University of California, Davis -- Department of Economics
ECN 122 : Game Theory    Professor Giacomo Bonanno
WINTER 2017 - THIRD MIDTERM EXAM

Answer all questions. If you don’t explain (= show your work for) your answers you will get no credit.

NAME:______________________________ University ID:____________________

CIRCLE THE NAME OF YOUR TA:  Yingxue Li  or  Johannes Matschke

If you don’t know the name of your TA, then write your Section Number: ________________

- By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

- If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [45 points] Consider the following situation of one-sided incomplete information. Players 1 and 2 are playing the following simultaneous game:

\[
\begin{array}{c|cc}
& C & D \\
\hline
A & z_1 & z_2 \\
B & z_3 & z_4 \\
\end{array}
\]

It is common knowledge between them that Player 2’s von Neumann-Morgenstern payoff function is as follows: \( \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ 85 & 100 & 85 & 0 \end{pmatrix} \). It is also common knowledge between them that Player 1 knows his own von Neumann-Morgenstern payoff function, but Player 2 is uncertain as to whether Player 1’s payoff function is \( \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ 0 & 0 & 1 & 0.5 \end{pmatrix} \) (she attaches probability \( \frac{1}{3} \) to this possibility) or \( \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ 0.85 & 0.85 & 1 & 0 \end{pmatrix} \) (she attaches probability \( \frac{4}{5} \) to this possibility). The beliefs of Player 2 are common knowledge between the two players.

(a) [15 points] Draw an extensive-form game that represents this situation of incomplete information (that is, apply the Harsanyi transformation).
(b) [20 points] Write the strategic form (or normal form) associated with the extensive form of part (a).

(c) [6 points] Does any player have any strategies that are strictly dominated? If Yes, then say which one(s) and name the strategy that dominates it (them).

(d) [4 points] Find the pure-strategy Nash equilibrium of this game.
2. [55 points] A referee tells Ann and Bob the following:
   “I will pick a number \( n \) from the set \{1,3,5\} and then write the two numbers \( n - 1 \) and \( n + 1 \) on two pieces of paper, shuffle them and hand one of them to Ann and one of them to Bob; each of you will get to see only your own piece of paper”.

(a) [4 points] List the set of possible states.

(b) [10 points] For each of Ann and Bob, use an information partition to represent their possible states of knowledge after they look at the piece of paper they are given.
(c) [10 points] Derive the corresponding common knowledge partition.

(d) [10 points] Suppose that the referee picks the number 5 and gives Ann the number 6 and Bob the number 4. What is the smallest event that is common knowledge between Ann and Bob? Give also a verbal description of that event.
(e) Let $E$ be the event “Bob’s number is 2”.

(e.1) [12 points] Find the events $K_A E$ (Ann knows E), $K_B E$ (Ann knows E), $Kᴬ Kᴮ E$ (Ann knows that Bob knows E) and $Kᴮ Kᴬ E$ (Bob knows that Ann knows E).

$$K_A E = \text{________________________} , \quad K_B E = \text{________________________} ,$$

$$Kᴬ Kᴮ E = \text{________________________} , \quad Kᴮ Kᴬ E = \text{________________________} ,$$

(e.2) [9 points] Find a state $\alpha$ such that $\alpha \in E$, $\alpha \in K_A E$, $\alpha \in K_B E$, $\alpha \in Kᴬ Kᴮ E$ and $\alpha \notin Kᴮ Kᴬ E$ (that is, at $\alpha$, E is true, Ann and Bob know E, Ann knows that Bob knows E, but it is not true that Bob knows that Ann knows E).