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Consider a simultaneous two-player second-price auction concerning a single, indivisible good. The game-frame is as follows: \( S_1 = S_2 = B \) where \( B = \{ p_1, p_2, \ldots, p_m \} \) is a finite set of positive numbers with \( p_k < p_{k+1} \) for every \( k \in \{1, \ldots, m-1\} \), the set of outcomes is the set of pairs \((i, p)\) where \( i \in \{1, 2\} \) is the winner of the auction and \( p \in B \) is the price that the winner has to pay, and the outcome function is as follows (\( b_i \) denotes the bid of Player \( i \)):

\[
f(b_1, b_2) = \begin{cases} (1, b_2) & \text{if } b_1 \geq b_2 \\ (2, b_1) & \text{if } b_1 < b_2 \end{cases}
\]

Let \( v_i \) be the value of the object to Player \( i \) (that is, Player \( i \) views getting the object as equivalent to getting $ v_i $). We shall consider various kinds of preferences. We state them in terms of Player 1, but the same definitions apply to Player 2. What is written in the box below applies to all three preferences (this is the “selfish” component; recall that \( x \succ_1 y \) means that Player 1 prefers outcome \( x \) to outcome \( y \)):

- **Player 1 is selfish and uncaring** if, in addition, her preferences are as follows (recall that \( x \sim_1 y \) means that Player 1 is indifferent between outcome \( x \) and outcome \( y \))
  - for every \( p \) and \( p', (2, p) \sim_1 (2, p') \);
  - for every \( p, (2, p) \sim_1 (1, v_1) \); and everything that follows from the above by transitivity.

(a) [5 points] Suppose that Player 1 is selfish and uncaring and \( v_1 = 80 \). How does she rank the following three outcomes: \((1, 72), (2, 60), (2, 35)\)? [Write your answer below]

- **Player 1 is selfish and benevolent** if, in addition, her preferences are as follows:
  - for every \( p \) and \( p', (2, p) \succ_1 (2, p') \) if and only if \( p < p' \);
  - \((2, p_m) \sim_1 (1, v_1) \); and everything that follows from the above by transitivity.

(b) [5 points] Suppose that Player 1 is selfish and benevolent and \( v_1 = 64 \). How does she rank the following three outcomes: \((1, 73), (2, 57), (2, 46)\)? [Write your answer below]
• Player 1 is **selfish and spiteful** if her preferences are as follows:
  for every $p$ and $p'$, $(2, p) \succeq_1 (2, p')$ if and only if $p > p'$;
  $(2, p_1) \sim_1 (1, v_1)$; and everything that follows from the above by transitivity.

(c) [5 points] Suppose that Player 1 is selfish and **spiteful** and $v_1 = 25$. How does she rank the following three outcomes: $(1,18), (2,50), (2,39)$? [Write your answer below]

(d) [10 points] Suppose that Player 1 is selfish and **uncaring**. Does she have a weakly or strictly dominant strategy? If your answer is Yes, say what that strategy is and state whether it is weak or strict dominance; if your answer is No prove it. [Write your answer below]

(e) [10 points] Suppose that Player 1 is selfish and **benevolent**. Is bidding $v_1$ a dominant strategy? Fully explain your answer. [Write your answer below]

(f) [10 points] Suppose that Player 1 is selfish and **spiteful**. Is bidding $v_1$ a dominant strategy? Fully explain your answer. [Write your answer below]
In parts (g)-(i) assume that $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$.

(g) [15 points] Suppose that it is common knowledge that both players are selfish and *uncaring*. Find all the pure-strategy Nash equilibria. [Write your answer below]

(h) Suppose that it is common knowledge that both players are selfish and *benevolent*.

(h.1) [5 points] Is $(3,5)$ a Nash equilibrium? Explain your answer. [Write your answer below]

(h.2) [15 points] Find all the pure-strategy Nash equilibria. [Write your answer below]

(i) Suppose that it is common knowledge that both players are selfish and *spiteful*.

(i.1) [5 points] Is $(3,5)$ a Nash equilibrium? Explain your answer. [Write your answer below]

(i.2) [15 points] Find all the pure-strategy Nash equilibria. [Write your answer below]