Insurance Markets

Consider an individual who has an initial wealth of $W$. With some probability $p$ he faces a loss of $x$ ($0 < x < W$). Thus his initial situation can be represented as a point in a two-dimensional diagram where we measure on the horizontal axis his wealth in the bad state (loss), denoted by $W_1$, and on the vertical axis his wealth in the good state (no loss), denoted by $W_2$. An insurance contract takes the individual from the initial point of no insurance to some other point.

An insurance contract can be described by a pair $(h, D)$, where $h$ is the **premium** and $D$ is the **deductible**. Note that the premium is paid in any case, that is both in the good state and in the bad state. Given a contract $(h, D)$, wealth in the good state will be $(W - h)$ and wealth in the bad state will be $W - h - D$. Thus if we represent a contract in the $(W_1, W_2)$ plane, then the premium is $(W - W_2)$ and the deductible is $(W_2 - W_1)$. For example, in Figure 1 Contract $A$ involves a premium of $5,000 - 4,500 = $500 and a deductible equal to $4,500 - 3,900 = $600$, and contract $B$ involves a premium of $(5,000 - 4,100) = $900 and zero deductible, that is, full insurance.

![Figure 1](image-url)
The expected profit from insurance contract \((h, D)\) is
\[
h - p(x - D) = h - px + pD \quad (1.a)
\]
If we represent the contract as a point in the \((W_1, W_2)\) plane, then the expected profit from the contract can be written as
\[
\frac{(W - W_2)}{h} - px + p\frac{(W_2 - W_1)}{D} = \frac{W - px - (1-p)W_2 - pW_1}{W_1 - W_2} \quad (1.b)
\]

An **isoprofit line** is a line that joins all the contracts that yield the same expected profit. Suppose that \(A = (W_{1A}, W_{2A})\) and \(C = (W_{1C}, W_{2C})\) are two contracts that yield the same profit, that is, \(\pi(A) = \pi(C)\).

The slope of the line joining these two contracts is:
\[
\frac{\text{rise}}{\text{run}} = \frac{W_{2A} - W_{2C}}{W_{1A} - W_{1C}}. \quad \text{Now, from (1.b) we get that } \pi(A) = W - px - (1-p)W_{2A} - pW_{1A} \quad \text{and } \pi(C) = W - px - (1-p)W_{2C} - pW_{1C}. \text{ Thus by setting } \pi(A) = \pi(C) \text{ we get: } -(1-p)(W_{2A} - W_{2C}) = p(W_{1A} - W_{1C}) \text{ so that }
\]
\[
\frac{\text{rise}}{\text{run}} = \frac{W_{2A} - W_{2C}}{W_{1A} - W_{1C}} = -\frac{p}{1-p}
\]

Hence the isoprofit line through points \(A\) and \(C\) is a straight line with slope \(-\frac{p}{1-p}\).

**EXAMPLE.** Let \(W = 16,000\), \(x = 3,600\) and \(p = \frac{2}{12}\). Consider a contract, call it \(A\), with premium \(h = 800\) and deductible \(D = 1,200\). Then the expected profit from this contract is
\[ \pi(A) = h - px + pD = 800 - \frac{2}{12} 3,600 + \frac{2}{12} 1,200 = 400. \] The slope of the isoprofit line through \( A \) is 

\[ -\frac{p}{1-p} = -\frac{\frac{7}{12}}{\frac{10}{12}} = -\frac{1}{\frac{10}{12}}. \]

Suppose that the premium is decreased by $200 but profits remain constant. Then we go from \( A = (14000, 15200) \) to \( C = (y, 15400) \) so that \( \text{rise} = -200 \). Thus, since 

\[ \frac{\text{rise}}{\text{run}} = -\frac{p}{1-p} = -\frac{1}{\frac{10}{12}}, \]

we have \( \frac{-200}{\text{run}} = -\frac{1}{\frac{10}{12}} \) hence \( \text{run} = +1,000, \) that is, \( 14,000 - y = 1,000, \) i.e. \( y = 13,000 \). Hence deductible at \( C \) is \( 15,400 - 13,000 = 2,400 \). So in the \((W_1, W_2)\) plane, \( C = (13000, 15400) \) or, in terms of premium and deductible, contract \( C \) is given by \( h = 600, D = 2,400 \).

