a simple extension of the standard model, the transportation sectors being simply additional sectors. However, from the world perspective, the effect of changes in the world prices $p$ must be analyzed by requiring that freight rates clear the market for international transport. I derive the comparative static model for this world transportation equilibrium in a setting where transportation services are provided both ways by competing domestic and foreign transportation sectors in a world consisting of two countries.

Survey of the Literature

Kemp (1964, chap. 10) surveyed the literature on costs of transportation up to 1960 and observed that not much progress had been made in integrating costs of transportation into the competitive general equilibrium theory of international trade. The reasons for this are that the jointness in the supply and in the demand for transport poses difficult problems, and the familiar geometric techniques cannot be employed when we increase the number of sectors to include transportation sectors. He concluded his survey with his own contribution, which is based on Negishi (1960). This effort led to Hadley and Kemp (1966); Woodland (1968); and Negishi (1972); establishing the existence and Pareto optimality of a world competitive equilibrium under free
international trade with transportation sectors that jointly supply outward and
inward carriage. The above contributions are in line with the literature in
general (world) equilibrium, and, since the equilibrium is not assumed to be
locally unique, they do not lend themselves to tractable comparative statics.

The literature on transportation within the standard Heckscher-Ohlin
model, with or without joint outputs and with more than one factor of produc-
tion, was summarized by Casas (1983). In Samuelson (1954), transportation
costs are a constant fraction of the production costs, independent of the
country that performs the transportation service. In the context of a two-sector
growth model, Herberg (1970) is more general in assuming that transport
services are produced by factors (of transportation) according to a constant
returns to scale transportation function, distinct from the production functions
for goods. But he also assumed that, in a two-commodity world, each country
transports its own imports and not its exports, thereby attributing a charac-
teristic of a nontraded good to the transportation service. Another implication
of this assumption is that the transport medium would be empty on the
outward journey. Cassing (1978), apparently independently, considered a
model that is a special case of Herberg's model. Only the imported good of
the home country incurs freight charges and the other good is transported
costlessly. This case corresponds to the cost of a round-trip voyage being fully
charged to the incoming shipment and nothing to the outward shipment, as
when the former exceeds the latter in volume and freight rates are by volume.
In his paper, Cassing compares his transportation model to the standard model
with two traded goods and one nontraded good. Whereas there are many
similarities between the two models and the transportation service in his
model is like a nontraded good, there is one essential difference between the
two models. As I show subsequently, the transportation model has qualitative
comparative staticic implications not possible under the standard model with
nontraded goods.

In light of the distinctions I will introduce in the HO models, transpor-
tation is characterized as a localized service with costs determined only by local
conditions. In contrast, the international transportation costs developed here
depend on worldwide economic conditions.

The paper is divided as follows. First, I define domestic prices and world
prices and the relations between them. Next, I describe the technology of
transportation in a world of two countries. In the fourth section, I define a
country's activity equilibrium for given freight rates, \( f \), and world prices, \( p \).
This is followed, in the fifth section, by the definition and comparative statics
of the world's transportation equilibrium when the world transport market is
cleared. In the sixth section, I list a variety of possible transportation equi-
libria in the HO model with transportation sectors. I complete the exercise by
stating the conditions and comparative statics of the world's general equi-
librium when the commodity markets are also cleared. I write this for an
arbitrary number of factors and goods, but it is restricted to two countries. The
model is comparable to a model with globally nontraded goods, that is, the
transportation services. But since the prices of these nontraded goods, that is,
the freight rates, \( f \), are to be exactly equal to the differences between the
domestic price of the import-competing goods and the world price (when
there are no tariffs), the model displays deviations from the Stolper-
Samuelson property not possible under a standard model with nontraded
goods.

**World Prices and Domestic Prices**

Assume a world of two countries, \( a \) and \( b \). Each country has a country center.
All production and consumption activities take place and all factors and goods
are priced at the country center. For country \( a \) these prices are \( w^a \) for factors
and \( p^a \) for goods. The port also is located at the country center.

*International transportation* is the activity taking an exported good from
the exporting country to the importing country. The factors of transportation
carrying out the international service may be from either country. There is
competition between countries for transportation between ports, but we will
assume that if some country performs an international transport activity, it
employs inputs of that country only.

In general, the transportation cost, \( f_k \), per unit of good \( k \) may vary from
good to good, dependent on volume, weight, gas or liquid or solid state,
temperature, or other dimensions that determine a differential in the technol-
gy of transportation. But we will assume constant returns to scale in all
transportation as well as in all production activities.

The world price, \( p_k \), of the \( k \)th good is defined as the FOB price at the
port of the exporting country. The domestic price of an imported good is the
CIF price. With \( f_k \), the international freight per unit of good \( k \) from
the exporting to the importing country, the relation between local price \( p_k^a \)
and world price \( p_k \) is

\[
p_k = p_k^a, \quad \text{if country } a \text{ exports good } k. \quad (1a)
\]

\[
p_k^a = p_k + f_k, \quad \text{if country } a \text{ imports good } k, \quad (1b)
\]

\[
p_k < p_k < p_k + f_k, \quad \text{if good } k \text{ is nontraded by country } a. \quad (1c)
\]

where the first inequality is the condition for not exporting and the second for
not importing.

