1. 

(a) Contract $A$ is a full-insurance contract that will be bought only by the $H$ types. To calculate the premium for contract $A$ solve \( \frac{4}{5}\sqrt{2500} + \frac{1}{5}\sqrt{1600} = \sqrt{2500 - h} \). The solution is $h = 196$. Thus the monopolist’s profits are $\pi_1 = [196 - \frac{1}{5} \times 900]N_H = 16N_H$.

(b) Contract $B$ is a full-insurance contract that will be bought by all types. To calculate the premium for contract $B$ solve \( \frac{9}{10}\sqrt{2500} + \frac{1}{10}\sqrt{1600} = \sqrt{2500 - h} \). The solution is $h = 99$. Thus the monopolist’s profits are $\pi_2 = [99 - \frac{1}{10} \times 900]N_H + [99 - \frac{1}{10} \times 900]N_L = 9N_H - 81N_L$.

(c) The $H$ types would choose the full-insurance contract $D$ while the $L$ types would choose the partial-insurance contract $C$. To compute the deductible for contract $C$ solve \( \frac{9}{10}\sqrt{2500 - 6.24} + \frac{1}{10}\sqrt{2500 - 6.24 - d} = \frac{9}{10}\sqrt{2500} + \frac{1}{10}\sqrt{1600} \). The solution is $d = 848.48$. Thus the monopolist’s profits are $\pi_3 = [190 - \frac{1}{10} \times 900]N_H + [6.25 - \frac{1}{10} (900 - 848.48)]N_L = 10N_H + 1.088N_L$.

2. Let $W_i(Q)$ be the willingness to pay of customer of type $i \in \{A, B, C\}$ for $Q$ units. Then

\[ W_A(Q) = 30Q - \frac{Q^2}{4}, \quad W_B(Q) = 15Q - \frac{Q^2}{8} \quad \text{and} \quad W_C(Q) = \frac{15}{2}Q - \frac{Q^2}{16}. \]

(a) Since $W_C(Q_1) - V_1 = -45$, $W_B(Q_1) - V_1 = 80$ and $W_A(Q_1) - V_1 = 330$, only customers of type $A$ and $B$ buy. Thus the firm’s profits will be $\Pi_1 = (50 + 60)[170 - 2(20)] = 14,300$.

(b) Since $W_C(Q_{21}) - V_{21} = 1$, $W_C(Q_{22}) - V_{22} = -42.25$, $W_B(Q_{21}) - V_{21} = 126$, $W_B(Q_{22}) - V_{22} = 126.5$, $W_A(Q_{21}) - V_{21} = 376$, $W_A(Q_{22}) - V_{22} = 464$, C-customers purchase the first package, while the others purchase the second package. Thus $\Pi_2 = 80[124 - 2(20)] + 110[211 - 2(30)] = 23,330$.

(c) Since $W_C(Q_{31}) - V_{31} = 0.75$, $W_C(Q_{32}) - V_{32} = -164$, $W_B(Q_{31}) - V_{31} = 169.5$, $W_B(Q_{32}) - V_{32} = 60$, $W_A(Q_{31}) - V_{31} = 507$, $W_A(Q_{32}) - V_{32} = 508$, B and C customers purchase the first package, while A customers purchase the second package. Thus $\Pi_3 = 140(168 - 60) + 50(388 - 112) = 28,920$.

(d) For linear pricing we first need to calculate aggregate demand. The demand functions are:
Thus aggregate demand is

\[
D(P) = \begin{cases} 
50D_A(P) + 60D_B(P) + 80D_C(P) & \text{if } 0 \leq P < 7.5 \\
50D_A(P) + 60D_B(P) & \text{if } 7.5 \leq P < 15 \\
50D_A(P) & \text{if } P \geq 15 
\end{cases}
\]

The profit function is \( \pi(P) = (P - 2)D(P) \). The maximum occurs at \( P = \frac{182}{17} = 10.706 \) with corresponding profits of \( \pi_{\text{linear}} = \Pi(10.706) = 25,769.412 \) (note that \( \Pi(7.5) = 22,275 \) and \( \Pi(15) = 19,500 \)). Thus with linear pricing the monopolist serves only types A and B.

\( \text{(e)} \) The monopolist would sell to each consumer a bundle containing the quantity at which MC crosses the consumer’s demand and charge a price for the bundle equal to the consumer’s willingness to pay for that quantity. Bundle for Type A: \( (Q = 56, V = 896) \) (with a profit per bundle of 784), bundle for type B: \( (Q = 52, V = 442) \) (with a profit per bundle of 338), bundle for type C: \( (Q = 44, V = 209) \) (with a profit per bundle of 121). Its total profits would be \( 50(784) + 60(338) + 80(121) = 69,160 \).
3. (a) Note that, for each type the cost of an extra unit of education exceeds the benefit (in terms of higher salary) of that extra unit. Thus each type will only consider three levels of education: 0, \(d\), and \(e\). The inequalities are as follows:

For type A:

\[
\begin{align*}
(1) \ [\text{e better than d}] & \quad 3 + \frac{e}{6} - \frac{e}{5} > 2 + \frac{d}{6} - \frac{d}{5} \\
(2) \ [\text{e better than 0}] & \quad 3 + \frac{e}{6} - \frac{e}{5} > 1 \\
\end{align*}
\]

For type B:

\[
\begin{align*}
(3) \ [\text{d better than e}] & \quad 2 + \frac{d}{6} - \frac{d}{4} > 3 + \frac{e}{6} - \frac{e}{4} \\
(4) \ [\text{d better than 0}] & \quad 2 + \frac{d}{6} - \frac{d}{4} > 1 \\
\end{align*}
\]

For type C:

\[
\begin{align*}
(5) \ [\text{0 better than e}] & \quad 1 > 3 + \frac{e}{6} - \frac{e}{2} \\
(6) \ [\text{0 better than d}] & \quad 1 > 2 + \frac{d}{6} - \frac{d}{2} \\
\end{align*}
\]

(b) Yes, when \(d = 10\) and \(e = 24\) all of the inequalities are satisfied.

(c) No, \(d = 10\) and \(e = 21\) inequality (3) is not satisfied (while the others are).

4. (a) Let \(q_i\) denote the probability of quality \(i\) (thus \(q_A = \frac{1}{8}\), etc.). We need to be able to find a \(p\) such that \(s_B \leq p < s_A\) (this is always possible since, by hypothesis, \(s_B < s_A\))

and \(p \leq \sum_{i \in \{B,C,D\}} \left( \frac{q_i}{q_B + q_C + q_D} \right) \alpha s_i\), that is, \(p \leq \frac{\alpha}{7}(3s_B + 3s_C + s_D)\). Thus the necessary and sufficient condition is \(s_B \leq \frac{\alpha}{7}(3s_B + 3s_C + s_D)\).

(b) We need to be able to find a \(p\) such that \(s_C \leq p < s_B\) (this is always possible since, by hypothesis, \(s_C < s_B\)) and \(p \leq \sum_{i \in \{C,D\}} \left( \frac{q_i}{q_C + q_D} \right) \alpha s_i\), that is, \(p \leq \frac{\alpha}{4}(3s_C + s_D)\). Thus the necessary and sufficient condition is \(s_C \leq \frac{\alpha}{4}(3s_C + s_D)\).