Each isoprofit line divides the plane into thee regions: (1) the line itself, where profits are constant and equal to some number \( \bar{\pi} \), (2) the region to the left of the line where profits are larger than \( \bar{\pi} \) and (3) the region to the right of the line where profits are less than \( \bar{\pi} \):

![Figure 3](image)

To see this, take any contract \( A \) and draw the isoprofit line that goes through \( A \) and take a point \( S \) vertically below \( A \) (then \( S \) has a higher premium and a lower deductible relative to \( A \)). We want to show that \( \pi(S) > \pi(A) \).

![Figure 4](image)
Now, \[ \pi(A) = W - px - (1-p)W_2 - pW_1^A \]
and \[ \pi(S) = W - px - (1-p)W_2^S - pW_1^A \] (since \( W_1^S = W_2^A \))
Thus \( \pi(S) - \pi(A) = (1-p)(W_2^A - W_2^S) > 0 \), since \( W_2^A > W_2^S \) and \( 0 < p < 1 \).

\[ \text{wealth in good state (no loss)} \]
\[ \text{probability of good state: } 1-p \]

Thus moving from \( A \) inside the shaded area profits increase (convex combination of no change and increase).

One particular isoprofit line is the line that corresponds to **zero profits**. This is the isoprofit line that goes through the No Insurance point \((W-x, W)\). In fact, we can think of this point as a contract with \( h = 0 \) and \( D = x \).

For example, if \( W = 5,000 \), \( x = 2,000 \) and \( p = 0.2 \), then \( \frac{p}{1-p} = \frac{0.2}{0.8} = -\frac{1}{4} \) and the zero-profit contracts are the points on the line of equation \( W_2 = 5,750 - \frac{W_1}{4} \) (if \( W_1 \) is reduced from 3,000 to 0, \( W_2 \) increases by \( \frac{1}{4} \times 3,000 = 750 \) from 5,000 to 5,750)

**The Consumer**

First we want to draw the indifference curves of the consumer in the \((W_1, W_2)\) plane. Let \( U(m) \) be the utility-of-money function and assume that \( U' > 0 \) and \( U'' < 0 \), that is, the consumer is risk averse. Consider a point \( A = (W_1^A, W_2^A) \) and let \( C = (W_1^C, W_2^C) \) be a point close to \( A \) and suppose that \( A \) and \( C \) lie on the same indifference curve. Then
\[ EU(A) = pU(W_1^A) + (1-p)U(W_2^A) \] and \( EU(A) = EU(C) \). If \( C \) is close to \( A \), then \( W_1^C \) is close to \( W_1^A \) so that
\[ U(W_1^C) = U(W_1^A) + U'(W_1^A) \times (W_1^C - W_1^A) \] (2.a)

Similarly, \( W_2^C \) is close to \( W_2^A \) so that
\[ U(W^C) = U(W^A) + U'(W^A) \times (W^C - W^A) \]  \hspace{1cm} (2.b)

Hence,

\[
EU(C) = pU(W^C) + (1 - p)U(W^C) = p\left[U(W^A) + U'(W^A) \times (W^C - W^A)\right] + (1 - p)\left[U(W^A) + U'(W^A) \times (W^C - W^A)\right] =
\]

\[
EU(A) + pU'(W^A)\left(W^C - W^A\right) + (1 - p)U'(W^A)\left(W^C - W^A\right) = 0 .
\]

Since \( EU(A) = EU(C) \), it follows that \( pU'(W^A)\left(W^C - W^A\right) + (1 - p)U'(W^A)\left(W^C - W^A\right) = 0 \).