Both the world prices, \( p = (p_k) \), and the world freight rates, \( f = (f_k) \), for
international transportation are determined in the world model. Correspondingly, in the single-country comparative static exercise below, \( p \) and \( f \) are given parametrically. For simplicity of notation, I assume that all goods are traded between the two countries. Define the diagonal matrices \( F^a \) and \( F^b \), where

\[
(F^a)_{kk} = \begin{cases} 1, & \text{if } a \text{ imports good } k, \\ 0, & \text{if } a \text{ exports good } k, 
\end{cases}
\]

\[
(F^b)_{kk} = \begin{cases} 1, & \text{if } b \text{ imports good } k, \\ 0, & \text{if } b \text{ exports good } k. 
\end{cases}
\]

Observe that \( F^a + F^b = I \), the identity matrix. With these definitions, price relations (1a) and (1b) state that the local prices satisfy

\[
p^a = p + F^a[e + \tau^a] \quad \text{and} \quad p^b = p + F^b[e + [p] \tau^b],
\]

where \([p]\) is the diagonal matrix with the prices \( p_1, p_2, \ldots, p_n \) on the diagonal and \( \tau^a, \tau^b \) are the ad valorem tariff tables of country \( a \) and \( b \) respectively.

The Transportation Technology

Transportation services are produced by bundles of factors, similar to production activities. Specialized factors, including quantifiable components of the infrastructure, may enter in the transportation function. Their total available supplies are listed under the endowment vector of the transporting country. We assume constant returns to scale in transportation as well as in production.

Hadley and Kemp (1966), in their general world equilibrium model, define the international transportation medium as a carrier of a certain number of cubic yards, outward for exports and inward on the return trip for imports. Goods are measured by the cubic yard, and exported goods of all descriptions can be stowed together up to the full volume capacity. Likewise, imported goods of any composition can be carried together on the inward part of the trip up to full capacity. Freight rates \( f \) are proportional to volume. It is a very flexible mode in being able to simultaneously carry any basket of goods. On the other hand, imputation of the transportation costs to both legs of the trip depends crucially on whether the cubic yardage in one direction is exactly equal to the cubic yardage in the other direction.

In comparison, here I retain the assumption that there are modes of transportation with some degree of substitutability between goods that can be carried on a journey, but this degree is less than perfect. For each trip, the exact mix of goods will be chosen optimally, dependent on the individual freight rates \( f \) and the prices of the factors operating the vessel. Flexible modes of transportation of this type could be called container carriers.

For some vessels and goods, there is no substitutability at all and it is optimal to carry a single good on each leg or one good on one leg and none on the return trip. These are bulk carriers or tankers, possibly distinct vessels for each different bulk or liquid good. It is often cost efficient for bulk carriers for coal and tankers for crude oil to return empty, in which case the imputation of the total cost is to the importer.

We will assume that there exist many different modes of transportation. Each mode is characterized by the vector of goods that can be carried jointly on the inward and outward journeys and/or by the vector of factors of transportation that are required on that journey. Following Chipman (1966), when there are at least as many distinct optimally chosen modes of transportation as the number of goods in the trade vector, we can define a cone of diversification in transportation, that is, the set of trade vectors that can be carried in fully loaded vessels. As long as the trade vector falls in this cone, all goods will have imputed to them some fraction of the costs of transportation. This imputation will be a smooth function of the parameters and of the traffic in the comparative static exercises.

The constant returns to scale assumption is justified on the usual grounds that, within one sector, many transportation firms operate identical technologies on a scale that is small relative to the total transportation in that sector. It also seems historically correct that there have always existed many different modes of transportation. An important consequence is that the imputation of the total costs of transportation to both legs of the journey is a property of the equilibrium solution that is robust under varying compositions of the trade flows.

I summarize the assumptions on the technology of transportation in the following. The international transport firm in the \( j \)th transport sector of country \( a \) operates a "vessel" of the \( j \)th type. The \( j \)th vessel of country \( a \) is described by the quantity \( B^{a}e_{ij} \) of good \( i \) that it carries from \( a \) to \( b \) or from \( b \) to \( a \) on a round-trip and by the input \( A_{ij}f \) of factor \( i \) required for the trip. In general, all these coefficients are chosen optimally in a convex set and thus may depend on domestic factor prices \( w^{a} \) and on international freight rates \( f \).

With these definitions, if the number of trips by firms in transportation sector \( j \) of country \( a \) is \( (e^{a}) \), the vector of goods transported internationally will be \( B^{a}e^{a} \) and, similarly, the vector of goods transported by country \( b \) is \( B^{b}e^{b} \).

The Partial Trading Equilibrium of a Single Country

If \( x^{a} = x^{a}(f, w^{a}, p, v^{a}, \tau^{a}) \) is country \( a \)'s demand vector for goods, the demand for international transport is \( F^{a}(x^{a} - y^{a}) \), where \( y^{a} \) is the production vector. The demand \( x^{a} \) is homogeneous of degree zero in its first three arguments, \( f \).
Assume that production sector $i$ produces good $i$ and only good $i$. $A^g$ is the matrix of factor-input coefficients in the producing sectors. Country $a$ does not necessarily produce all goods. If $y^s$ is the subvector of positive components of $y^a$, we write $y^s = P^o y^o$. where $P^o$ is the production pattern matrix, with its $i$th row having a single nonzero entry, 1, if the $i$th good is produced, and 0 otherwise. With all goods produced, $P^o$ is the identity matrix.

