1. (a) Alternative $a$ gets $3 + 2 + 1 = 6$ points, $b$ gets $2 + 3 + 3 + 2 = 7$ points and $c$ gets $1 + 1 + 3 = 5$ points. Thus the social ranking is
\[ b \\
 a \\
 c \]

(b) Each alternative gets 6 points. Thus the social ranking is: society is indifferent among all three alternatives ($a \sim_S b \sim_S c$).

(c) The Borda count satisfies Rationality, Non-dictatorship and Unanimity.

(d) The Borda count violates Full Domain (also called Freedom of expression) because it does not allow expression of indifference between two or more alternatives. The Borda count also violates Independence of Irrelevant Alternatives. To see this, consider the profile of preferences of part (b) which yields, in particular, $a \sim_S b$. The social ranking of $a$ versus $b$ does not only depend on how the individuals rank $a$ versus $b$. For example, if individual 3 had reported the ranking
\[ b \\
 a \\
 c \]
(which has the same ranking of $a$ versus $b$), then the social ranking would have been
\[ a \\
 b \\
 c \]
so that, in particular, $a \succ_S b$.

2. (a) With the Borda count and sincere voting $x$ gets 22 points, $a$ gets 17, $b$ gets 16 and $c$ gets 15. Thus the social ranking is
\[ x \\
 a \\
 b \\
 c \]

If, after the election, $x$ drops out then the next best candidate will be chosen, that is candidate $a$.

(b) Eliminating $x$ from the above profile we have:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
</tr>
<tr>
<td>$b$</td>
<td>$c$</td>
<td>$a$</td>
<td>$b$</td>
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</tr>
<tr>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>$a$</td>
</tr>
</tbody>
</table>

and using the Borda count with this profile we have that $a$ gets 13 points, $b$ gets 14 and $c$ gets 15. Thus the social ranking becomes
that is, a complete reversal of the previous one! The winner is now \( c \), who was the lowest ranked candidate before!

3. (a)

(b) Take the Condorcet paradox and suppose that the agenda is to choose first between A and B and then the winner is put up against C.

<table>
<thead>
<tr>
<th>1’s preferences</th>
<th>2’s preferences</th>
<th>3’s preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

If everybody votes sincerely, then in the vote between A and B, the winner is A and in the final vote between A and C the winner is C, which is 1’s worst candidate. If 1 voted as if his true preferences were BAC, then he will vote for B in the first round, so that (if everybody else votes sincerely) the winner is B and then in the final vote between B and C, the winner is B, whom 1 prefers to C.