HOMEWORK 7  (for due date see the web page)

There are three types of workers: 1, 2 and 3. The proportion of type $i$ ($i \in \{1, 2, 3\}$) in the population of workers is $p_i$, with $0 < p_i < 1$ (and, of course, $p_1 + p_2 + p_3 = 1$). A newly set-up firm wishes to hire workers. The firm has the following information:

1. A type $i$ worker would be able to produce $x_i$ units of output per year, with $0 < x_3 < x_2 < x_1$. The firm expects that it will be able to sell each unit of output for $SR$. Labor costs are the only costs for the firm.

2. Workers of type 1 will apply for a job with the new firm if and only if they expect to earn $w_H$ or more (per year), while workers of type 2 and 3 will apply if and only if they expect to earn $w_L$ or more (per year) where $0 < w_L < w_H < w_L \frac{x_2}{x_3}$.

The firm is considering two options: (1) offer to hire workers at a fixed yearly salary $w$, or (2) offer to hire workers at a piece rate $b$ (that is, the worker will be paid $b$ for each unit of output she produces). When a worker applies for a job, the firm is unable to tell what type the applicant is. On the other hand, each applicant knows her own productivity (i.e. an applicant of type $i$ knows that she will be able to produce $x_i$ units of output in one year). The firm is risk neutral and is planning to hire a total of $N$ workers. Assume that there are at least $N$ workers of each type.

(a) If the firm decides to offer a fixed salary, what salary should it offer?

(b) If the firm decides to offer a piece-rate, what is the optimal piece rate? Carefully distinguish between the following cases: (b.1) $w_H < w_L \frac{x_1}{x_2}$ and (b.2) $w_L \frac{x_1}{x_2} < w_H < w_L \frac{x_1}{x_3}$.

For each of the following cases find the optimal policy for the firm (i.e. determine the optimal fixed salary - if a fixed salary is the best option - or the optimal piece rate - if a piece rate is the best option).

(c) $p_1 = \frac{1}{6}, p_2 = \frac{3}{6}, p_3 = \frac{2}{6}; x_1 = 42, x_2 = 40, x_3 = 38; R = 54; w_L = 900, w_H = 924.$

(d) $p_1 = \frac{2}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{4}; x_1 = 16, x_2 = 15, x_3 = 12; R = 40; w_L = 225, w_H = 252.$