Tariff revenues are $R = \tau^o |p|(x^a - y^a)$. If $R$ is redistributed to consumers, the equality between consumers' expenditures, $p^o x^a$, and national product plus tariff revenues, $p^o x^a + f' B^2 y^a + R$, implies

$$p'(x^a - y^a) + f'[F^o(x^a - y^a) - B^o y^a] = 0.$$ (3)

Trade in goods valued at international prices, $p$, plus net earnings in international transport services valued at international freight rates, $f$, are in balance. This is the current account condition.

In the interest of brevity, I restrict all definitions and conditions to strictly positive variables with all goods traded between $a$ and $b$.

**Activity Equilibrium of a Single Country**

Given international freight rates, $f$, world prices, $p$, inelastically supplied endowments, $w^o$, tariff rates, $\tau^o$, and optimally chosen smooth consumption bundles, $x^o(f, w^o, p, v^o, \tau^o)$,

1. the trading pattern matrix $F^o$ with $F^o(x^o - y^o) \geq 0$, the production pattern matrix $P^o$ with $y^o = P^o y^o$, the positive domestic factor prices, $w^o$, domestic production levels, $y^o$, and international transport trips, $x^o$
2. country $a$'s activity equilibrium if the factor markets are cleared and profits are maximized in all production and transportation activities, that is, if

$$A^g y^o + A^g x^o = y^o = 0, \quad y^o = P^o y^o;$$ (4.1)

$$w^o A^g = P^o y^o = 0, \quad \text{with} \quad P^o = (p + F^o f + |p| \tau^o)' P^o;$$ (4.2)

$$w^o A^g = f' B^o y^o = 0.$$ (4.3)

The equilibrium conditions have straightforward interpretations. Conditions (4.1) place the endowment vector inside the endowment cone of diversification to clear the factor markets. The other conditions are the profit maximizing conditions under constant returns to scale. Conditions (4.2) state the role of trade as the equalizer of prices of homogeneous domestic and foreign goods, after proper accounting for transport costs and tariffs. Nominal full employment GNP, as defined in Samuelson (1953), is a function $G^o(f, p^o, v^o)$ of international freight rates, $f$, domestic prices, $p^o$, and endowments, $v^o$.

**The Comparative Statics of Country $a$'s Activity Equilibrium**

The difference between the transportation model and the standard HO model can be seen from the comparative static relations. Letting $S^o$ be the matrix of factor substitutions in production and transportation firms, from equilibrium conditions (4.1) through (4.3), the comparative static system is

$$
\begin{pmatrix}
S^o & A^g & A^g \\
A^g & 0 & 0 \\
A^g & 0 & 0
\end{pmatrix}
\begin{pmatrix}
dw^o \\
dy^o \\
dz^o
\end{pmatrix}
= \begin{pmatrix}
P^o (F^o df + (I + |p| \tau^o) dp + |p| d\tau^o) \\ B^o df
\end{pmatrix}.

$$ (5)

where $|\tau^o|$ is the diagonal matrix with the tariff rates on the diagonal.

Country $a$'s activity equilibrium is unique only if the number of factors is not less than the number of goods produced plus the number of transportation sectors. With two factors, we could have a unique equilibrium with one production sector and one transport sector. When $A^o = (A^g, A^g)$ is square and nonsingular, we have

$$
\begin{pmatrix}
\frac{\partial w^o}{\partial v^o} & \frac{\partial w^o}{\partial \tau^o} \\
\frac{\partial y^o}{\partial v^o} & \frac{\partial y^o}{\partial \tau^o} \\
\frac{\partial z^o}{\partial v^o} & \frac{\partial z^o}{\partial \tau^o}
\end{pmatrix}
= \begin{pmatrix}
(A^o)^{-1} P^o & P^o (I + |p| \tau^o) \\
B^o & 0
\end{pmatrix}
\begin{pmatrix}
A^o \\
D^o
\end{pmatrix},

$$ (6)

with $D^o = -(A^o)^{-1} S^o (A^o)^{-1}$, a positive, semidefinite matrix.
The assumptions made below, under which the world transportation equilibrium of country $a$ and country $b$ is unique, will imply that the country activity equilibria may not be unique for either or for both countries.

**Example.** In a $2 \times 2$ model, if country $a$ exports good 1 but does not produce good 2, we have $P^a = (1\ 0)$, $F^a = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. Assume there is one transportation sector carrying positive amounts of both goods 1 and 2 on a round-trip and also assume factor 1 is relatively intensive in the production sector. Then we have

$$\text{sign} \begin{pmatrix} \frac{\partial w^a}{\partial y^a} & \frac{\partial w^a}{\partial z^a} \\ \frac{\partial w^a}{\partial z^a} & \frac{\partial w^a}{\partial (f', p', \tau^a)} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (- - + 0 + 0) \\ (+ + - 0 - 0) \end{pmatrix} \begin{pmatrix} + - \\ - + \end{pmatrix}$$

implying an increase in either freight rate $f_1$ or $f_2$ increases the factor price $w^a$ and the number of transportation trips $z^a$, while lowering $w^a$ and the production of good 1. A subsidy on exported good 1 raises the production and lowers the transport activity, whereas a tariff on imported good 2 has no effect, since there is no domestic import competitor.

**World Transportation Equilibrium**

World prices, $p$, and international freight rates, $f$, are not independent variables. Clearly, the goods markets cannot be in equilibrium without the transport markets being in equilibrium. The equilibrium values of $f$ and $p$ are determined in the simultaneous process that clears both the commodity world markets and the markets for international transport.