Thus

\[
\text{rise } \frac{\text{run}}{W^C - W^A} = -\frac{p}{1 - p} \frac{U'(W^A)}{U'(W^A)}. \text{ Hence}
\]

The slope of the indifference curve at point \((W_1, W_2)\) is

\[ -\frac{p}{1 - p} \frac{U'(W^A)}{U'(W^A)} \]

By strict concavity of the utility function, if \( m_1 < m_2 \) then \( U'(m_1) > U'(m_2) \). Thus

- Above the 45° line (where \( W_1 < W_2 \)), the slope of the indifference curve is greater \textit{in absolute value} than the slope of the isoprofit line that goes through that point, that is,

\[
\frac{p}{1 - p} \frac{U'(W^A)}{U'(W^A)} > \frac{p}{1 - p}
\]

This is because \( W_1 < W_2 \) (we are above the 45° line) and thus \( U'(W^A) > U'(W^A) \), so that

\[ \frac{U'(W^A)}{U'(W^A)} > 1 . \]

- Along the 45° line (where \( W_1 = W_2 \)), the slope of the indifference curve is equal to the slope of the isoprofit line that goes through that point, that is,

\[
-\frac{p}{1 - p} \frac{U'(W^A)}{U'(W^A)} = -\frac{p}{1 - p}
\]

Putting together what we found above, namely that

(1) at a point \( A \) above the 45° line the slope of the indifference curve is greater \textit{in absolute value} than the slope of the isoprofit line that goes through that point, and

(2) moving away from a point \( A \) above the 45° line in the direction between the vertical direction downwards and the direction of the isoprofit line (which has slope \( -\frac{p}{1 - p} \)), profits increase.

Thus profits for the insurance company increase as we move along the indifference curve that goes through point \( A \) towards the 45° line, as shown in the following figure.
What is the maximum premium, call it $h^*$, that an individual would be willing to pay for full insurance? It is the solution to the equation

$$U(W - h) = pU(W - x) + (1 - p)U(W)$$

For example, if $W = 1,600$, $x = 700$, $p = \frac{1}{10}$ and $U(W) = \sqrt{W}$ then $h^*$ is given by the solution to

$$\frac{\sqrt{1,600} - h}{10} \leq \frac{\sqrt{1,600 - 700} + \frac{9}{10} \sqrt{1,600}}{10} = 39$$

which is $h^* = 79$.

Next we show that $px < h^*$. To begin with, note that, by definition of risk-aversion, the expected utility of a lottery is less than the utility of the expected value:

$$pU(W - x) + (1 - p)U(W) < U(W - px) \quad (1)$$

By definition of $h^*$,

$$U(W - h^*) = pU(W - x) + (1 - p)U(W) \quad (2)$$

Thus from (1) and (2) we get

$$U(W - h^*) < U(W - px).$$

Since $U$ is increasing, it follows that

$$W - h^* < W - px \quad \text{that is,} \quad px < h^*$$

Next we show that $h^* < x$. First of all, note that
\[ U(W-x) = pU(W-x) + (1-p)U(W-x) < pU(W-x) + (1-p)U(W) = U(W-h^*) \]

That is,

\[ U(W-x) < U(W-h^*) \]

Which implies, since \( U \) is increasing, that

\[ h^* < x. \]

This can be shown graphically as follows.

Note: (1) \( h^* < x \)

(2) \( h^* > px \), i.e. risk-averse person willing to pay a premium which is higher than the expected loss.

**Figure 7**

Two observations:

1. We have assumed that the granting of full insurance does not affect the value of \( p \) (the probability of loss). This, however, is often not true. If you are insured, you don’t face a risk and therefore you exert less effort or care in trying to prevent a loss (e.g. you are less careful about locking your bike or locking the front door of your house).

2. The fact that \( h^* > px \) is what makes the sale of insurance policies a profitable business. If the probability of losses is independent across consumers and the insurance company sells a large number \( N \) of policies, then, by the law of large numbers in probability
theory, it will have to make payments of $pxN; however, it will be collecting $h^*, thus making a profit of $(h^* - px)N$ (assuming no other costs: an assumption that we will relax later).

**CHOOSING A CONTRACT FROM A MENU**

Often insurance companies offer a menu of possible contracts, not just one contract. Typically consumers have a choice between a higher premium and higher coverage or a lower premium and lower coverage.

