1. The following game has two Nash equilibria: (T,L) and (B,R). The latter is the unique iterated weak dominant strategy equilibrium and it is strictly Pareto dominated by (T,L).

\[
\begin{array}{c|cc}
\text{Player I} & \text{T} & \text{B} \\
\hline
\text{L} & 2, 3 & 2, 1 \\
\text{R} & 0, 2 & 1, 2 \\
\end{array}
\]

2. Recall that \( n > 1 \) and that a player can win only if he/she bids a positive amount. Let \((x_1, ..., x_n)\) be the bids. Let \( x_i \) be a highest bid, that is, \( x_i \geq x \) for all \( x \in \{x_1, ..., x_n\} \).

If \( x_i > 0 \), it cannot be that \( x_i = x_j \) for some \( j \neq i \) (nobody wins and i and j have to pay a positive amount). If \( x_i > 0 \) and there is a \( j \neq i \) such that \( 0 < x_j < x_i \) then it is not a Nash equilibrium (player \( j \) would do better by reducing his bid to zero). Thus it must be \( x_j = 0 \) for every \( j \neq i \). But then in order for this to be a Nash equilibrium \( x_i \) must be the smallest possible amount, that is 1 cent (otherwise player \( i \) can increase his payoff by reducing his bid). But the situation where \( x_i \) is equal to 1 cent and \( x_j = 0 \) for every \( j \neq i \) is not a Nash equilibrium because a player \( j \neq i \) can increase her payoff by bidding 2 cents.

If, on the other hand, \( x_i = 0 \), then any player can increase his payoff by bidding 1 cent.

Thus there are no Nash equilibria.

3. The set of strategies of player \( i \) is \( S_i = [0, \infty) \) and the payoff functions are

\[
\pi_1(t_1, t_2) = \begin{cases} -t_1 & \text{if } t_1 < t_2 \\ \frac{v_1}{2} - t_1 & \text{if } t_1 = t_2 \\ v_1 - t_2 & \text{if } t_1 > t_2 \end{cases} \quad \text{and} \quad \pi_2(t_1, t_2) = \begin{cases} -t_2 & \text{if } t_2 < t_1 \\ \frac{v_2}{2} - t_2 & \text{if } t_2 = t_1 \\ v_2 - t_1 & \text{if } t_2 > t_1 \end{cases}
\]

Let \((t_1, t_2)\) be a pair of strategies. If \( t_1 = t_2 \) then by conceding slightly later than \( t_1 \) Player 1 can obtain the object in its entirety instead of getting just half of it, so this is not an equilibrium. If \( 0 < t_1 < t_2 \) then Player 1 can increase her payoff to zero by deviating to \( t_1 = 0 \). Finally, if \( 0 = t_1 < t_2 \) then Player 1 can increase her payoff by deviating to a time slightly after \( t_2 \) unless \( v_1 - t_2 \leq 0 \). Similarly for \( 0 = t_2 < t_1 \) to constitute an equilibrium we need \( v_2 - t_1 \leq 0 \). Hence \((t_1, t_2)\) is a Nash equilibrium if and only if either \( 0 = t_1 < t_2 \) and \( t_2 \geq v_1 \) or \( 0 = t_2 < t_1 \) and \( t_1 \geq v_2 \).