As in the models without transportation, we can proceed in a stepwise fashion, conceptually clearing first the markets for transport services, assuming that general equilibrium world prices $p$ are given. I will define the resulting model as the set of conditions defining the world's transportation equilibrium with the world consisting of the two countries, $a$ and $b$.

**The World's Transportation Equilibrium**

Given world prices, $p$, inelastically supplied endowments, $v^a$ and $v^b$, tariff rates, $\tau^a$ and $\tau^b$, and optimally chosen smooth consumption bundles $x^a = x^a(f, w^a, p, v^a, \tau^a)$ and $x^b = x^b(f, w^b, p, v^b, \tau^b)$,

1. The trading pattern matrix $F^a$ with $F^a(x^a - v^a) \geq 0$, the production pattern matrix $P^a$ with $y^a = P^a y^a$, the positive domestic factor prices $w^a$, domestic production levels $y^a$, and international transportation trips $z^a$.
2. The trading pattern matrix $F^b$ with $F^b(x^b - v^b) \geq 0$, the production pattern matrix $P^b$ with $y^b = P^b y^b$, the positive domestic factor prices $w^b$, domestic production levels $y^b$, and international transportation trips $z^b$.
3. The international freight rates, $f$.

are the world's transportation equilibrium if they are country $a$ and country $b$ activity equilibria and the international transportation markets are cleared, that is, if

$$w^a A^a f^a B^a = 0. \quad (7a)$$

$$w^b A^b f^b B^b = 0. \quad (7b)$$

$$F^a(x^a - P^a y^a) - B^a z^a + F^b(x^b - P^b y^b) - B^b z^b = 0. \quad (7ab)$$

$$A^a y^a + A^a z^a - v^a = 0, \quad \text{with} \quad y^a = P^a y^a. \quad (8a)$$

$$A^b y^b + A^b z^b - v^b = 0, \quad \text{with} \quad y^b = P^b y^b. \quad (8b)$$

$$w^a A^a - p^a P^a = 0, \quad \text{with} \quad p^a P^a = (p + F^a f + |p| \tau^a)' P^a. \quad (9a)$$

$$w^b A^b - p^b P^b = 0, \quad \text{with} \quad p^b P^b = (p + F^b f + |p| \tau^b)' P^b. \quad (9b)$$

I have written the equations in the order of the profit maximizing conditions in transport activities (7a) and (7b), the world transport market clearance conditions (7ab), the local factor market clearance conditions (8a) and (8b), and the profit maximizing conditions in production (9a) and (9b).
Define the global variables and matrices as

\[ z = \begin{pmatrix} z^a \\ z^b \end{pmatrix}, \quad w = \begin{pmatrix} w^a \\ w^b \end{pmatrix}, \quad y_p = \begin{pmatrix} y^a_p \\ y^b_p \end{pmatrix}, \quad x = \begin{pmatrix} x^a \\ x^b \end{pmatrix}, \]

\[ v = \begin{pmatrix} v^a \\ v^b \end{pmatrix}, \quad \tau = \begin{pmatrix} \tau^a \\ \tau^b \end{pmatrix}, \]

\[ B = \begin{pmatrix} B^a & B^b \end{pmatrix}, \quad F = \begin{pmatrix} F^a & F^b \end{pmatrix}, \quad S = \begin{pmatrix} S^a & 0 \\ 0 & S^b \end{pmatrix}, \quad \text{and} \]

\[ \Lambda_p = \begin{pmatrix} \Lambda^a_p & 0 \\ 0 & \Lambda^b_p \end{pmatrix}, \quad \Lambda_F = \begin{pmatrix} \Lambda^a_F & 0 \\ 0 & \Lambda^b_F \end{pmatrix}, \quad P = \begin{pmatrix} P^a & 0 \\ 0 & P^b \end{pmatrix}. \]

In terms of these world variables, the competitive profit-maximizing conditions (7a) and (7b) are \( w' A_F = f' B \), and the transport market clearance conditions (7ab) are \( F(x - P y_p) = Bz \). That is, the world transport demand vector \( F(x - P y_p) \) falls in the cone of diversification spanned by the transport sectors of both countries together.

Below, I restrict the analysis to those world transportation equilibria where

1. \( B \) is square and nonsingular, that is, I assume there are as many active transport modes in the world as there are goods, and the optimum bundles carried on the vessels of country \( a \), if any, are of a different composition than those carried on vessels of country \( b \), and
2. the number of factors of production in country \( a \) plus the number of factors in country \( b \) is not less than the number of goods produced in country \( a \) plus the number of goods produced in country \( b \).

The Comparative Statics of the World Transportation Equilibrium

Totally differentiating the equilibrium relations, we have the following comparative static system.

\[
\begin{pmatrix}
0 & -B' & A_F' & 0 \\
-B & F & \frac{\partial x}{\partial f'} & F \frac{\partial x}{\partial w'} & -FP \\
A_F & 0 & S & A_p \\
0 & -P'F' & A_F' & 0
\end{pmatrix}
\begin{pmatrix}
dz \\
df \\
dw \\
dy_p
\end{pmatrix}
= \begin{pmatrix}
0 \\
-F \frac{\partial x}{\partial p'} dp - F \frac{\partial x}{\partial \omega'} d\omega \\
P' \left((l + |\tau^a|) dp + |p| d\tau^a\right) \\
P' \left((l + |\tau^b|) dp + |p| d\tau^b\right)
\end{pmatrix},
\]

where \( \omega' = (v', \tau^a', \tau^b') \). Observe that with the demand \( x \) homogeneous of degree zero in \( f, p, w \) and the marginal costs homogeneous of degree one in \( w \), equation (11) is satisfied by the equilibrium values \( (z, f, w, y_p) \) for given \( p \) and \( v \), before the changes in \( p \) and \( \omega \).

