The Hurwicz optimism index

The MaxiMin criterion focuses on the worst outcome for each act, while the MaxiMax criterion focuses on the best outcome. Hurwicz suggested a more general criterion that takes into account both the worst and the best outcome. Let $\alpha$ be the weight attached to the worst outcome and $(1 - \alpha)$ the weight attached to the best outcome, with $0 \leq \alpha \leq 1$. Then we can call $\alpha$ an index of pessimism and $(1 - \alpha)$ an index of optimism. For example, consider the following decision problem, where utilities are von Neumann-Morgenstern utilities:

<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>$a_2$</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$a_3$</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Somebody with index $\alpha$ evaluates act $a_1$ by taking the weighted average

$$H_\alpha(a_1) = 0 \alpha + 10(1 - \alpha) = 10 - 10\alpha,$$

evaluates act $a_2$ by taking the weighted average

$$H_\alpha(a_2) = 2\alpha + 8(1 - \alpha) = 8 - 6\alpha$$

and assigns the value 4 to $a_3$ ($H_\alpha(a_3) = 4\alpha + 4(1 - \alpha) = 4$) and then chooses the act that gives the highest value. For example, if $\alpha = \frac{3}{5}$ then

$$H_\frac{3}{5}(a_1) = 10 - 10 \frac{3}{5} = 4, \quad H_\frac{3}{5}(a_2) = 8 - 6 \frac{3}{5} = \frac{40 - 18}{5} = \frac{22}{5} = 4.4 \quad \text{and} \quad H_\frac{3}{5}(a_3) = 4$$

and thus chooses $a_2$. Note that $\alpha = 0$ corresponds to extreme optimism and thus to the MaxiMax criterion and $\alpha = 1$ corresponds to extreme pessimism and corresponds to the MaxiMin criterion. Thus the Hurwicz criterion is a generalization that incorporates both as special cases.