**EXAMPLE.** Consider an individual whose initial wealth is $1,000. He faces a potential loss of $400, with probability $\frac{1}{5}$. Suppose that the insurance company offers the following options:

<table>
<thead>
<tr>
<th>Premium</th>
<th>Deductible</th>
</tr>
</thead>
<tbody>
<tr>
<td>contract 1</td>
<td>$82</td>
</tr>
<tr>
<td>contract 2</td>
<td>$62</td>
</tr>
<tr>
<td>contract 3</td>
<td>$40</td>
</tr>
</tbody>
</table>

The individual’s utility-of-money function is $U(m) = \sqrt{m}$. Which contract would he choose?

No insurance:

$$
\left( \begin{array}{cc} 1000 & 600 \\ 0.8 & 0.2 \end{array} \right) = \left( \begin{array}{c} 0.8 \times 1000 + 0.2 \times 600 \end{array} \right) = 30.197
$$

Expected Utility

Full insurance:

$$
\left( \begin{array}{cc} 1000 - 82 & 0 \\ 1 & 1 \end{array} \right) = \left( \begin{array}{c} 0.8 \times 918 + 0.2 \times 838 \end{array} \right) = 30.299
$$

Decuctible of 100:

$$
\left( \begin{array}{cc} 1000 - 62 & 1000 - 100 - 62 \\ 0.8 & 0.2 \end{array} \right) = \left( \begin{array}{c} 0.8 \times 938 + 0.2 \times 838 \end{array} \right) = 30.291
$$

Decuctible of 200:

$$
\left( \begin{array}{cc} 1000 - 40 & 1000 - 200 - 40 \\ 0.8 & 0.2 \end{array} \right) = \left( \begin{array}{c} 0.8 \times 960 + 0.2 \times 760 \end{array} \right) = 30.301
$$

Thus the best option is contract 3 (deductible of $200) and the next best option is full insurance.

Now suppose that the consumer can choose any contract from the set of contracts that yield zero profits. Then for each insurance policy the issuer collects premium $h$ and expects to pay out $p(x - D)$. Thus zero profits means that $h$ and $D$ are such that

$$
h = p(x - D).
$$

Suppose that customers are free to choose any premium-deductible combination $(h,D)$ that satisfies the above equation.

What premium-deductible combination $(h,D)$ would a risk-averse individual choose? Clearly, once you choose $D$, the premium is determined by the equation $h = p(x - D)$. Thus
Wealth if no loss: \[ W - h = W - p(x - D) = W - px + pD \]

Wealth if loss: \[ W - h - D = W - p(x - D) - D = W - px + pD - D = W - px - (1 - p)D \]

Thus expected utility from policy \((h, D)\) is

\[ f(D) = p U(W - px - (1 - p)D) + (1 - p) U(W - px + pD) \]

The individual will choose \(D\) to maximize \(f(D)\). A necessary condition is \(f'(D) = 0\). Now

\[ f'(D) = p U'(W - px - (1 - p)D) - (1 - p) + (1 - p) U'(W - px + pD) (p) \]

Thus we need

\[ -(1 - p) p U'(W - px - (1 - p)D) + (1 - p) p U'(W - px + pD) = 0 \]

that is,

\[ U'(W - px + pD) = U' \left( \frac{W - px - (1 - p)D}{W - px + pD - D} \right) \]

Since \(U\) is strictly concave, the slope of \(U\) is different at any two different points. Thus the only way to satisfy the equation \(U'(x) = U'(y)\) is to have \(x = y\). Thus to satisfy the above equation we need

\[ W - px + pD = W - px + pD - D \]

and this requires \(D = 0\). Thus the individual would choose full insurance. [Of course we already knew that, because it is only along the 45° line that there is a tangency between the indifference curve and the zero-profit line: on the 45° line the slope of both is \(-\frac{p}{1 - p}\).]

**Case 1: THE INSURANCE INDUSTRY IS A MONOPOLY**

The monopolist will want to offer a contract that lies on the indifference curve through the No Insurance point (if it offered a contract above that indifference curve it would not be maximizing its profits, because coming vertically down from that point to the indifference curve would yield an increase in profits). Furthermore, as seen above, moving along the indifference curve towards the 45° line profits increase. Thus the monopolist would want to offer full insurance at premium \(h^*\).