The comparative static system (11), for the two countries together, and the nontraded goods model for a single country share a common feature. The markets for transportation in (11) and the markets for the nontraded goods are cleared locally. For a comparison, Woodland (1982, 227) uses comparable notation for the nontraded goods model. However, there is also an essential difference between the two models. In the nontraded goods models there is a null matrix in the (2,4) and (4,2) submatrices at (11), instead of the matrices \(-FP\) and \(-P'F\), respectively, in the transportation model here. Rows 4 in the system above state that the transportation costs \( f \) are equal to the differences between the domestic production costs and the world prices of imported goods that are also produced at home. This relation does not exist in the standard nontraded goods model, where the prices of the nontraded goods are not so restricted. It has important qualitative differences, as illustrated subsequently.

I first state the solution of equation (11). With \( B \) nonsingular, eliminate the top two rows and the variables \( dz \) and \( df \). In terms of an assumed change \( dp \) in world prices and for a given change \( d\omega \) in the endowments and tariffs, we have, recursively,

\[
\begin{pmatrix}
d\omega \\
dy_p
\end{pmatrix} = \begin{pmatrix} \bar{S} & \bar{A} \\ \bar{A}' & 0 \end{pmatrix}^{-1} \times
\begin{pmatrix}
dz \\
df
\end{pmatrix},
\]

\[
\begin{pmatrix}
dv \\
-dA_F B^{-1} F \left( \frac{\partial x}{\partial p'} dp + \frac{\partial x}{\partial \omega'} d\omega \right) \\
P' \left((l + |\tau^a|) dp + |p| d\tau^a\right)
\end{pmatrix}, \quad \text{and}
\]

\[
dz = B^{-1} F \Delta e, \quad df = B'^{-1} A^*_p dw. \quad (12b)
\]
where we assumed that the inverse matrix exists, and

\[ \dot{S} = S + \dot{A}_F B^{-1} F \left( \frac{\partial x}{\partial j} B^{-1} A_F' + \frac{\partial x}{\partial w'} \right), \]  
(13a)

\[ \dot{A} = A_F - A_F B^{-1} F P, \]  
(13b)

\[ \Delta e = \left[ \frac{\partial x}{\partial p} dp + \frac{\partial x}{\partial \omega} d\omega + \left( \frac{\partial x}{\partial j} B^{-1} A_F' + \frac{\partial x}{\partial w'} \right) dw - P dy_p \right]. \]  
(13c)

The relations express the world’s transportation equilibrium conditions and differ from the standard conditions without transportation only because the clearance of the world transport markets is maintained at all times.

1. With changing factor prices, the matrix \( \dot{S} \) of equation (13a) is the usual substitution matrix \( S \) in production and transportation sectors plus the rates of change in factor demand,

\[ A_F B^{-1} F \left( \frac{\partial x}{\partial j} B^{-1} A_F' + \frac{\partial x}{\partial w'} \right), \]

needed for the provision of the rates of change.

\[ F \left( \frac{\partial x}{\partial j} B^{-1} A_F' + \frac{\partial x}{\partial w'} \right), \]

in transport demand.

2. A column of the factor-input coefficient matrix \( \dot{A} \) of equation (13b) is equal to the factor-input coefficient matrix \( A_F \) in production if the corresponding good is exported by the country. If the good is imported and also produced in the country, we subtract the column of the factor-input coefficients needed to transport the good. This construct is to measure the net change in the demand for factors if the production levels, \( y_p \), change and we keep the world transport markets cleared. The net change in the demand for factors needed to produce locally one unit of output of an imported good is equal to the difference between the factors needed in production and the factors saved in transportation.

3. With these adjustments for keeping equilibrium in the transport market, system (12a) is the comparative static system linking domestic factor prices and output levels to international prices, endowments, and tariff tables. If \( p \) changes, the change in transport demand before the change in production \( y_p \) is

\[ \frac{\partial x}{\partial p}, dp, \]

for which

\[ B^{-1} F \frac{\partial x}{\partial p}, dp \]

trips are needed. Therefore a change in \( p \) is equivalent to a change of

\[ -A_F B^{-1} F \frac{\partial x}{\partial p}, dp \]

in the endowments.

4. The vector \( \Delta e \) of equation (13c) lists the changes in the trade vectors of country \( a \) and \( b \). The corresponding changes in the demand for transport are \( F \Delta e \), which, under equilibrium conditions, are equal to the changes in the supply of transport services \( Bd \), as stated at equation (12b).

