1. (a) Brad

<table>
<thead>
<tr>
<th>Ann</th>
<th>Red</th>
<th>Black</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>Ann has $4, Brad has $8</td>
<td>with probability 1/3 Ann has $12 and Bob has $0, with probability 2/3 Brad has $12 and Ann has $0</td>
</tr>
<tr>
<td>Black</td>
<td>with probability 1/3 Ann has $12 and Bob has $0, with probability 2/3 Brad has $12 and Ann has $0</td>
<td>Ann has $4, Brad has $8</td>
</tr>
</tbody>
</table>

(b) Since Ann is averse to risk (prefers $x to a lottery with expected value $x), she prefers $4 for sure to the lottery $\left( \frac{12}{3}, \frac{0}{3} \right)$ which has an expected value of $4$. Thus she prefers match [the outcome associated with (R,R) and (B,B)] to mismatch (the outcome associated with (R,B) and (B,R)]. On the other hand, since Brad is risk-loving (prefers a lottery with expected value $x$ to $x$ for sure), Brad prefers mismatch to match.

(c) The reduced form is as follows.

<table>
<thead>
<tr>
<th>Brad</th>
<th>Red</th>
<th>Black</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>$U(4) = 15$, $V(8) = 64$</td>
<td>$\frac{1}{3}U(12) + \frac{2}{3}V(0) = 13$, $\frac{1}{3}V(0) + \frac{2}{3}V(12) = 96$</td>
</tr>
<tr>
<td>Black</td>
<td>$\frac{1}{3}U(12) + \frac{2}{3}V(0) = 13$, $\frac{1}{3}V(0) + \frac{2}{3}V(12) = 96$</td>
<td>$U(4) = 15$, $V(8) = 64$</td>
</tr>
</tbody>
</table>

(d) There are no pure-strategy equilibria. The mixed-strategy equilibrium is found by solving:

$$15q + 13(1 - q) = 13q + 15(1 - q)$$
$$64p + 96(1 - p) = 96p + 64(1 - p)$$

The solutions are $p = 1/2$ and $q = 1/2$. Thus each player chooses Red and Black with equal probability.

(e) The equilibrium payoff is 14 for Ann and 80 for Brad.

2. (a) To find subgame-perfect equilibria first we solve the subgame on the left:

<table>
<thead>
<tr>
<th>Player 2</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 0</td>
<td>0, 1</td>
</tr>
<tr>
<td>B</td>
<td>0, 2</td>
<td>2, 0</td>
</tr>
</tbody>
</table>

There is no pure-strategy Nash equilibrium. Let $p$ be the probability of A and $q$ the probability of C. Then at a Nash equilibrium it must be that $q = 2(1 - q)$ and $2(1 - p) = p$. Thus there is a unique Nash equilibrium given by $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$. Next consider the subgame on the right, whose strategic form is as follows:
This game has three pure-strategy Nash equilibria: \((E,H,L)\), \((E,G,L)\) and \((F,H,M)\). Thus each of the following is a subgame-perfect equilibrium:

\[
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 1 & 1 & 0
\end{pmatrix},
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix} \quad \text{and} \quad
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 1 & 0 & 1
\end{pmatrix}.
\]

To find more subgame-perfect equilibria, note that in the subgame on the right - for Player 3 L is strictly better than M against any pair of strategies of Players 1 and 2 except for \((F,H)\). Thus if \(P(E) > 0\) and/or \(P(G) > 0\) then Player 3’s best reply is \(P(L) = 1\). However, we know that \((F,H,L)\) is not a Nash equilibrium. Thus if there are Nash equilibria with \(P(E) > 0\) and/or \(P(G) > 0\) then they must be such that \(L\) is played with probability 1. When Player 3 plays \(L\) with probability 1 then for Player 2 \(E\) is strictly better than \(F\); thus candidates for Nash equilibrium must be of the form

\[
\begin{pmatrix}
E & F & G & H & L & M \\
1 & 0 & p & 1 - p & 1 & 0
\end{pmatrix}.
\]

Indeed, for every \(p \in [0,1]\), such a strategy profile is a Nash equilibrium. Thus in addition to the three subgame-perfect equilibria given above, the following are also subgame-perfect equilibria, for every \(0 < p < 1\):

\[
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 & p & 1 - p & 1 & 0
\end{pmatrix}.
\]

Finally, when Players 1 and 2 play \(F\) and \(H\), respectively, Player 3 is indifferent between \(L\) and \(M\) and thus is willing to mix. Let Player 3’s strategy be \(\begin{pmatrix}L & M \\ q & 1 - q\end{pmatrix}\); then for Player 1 \(F\) must be at least as good as \(E\), that is, \(1 \geq 2q\) or \(q \leq \frac{1}{2}\) and for Player 2 \(H\) must be at least as good as \(G\), that is, \(2 \geq 3q + 1 - q\), that is, \(q \leq \frac{1}{2}\). Thus, for every \(q \leq \frac{1}{2}\) the following is a Nash equilibrium of the
subgame: \[
\begin{pmatrix}
E & F & G & H & L & M \\
0 & 1 & 0 & 1 & q & 1-q
\end{pmatrix},
\]
so that the following are subgame-perfect equilibria of the entire game:

\[
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 1 & q & 1-q
\end{pmatrix}
\]
for every \( q \leq \frac{1}{2} \).

(b) Since choice M is strictly dominated by L, there cannot be a weak sequential equilibrium where Player 3 plays M. \[
\begin{pmatrix}
A & B & C & D & E & F & G & H & L & M \\
\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 1 & 0 & 0
\end{pmatrix}
\]
with beliefs \( \mu(x) = \frac{2}{3}, \mu(y) = \frac{1}{3} \) and \( \mu(w) = 1, \mu(z) = 0 \) is a weak sequential equilibrium.