Note: this exam was based on the material covered in class on January 20, 2017 and in the textbook pp. 22-24.

(a) $(2, 33) \sim (2, 58) \succ (1, 70)$.

(b) $(1, 75) \succ (2, 51) \succ (2, 62)$.

(c) $(2, 47) \succ (2, 34) \succ (1, 21)$.

(d) This is the standard case where Vickrey’s theorem applies: $b = v_i$ is a weakly dominant strategy.

(e) Recall that $p_i < v_i < p_m$. Bidding $v_i$ is not a dominant strategy: if Player 2 bids $p_m$ then the outcome is $(2, v_i)$ and Player 1 would prefer bidding $p_1$, since – by benevolence – $(2, p_1) \succ (2, v_i)$.

(f) Recall that $p_i < v_i < p_m$. We need to distinguish two cases. Case 1: $v_i > p_2$; in this case bidding $v_i$ is not a dominant strategy: if Player 2 also bids $v_i$ then the outcome is $(1, v_i)$ and, since $(1, v_i) \sim (2, p_1) \prec (2, p_2)$, Player 1 would prefer bidding $p_2$, inducing the outcome $(2, p_2)$. Case 2: $v_i \leq p_2$; in this case bidding $v_i$ is not a dominant strategy: if Player 2 bids $p_3$ (recall that $m > 3$) then the outcome is $(2, v_i)$ and Player 1 would prefer bidding $p_3$, inducing the outcome $(2, p_3)$ which she prefers to $(2, p_2)$ which, in turn, is at least as good as $(2, v_i)$ (since $v_i \leq p_2$).

(g) The assumption is that it is common knowledge that both players are selfish and uncaring and $B = \{3, 4, 5 = v_1, 6, 7 = v_2\}$. Since bidding one’s own true value is a weakly dominant strategy, $(5, 7)$ is a Nash equilibrium; however it is not the only Nash equilibrium. All of the following are Nash equilibria: $(3, 7)$, $(4, 7)$, $(5, 7)$, $(6, 7)$, $(3, 6)$, $(4, 6)$, $(5, 6)$, $(3, 5)$, $(4, 5)$, $(7, 3)$, $(7, 4)$ and $(7, 5)$. Thus a total of 12 equilibria.

(h) The assumption is that it is common knowledge that both players are selfish and benevolent and $B = \{3, 4, 5 = v_1, 6, 7 = v_2\}$.

(h.1) $(5, 7)$ is not a Nash equilibrium because the associated outcome is $(2, 5)$; by benevolence, $(2, 3) \succ (2, 5)$ and Player 1 can induce outcome $(2, 3)$ by reducing her bid from 5 to 3.

(h.1) The Nash equilibria are: $(3, 5)$, $(3, 6)$, $(3, 7)$ and $(7, 3)$.

(i) The assumption is that it is common knowledge that both players are selfish and spiteful and $B = \{3, 4, 5 = v_1, 6, 7 = v_2\}$.

(i.1) $(5, 7)$ is not a Nash equilibrium because the associated outcome is $(2, 5)$; by spitefulness, $(2, 6) \succ (2, 5)$ and Player 1 can induce outcome $(2, 6)$ by increasing her bid from 5 to 6.

(i.2) The Nash equilibria are: $(6, 7)$, $(5, 6)$, $(4, 5)$. 