I summarize this section as follows. With \( B \) nonsingular and the bordered substitution matrix \( \dot{S} \) nonsingular, the fundamental system (12a) relating domestic factor prices and output levels to endowments, world prices \( p \) and tariff tables is of the same general structure as the single-country comparative systems in HO models without transportation. But here, all variables pertain to two countries and there are possible interactions. If \( \dot{A} \) is nonsingular, implying the number of factors is equal to the number of production sectors in the two countries combined, we have the relations

\[
\left( \begin{array}{c}
\frac{\partial w}{\partial \nu'} \\
\frac{\partial w}{\partial \tau'} \\
\frac{\partial y_p}{\partial \nu'} \\
\frac{\partial y_p}{\partial \tau'}
\end{array} \right) =
\left( \begin{array}{cccc}
0 & (\dot{A}')^{-1} P' & (I + \tau)^{1/2} & [p] \\
(\dot{A})^{-1} (I - A_F B^{-1} F \frac{\partial x}{\partial \nu'}) & (I + \tau^b) & 0 & [p]
\end{array} \right)
\]
Examples of World Trading Equilibria

We construct four different trading equilibria, restricting all examples to HO models with two goods, two factors, and our two countries, a and b. Country a imports good 2 and country b imports good 1, so that

\[ F^a = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad F^b = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \]

Example 1: (Herberg-Cassing) HO Countries Carry Their Imports

Assume each country produces both goods and transports its own imports. With country a importing good 2, choose the units of the goods such that bulk carriers of a country carry one unit of the imported good, that is

\[ P^a = P^b = I_2, \quad B^a = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad B^b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \]

The freight rates are \( f_2 = w^a_A \gamma, f_1 = w^b_A \delta, \) and are determined by the factor prices of the carrying country. The slopes of demand are

\[ \frac{\partial x^a_2}{\partial f_2} = 0, \quad \frac{\partial x^b_1}{\partial f_2} = 0 \]

since \( f_1 \) and \( f_2 \) do not determine the prices \( p^a \) and \( p^b \) respectively. The comparative static relations (14) hold with

\[ \mathbf{\hat{S}} = \mathbf{S} + \mathbf{A}_F \begin{pmatrix} \frac{\partial x^a_2}{\partial f_2} & 0 \\ 0 & \frac{\partial x^b_1}{\partial f_1} \end{pmatrix} A_F \quad \text{and} \quad \mathbf{\hat{A}} = \begin{pmatrix} \mathbf{A}_F - (0 & \mathbf{A}_F) \\ 0 & \mathbf{A}_F - (\mathbf{A}_F \ 0) \end{pmatrix}. \tag{15} \]

In the Herberg-Cassing model, both \( \mathbf{\hat{S}} \) and \( \mathbf{\hat{A}} \) are block-diagonal matrices. The fundamental system (12a) consists of two separable single-country systems, with each country producing the transport service for the imported good as a nontraded good. If, in both countries, factor 1 is used relatively intensively in production sector 1 and the factor inputs in transportation are relatively small, that is, if the factor 2 input in production exceeds that in transportation, and if

\[ (\mathbf{A}_F)^{-1} \mathbf{A}_F < (1, 1), \quad \text{and} \quad (\mathbf{A}_F)^{-1} \mathbf{A}_F < (1, 1). \tag{16} \]

both matrices in \( \mathbf{\hat{A}} \) have inverses with positive diagonal and negative off-diagonal elements. The traditional Stolper-Samuelson relations hold under those conditions.

On the other hand, \( \mathbf{\hat{A}} \) could have a negative element when a factor-input coefficient in production is less than the factor-input coefficient in transportation of the imported good. Then system (14) implies that an increase in an international price could increase both nominal factor prices, as shown in figure 1. The explanation is that an increase in the world price, \( p_1 \), raises \( w^a_f \) (and lowers \( w^b_f \)), which raises the transport cost \( f_2 \), which raises the domestic price \( p_2 \), which raises \( w^b_f \) (and lowers \( w^a_f \)). With good 2 the imported good, \( p_2 = p^a_f - f_2 \), the local one-dollar revenue isoquant in sector 2 consists of the cost minimizing factor bundles in production minus those in transportation and is obtained geometrically by subtracting the points along the two-unit isoquants that have equal marginal rates of substitution. This revenue isoquant must have a nonincreasing marginal rate of substitution along its path, but may have a kink. If the cost of the inputs in transportation is greater than the cost of domestic production, the good is nontraded. A simultaneous increase in both factor prices is not possible under a standard HO model with a nontraded good. It is possible if traded and nontraded goods are produced jointly in some sectors.
Example 2: HO Countries Carry Their Exports

If each country transports its exported good, we have

\[ P^a = P^b = I_2, \quad B^a = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad B^b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \]

The freight rates \( f_1 = \omega^a A^a \) and \( f_2 = \omega^b A^b \) are determined by the factor prices in the exporting country. The world substitution matrix \( \hat{S} \) is the same as in the Herberg-Cassing model, but the world factor-input coefficient matrix,

\[ \hat{A} = \begin{pmatrix} A^a & -A^a & 0 \\ 0 & A^b & \end{pmatrix}, \tag{17} \]

corresponding to the notion that increasing the production of good 2 in country \( a \) requires inputs in production there, but saves inputs otherwise needed for transportation in country \( b \).

Let \( \alpha = (A^a)^{-1} A^a \) and \( \beta = (A^b)^{-1} A^b \). Assume factor 1 is used intensively in production sector 1 relative to production sector 2. We have:

1. \( \alpha_1 > 0, \alpha_2 > 0 \), if and only if in country \( a \) factor 1 is also used intensively in production sector 1 relative to the transportation sector and in the transportation sector relative to production sector 2.
2. \( \alpha_1 > 0, \alpha_2 < 0 \), if and only if in country \( a \) factor 1 is used intensively in the transportation sector relative to both production sectors.
3. \( \alpha_1 < 0, \alpha_2 > 0 \), if and only if in country \( a \) factor 1 is used intensively in both production sectors relative to the transportation sector.