**Case 2: FREE ENTRY in the Insurance Industry leads to ZERO PROFITS**

A contract that yields zero profit is called a **fair contract** and the zero profit line is called the **fair odds** line. Recall that the zero profit line is the straight line that goes through the No Insurance point and has slope \(-\frac{p}{1 - p}\).
Define an equilibrium in a competitive insurance industry as a situation where every firm makes zero profits and no firm (existing or new) can make positive profits by offering a new contract. What contract(s) will be offered in this industry?

First of all, we show that at least one firm must offer the full-insurance contract (given by \( h = px \) and \( D = 0 \)). Suppose not. Then take a contract on the zero-profit line that is offered and that some consumers are buying. Since it is not the full-insurance contract, it must lie above the 45° line, like contract \( A \) in Figure 8 below. Hence the slope of the indifference curve through \( A \) is steeper than the zero-profit line and, therefore, there is a contract \( B \) that lies below the zero-profit line (hence \( \pi(B) > 0 \)) and above the indifference curve through \( A \), so that a firm that offered contract \( B \) would attract all the consumers who were buying \( A \) and make positive profit.

Secondly, since the full-insurance contract is offered by at least one firm, every consumer purchases that contract (because any other contract that is offered must be on the zero-profit line and, as just shown, it must lie on a lower indifference curve that the full insurance contract.

**Figure 8**

Secondly, since the full-insurance contract is offered by at least one firm, every consumer purchases that contract (because any other contract that is offered must be on the zero-profit line and, as just shown, it must lie on a lower indifference curve that the full insurance contract.

Adverse Selection in Insurance Markets

Suppose that there are two types of individuals. They are all identical in terms of the initial wealth (or wealth in the good state), denoted by \( W \), and in terms of the potential loss that they face, denoted by \( x \). They also have the same utility-of-money function \( U \). What they differ in is the probability of loss: it is \( p_H \) for type H (high-risk) individuals and \( p_L \) for type L (low-risk) individuals with \( 1 > p_H > p_L > 0 \). Then type-H individuals have steeper indifference curves than type-L individuals. In fact, fix an arbitrary point \((W_1, W_2)\). As we saw above, the slope of the indifference curve going through this point is

\[
\frac{dW_2}{dW_1} = -\frac{p}{1-p} \frac{U'(W_1)}{U'(W_2)}
\]

Thus for type-H individuals
it is \( -\frac{p_H}{1-p_H} \frac{U'(W_1)}{U'(W_2)} \) and for type-L individuals it is \( -\frac{p_L}{1-p_L} \frac{U'(W_1)}{U'(W_2)} \). From \( p_H > p_L \) we get that 
\[ 1 - p_H < 1 - p_L. \]
Hence 
\[ \frac{p_L}{1-p_L} < \frac{p_H}{1-p_H} < \frac{p_H}{1-p_L} \]
where the latter inequality follows from \( p_H > p_L \).

![Figure 9](image)

[Note: if we measured wealth in good state on the horizontal axis, then the opposite would be true: the L type indifference curve would be steeper than the H type.]

Suppose that the insurance companies cannot tell who is who. However it is known that the proportion \( q_H \) are of high risk and the proportion \( (1 - q_H) \) are of low risk, with \( 0 < q_H < 1 \). Let \( N \) be the total number of potential customers, so that \( q_H N \) are of type \( H \) and \( (1 - q_H)N \) are of type \( L \).

Let \( h_H^* \) be the maximum premium that the \( H \) people would be willing to pay for full insurance and \( h_L^* \) be the maximum premium that the \( L \) people would be willing to pay for full insurance:

**Case 1: MONOPOLY**

What policy or policies would the monopolist want to offer? There are three options.