Identical conditions in country \( b \) hold for the signs of the components of \( \beta \). Define the factor-input coefficient matrices as

\[ Z^a = (A^a)^1 (A^a)^2 - \beta_1 A^a, \]

\[ Z^b = (A^b)^1 - \alpha_2 A^b \]

in terms of which we can write

\[ (\hat{A})^{-1} = \begin{pmatrix} (Z^a)^{-1} & (\alpha \alpha)(Z^b)^{-1} \\ 0 & (Z^b)^{-1} \end{pmatrix}. \tag{18} \]

An interesting sign pattern is obtained if, in country \( a \), factor 1 is intensive in transportation relative to both production sectors and, in country \( b \), factor 2 is intensive in transportation relative to both production sectors. Under these conditions, all four \( 2 \times 2 \) submatrices of \( (\hat{A})^{-1} \) have positive diagonal and negative off-diagonal elements and the Stolper-Samuelson property is preserved.

In general, a great variety of signs is possible. For given world prices, \( p \), factor prices, \( \omega^m \), depend on the technology of both countries and transportation costs are not determined by the domestic conditions in one country alone (as for a nontraded good). We verify that if the factor inputs in transportation are small relative to those in production, the production characteristics of the model are those of two separable production systems.

Example 3: HO Countries Carry Both Imports and Exports

Within the standard \( 2 \times 2 \times 2 \) HO model, assume each country produces and transports both goods. We have

\[ P^a = P^b = I_2, \quad B = (B^a \quad B^b) \]

is nonsingular.

With \( (\omega^a A^a, \omega^a A^a) = f^i B \), the international freight rates, \( f_i \) are determined by cost conditions in both countries. The comparative static relations (14) hold with

\[ \hat{S} = S + A^a B^{-1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial x}{\partial w} B^{-1} A^a + \frac{\partial x}{\partial w^m} \end{pmatrix}, \quad \text{and} \tag{19a} \]

\[ \hat{A} = \begin{pmatrix} A^a + (0 - b^{12} A^a) \frac{\partial x}{\partial w} & (-b^{11} A^a & 0) \\ 0 - b^{22} A^a & A^b + (-b^{21} A^a & 0) \end{pmatrix}. \tag{19b} \]

where \( B^{-1} = (b^{ij}) \). The matrix \( (\hat{A})^{-1} \) is of the same form as matrix (18) after replacing, in matrix (17), the second column of \( A^a \) and the first column of \( A^b \) by their expression in matrix (19). Again, we subtract transportation-input coefficients from the production coefficients for imported goods. Definite signs can be obtained under the conditions of the preceding example.

Another possible equilibrium is when country \( a \) carries all the goods. In this case, the matrix \( \hat{A} \) is upper block-triangular. We consider one more special case.

Example 4: Countries Specialized in Production Carry Both Imports and Exports

With country \( a \) producing good 1 and country \( b \) producing good 2, assume that both countries carry both goods. We have
\[
P^a = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad P^b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad B = (B^a \quad B^b) \text{ is nonsingular},
\]
with \(w'\Lambda_f = f'B\). The comparative static relations (12a) hold with
\[
\dot{S} = S + A_f \dot{B}^{-1}(0 \quad 0 \quad 1 \quad 0) \left( \frac{\partial x}{\partial f} B^{-1} \dot{A}_f' + \frac{\partial x}{\partial w} \right), \quad \text{(20a)}
\]
\[
\dot{A} = A_p \begin{pmatrix} A_f' & 0 \\ 0 & A_f' \end{pmatrix}, \quad \text{(20b)}
\]
since there are no active import competing industries in either country. The factor-input coefficient matrix \(A\) is a \(4 \times 2\) matrix, yet the equilibrium is locally isolated if the bordered substitution matrix \(\dot{S}\) at system (12a) is nonsingular. This is comparable to the single country comparative static model when the country is specialized, that is, the number of factors exceeds the number of goods produced. This is an example where the country activity equilibrium and the corresponding comparative static system (5) are easily analyzed, there being two sectors in transportation and production activities combined. However, in the world’s transportation equilibrium, the model and analysis is one of specialization. Coefficients of the substitution matrix \(\dot{S}\) enter into the relationship between international values, \(p\), and national values, \(w\).

The World's General Equilibrium

Given inelastically supplied world endowments, \(v\), tariff rates, \(\tau\), and optimally chosen smooth consumption bundles, \(x = x(f, w, p, v, \tau)\),

1. the world trading pattern matrix \(F\) with \(F(x - y) > 0\), the production pattern matrix \(P\) with \(y = Py\), the positive domestic factor prices \(w\), domestic production levels \(y_p\), and international transportation trips \(z\), and

2. the world goods prices, \(p\).

are the world’s general equilibrium if they are the world’s transportation equilibrium and the world goods markets are cleared, that is, if conditions (7), (8), and (9) hold, and

\[
e^a + e^b = 0, \quad \text{(21)}
\]
where \(e^a = x^a - y^a\) and \(e^b = x^b - y^b\).

Totally differentiating equilibrium conditions (21), we have the comparative static system

\[
\frac{\partial(x^a + x^b)}{\partial f} df + \frac{\partial(x^a + x^b)}{\partial w} dw + \frac{\partial(x^a + x^b)}{\partial p} dp
\]
\[- (P^a dy_p + P^b dy_p) = - \frac{\partial(x^a + x^b)}{\partial \omega} d\omega, \quad \text{(22)}
\]

where \(df, dw, \) and \(dy_p\) are the changes in the world’s transportation equilibrium quantities, \(f(p, v, \tau), w(p, v, \tau), \) and \(y_p(p, v, \tau),\) stated at (12a) and (12b). Substituting these quantities into equation (22), we have the equilibrium world price changes, \(dp\), solve

\[
\frac{\partial(e^a + e^b)}{\partial p} dp = - \frac{\partial(e^a + e^b)}{\partial \nu} dy - \frac{\partial(e^a + e^b)}{\partial \tau} d\tau, \quad \text{(23)}
\]

where

\[
e^a(p, v, \tau) = \ddot{x}^a(p, v, \tau) - y^a(p, v, \tau),
\]
\[
e^b(p, v, \tau) = \ddot{x}^b(p, v, \tau) - y^b(p, v, \tau),
\]

with

\[
\ddot{x}(p, v, \tau) = x(B^{\text{tr}} A_f' w(p, v, \tau), w(p, v, \tau), p, v, \tau) \quad \text{and}
\]
\[
y(p, v, \tau) = Py(p, v, \tau).
\]

the vector of demand and production in the world’s transportation equilibrium of both countries.

Under the assumptions stated above and with the price slopes of \(e\) in equation (23) having rank \(n - 1\), we conclude that, with \(p_a = 1\), the world’s general equilibrium prices are a function \(p = p(v, \tau)\) with derivatives determined by equation (23). These are standard conclusions of the models without transportation. The difference is that, here, the equilibrium factor prices, \(w(p, v, \tau),\) and output levels, \(y(p, v, \tau),\) implicit in \(e\) are the world’s transportation equilibrium variables and not the individual country-activity equilibrium variables. Likewise, full employment GNP \(G^*(f, v^*, \nu^*)\) of country \(a\) is a function \(G^a(v, \nu)\) under general equilibrium.

Remark. With two goods, the world’s transportation equilibrium trade flows \(\ddot{e}^* = x^*(p, v^*, \tau^*), y^*(p, v^*, \tau^*)\) and \(\ddot{e}^b = x^b(p, v^*, \tau^*) - y^b(p, v^*, \tau^*)\) as functions of \(p\) are the offer curves. Since these flows contain features of the nontraded goods model, the offer curves may bend back.
on themselves and the analysis based on geometric regularity of these curves must correspond to special implicit assumptions.

Conclusions

As analyzed in Hadley and Kemp (1966), transportation is a complex general equilibrium problem. Because of this, it is not surprising that not more effort has been spent on it. This paper makes a contribution by placing internationally competitive transportation sectors in the heart of the standard Heckscher-Ohlin model. I have been careful in stating as many of the results as possible for the model with an arbitrary number of factors and goods, but I restricted the model to two countries. The reason is simply that the optimal traffic flows become very complicated when there are more than two countries, and my analysis does not extend to multiple circuits.

I analyzed the implications of transport services being supplied by internationally competing transport firms. In general, freight rates are determined by economic costs both here and abroad. I showed how my model contains the Herberg-Cassing model as a special case. Whether one country performs the shipping operations or both, is difficult to predict, since it is an equilibrium condition (as stated above), and there may be multiple equilibria. Naturally, in equilibrium, countries pursue their comparative advantage as defined and analyzed, for example, in Deardorff (1979) and (1980).3

The interpretation of the specifics of the model is left to the reader. It seems compelling that there are transportation-specific factors in the economy besides land and labor. The economic historian or development economist in us surely knows the importance of transportation in trade between countries.

NOTES

This paper states my current view on the topic of transportation in trade models and I dedicate it to my teacher, mentor, colleague, and friend Murray C. Kemp. It contains some parts of a working paper presented at Groningen, University College Dublin, Monash, and Newcastle. An earlier draft was also presented at a seminar in Australia, in the Australian winter of 1986, organized by Alan Woodland and Murray Kemp, to whom I owe some useful suggestions and conceptual improvements. Robert Conlon helped me with some of the academic descriptions of how transportation operates in practice, and Alfonso De Jonghe, at a visit to the port of Antwerp, gave me an introduction to the commercial organization of shipping and to the stacks of volumes on agreements about tariffs by the Gulf European Freight Association. (U.S. Gulf of Mexico Ports to Ports in Europe). All remaining misconceptions, errors, or irrelevancies in the paper are due to my own stubborn pursuit of this topic.

1. The effect of transportation costs in a Ricardo-Torrens model with a single input and multiple goods was understood by Harberler (1936). Later, this was developed in more detail by Dornbusch, Fisher, and Samuelson (1977) and by Houthakker (1976). In these models, tariffs and transportation rates have comparable implications, and, because of their simplicity, the Ricardo-Torrens models often serve as a basis for empirical and policy-oriented studies, such as Conlon (1985).

2. In reality, besides volume, such considerations as weight are involved in container carriers, and different container carriers are built to accommodate special demands, such as refrigeration or temperature stability.

3. Besides the problem of multiplicity, I also leave open the question of to what extent Deardorff's comparative advantage concept is applicable here. In my model, the production with transportation value isoquants can be entirely different from the production isoquants before trade, as illustrated in fig. 1, and offer curves may bend backward.

REFERENCES


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Part 2
Tariffs, Quotas, and Trade Policies