**OPTION 1.** Offer only one contract, which is attractive only to the \( H \) type. In this case the monopolist will want to offer a contract which is on the indifference curve of the \( H \) type that goes through the No Insurance point. Since profits increase along that indifference curve moving towards the \( 45^\circ \) line, the profit-maximizing contract under Option 1 is the full insurance contract with premium \( h_H^* \) and the corresponding profits will be:

\[ \pi_1 = q_H N(h_H^* - p_H x) \]
OPTION 2. Offer only one contract, which is attractive to both types. In this case the monopolist will want to offer a contract which is on the indifference curve of the $L$ type that goes through the No Insurance point. However, it is not optimal to offer full insurance (at premium $h_L^\ast$). To see this, note that when both types apply, profits from a contract $(h, D)$ are given by

$$\pi = N\left[h - \bar{p}(x - D)\right]$$

where $\bar{p} = q_H p_H + (1 - q_H) p_L$ is the average probability of loss. Clearly, $p_L < \bar{p} < p_H$. Thus the isoprofit line that goes through contract $(h, D)$ is the straight line with slope $-\frac{\bar{p}}{1 - \bar{p}}$ and, since $p_L < \bar{p} < p_H$, $\frac{\bar{p}}{1 - \bar{p}} > \frac{p_L}{1 - p_L}$. Thus, since the slope of the $L$-indifference curve along the $45^\circ$ line is $-\frac{p_L}{1 - p_L}$, the isoprofit line is steeper than the indifference curve at the full insurance contract and, therefore, there is a contract, like $B$ in Figure 10 below, that yields higher profits than the full insurance contract $A$.

![Figure 10](image)

The same argument applies to any other point on the indifference curve of the $L$ type that goes through the No Insurance point at which the slope of the indifference curve is less than $-\frac{\bar{p}}{1 - \bar{p}}$. Similarly, a contract on the indifference curve at which the slope of the indifference curve is larger than $-\frac{\bar{p}}{1 - \bar{p}}$, cannot be optimal (moving to the right towards the $45^\circ$ line would increase profits). Thus the best contract under Option 2 is that contract on the indifference curve of the $L$ type that goes through the No Insurance point at which the slope is equal to $-\frac{\bar{p}}{1 - \bar{p}}$.

There is no need to compute the optimal contract under Option 2, because we will show later that **Option 2 is never optimal**.
**OPTION 3.** Offer two contracts, one targeted to the $H$ type and the other targeted to the $L$ type. Then, by the usual argument, the contract targeted to the $H$ type must be a full insurance contract. Then the **first constraint** the monopolist faces is that the premium $h$ for the full insurance policy targeted to the $H$ type must be $h \leq h^*_H$. The **second constraint** is that the other policy must be less attractive than the full insurance policy for the $H$ type, that is, it must lie below the indifference curve going through the full insurance policy. The **third constraint** is that the policy targeted to the $L$ type must be attractive to them, that is, it cannot lie below their indifference curve that goes through the No Insurance point. Since profits from the $L$ type increase along this indifference curve moving towards the $45^\circ$ line, the contract targeted to them must be the contract that lies at the intersection of the two indifference curves (see Figure 11 below).

Let $L = (h_L, D_L)$ be the policy targeted to the $L$ types and $H = (h_H, 0)$ the policy targeted to the $H$ types and suppose that all these constraints are satisfied. Then profits will be

$$\pi_3 = (h_H - p_H x)q_H N + (h_L - p_L x + p_L D_L)(1 - q_H) N.$$

Now, **Option 3 yields higher profits than Option 2.** To see this, start with the pooling contract of Option 2 (point $B$ in Figure 12 below) and draw the indifference curve for the $H$ type that goes through that contract. Let $C$ be the contract at the intersection of this indifference curve and the $45^\circ$ line. Then profits from the $H$ type will be higher at $C$ than at $B$ (profits increase along an indifference curve when moving towards the $45^\circ$ line). If the firm offers a full-insurance contract with a premium slightly lower than the premium associated with $C$, then the $H$ people will switch.
from $B$ to $C$, while the $L$ people will stay at $B$. Thus profits from the $L$ people won’t change, but profits from the $H$ people will increase. Hence the original pooling contract $B$ is not optimal.

In conclusion,

- when $q_H$ is close to 1, the monopolist will offer only the full-insurance contract with premium $h_H$.
- when $q_H$ is not close to 1, the monopolist will offer two contracts as explained under Option 3.

Case 2: COMPETITIVE INDUSTRY

Consider now a competitive industry where free entry leads to zero profits. Define an equilibrium as a set of contracts such that (1) every firm makes zero profits and (2) no (existing or new) firm could make positive profits by introducing a new contract.

Now, could there be a pooling equilibrium where only one contract is offered (with non-negative deductible, so that $W_1 \leq W_2$), everybody buys it and the firms make zero profits? The answer is No. Let $A$ be such a contract. By the crossing property of the indifference curves there is a contract $B$ which is between the two indifference curves, so that $B$ would be preferred by the $L$ type but not by the $H$ type and therefore would attract only and all the $L$ types.
Contract $A$ consists of a premium $h_A > 0$ and deductible $D_A > 0$. Since it is on the average fair odds line, $h_A - \bar{p}(x-D_A) = 0$. Since $p_L < \bar{p}$, it follows that $h_A - p_L(x-D_A) > 0$. Choose a contract $B = (h_B, D_B)$ between the two indifference curves (as shown in the picture above) with $h_B < h_A$ and $D_B > D_A$ but small enough so that $h_B - p_L(x-D_B) > 0$ (such a contract exists because the function $f(h,D) = h - p_L(x-D)$ is continuous and $f(h_A, D_A) = h_A - p_L(x-D_A) > 0$). Then a firm offering such a contract would attract all and only the $L$ types and make positive profits.

Thus if there is a zero-profit equilibrium it must be an equilibrium with at least two contracts. Such an equilibrium is called a separating equilibrium if all the $L$ types buy one contract and all the $H$ types buy a different contract. What would such an equilibrium look like with exactly two contracts? The zero-profit equilibrium requires that the $H$-contract be on the fair odds line for the $H$ type, that is, on the line with slope $-\frac{p_H}{1-p_H}$, and that the $L$-contract be on the fair odds line for the $L$ type, that is, on the line with slope $-\frac{p_L}{1-p_L}$.
wealth in good state (no loss) probability of good state: $1 - p$

wealth in bad state (loss of $x$) probability of bad state: $p$

By the argument used above, if the H type is not offered full insurance, then somebody could step in and offer a full-insurance contract attractive to the H type and make positive profits (profits from the $H$ type increase when traveling along an indifference curve towards the 45° line). Thus the contract designed for the H type must be on the 45° line. It is shown as point H in the following diagram.

An analogous full insurance contract for the L type (given by the intersection of the 45° line and the fair odds line for the L type) cannot be offered, because such a contract would be more attractive than contract $H$ for the $H$ type, everybody would buy it and it would yield negative profits (because, when everybody buys the same contract the relevant fair odds line is the average one, which is steeper than the $L$ one). The incentive compatibility constraint for the $H$ type requires the contract designed for the $L$ type to be below or on the indifference curve of the $H$ type that goes through the full insurance contract $H$. The zero profit condition requires it to be on the fair odds line of the $L$ type as close as possible to the 45° line (because traveling along the $L$ indifference curve towards the 45° line increases profits from the $L$ type). Such a point is point $L$ in the above figure. The $L$ types prefer contract $L$ (because of the way the $L$ and $H$ indifference curves cross at the $H$ contract: see Figure 15 below).
Figure 15

Is this an equilibrium? It depends on the position of the fair odds line. Consider contract $P$ in Figure 16 below. It is more attractive than $L$ and $H$ for both groups, thus a firm offering it would attract both types. Since point $P$ lies below the average fair odds line, a firm offering it would make positive profits. Hence the pair $L$ and $H$ would not be an equilibrium (on the other hand we know from the previous analysis that $P$ cannot be an equilibrium either, because there cannot be a pooling equilibrium).
wealth in good state (no loss) probability of good state: $1 - p$

wealth in bad state (loss of $x$) probability of bad state: $p$

**Figure 16**

Thus for a separating equilibrium it must be the case that the average fair odds line be below (or at most tangent to) the indifference curve of the $L$ type through contract $L$. This amounts to saying that the fraction of $H$ type in the population is sufficiently high.