1) The cost of attending your college has once again gone up. Although you have been told that education is investment in human capital, which carries a return of roughly 10% a year, you (and your parents) are not pleased. One of the administrators at your university/college does not make the situation better by telling you that you pay more because the reputation of your institution is better than that of others. To investigate this hypothesis, you collect data randomly for 100 national universities and liberal arts colleges from the 2000–2001 U.S. News and World Report annual rankings. Next you perform the following regression

\[
\widehat{Cost} = 7,311.17 + 3,985.20 \times \text{Reputation} - 0.20 \times \text{Size} \\
+ 8,406.79 \times \text{Dpriv} - 416.38 \times \text{Dlibart} - 2,376.51 \times \text{Dreligion}
\]

\[
(2,058.63) \quad (664.58) \quad (0.13) \\
(2,154.85) \quad (1,121.92) \quad (1,007.86)
\]

\[R^2 = 0.72, \; \text{SER} = 3,773.35\]

where Cost is Tuition, Fees, Room and Board in dollars, Reputation is the index used in U.S. News and World Report (based on a survey of university presidents and chief academic officers), which ranges from 1 ("marginal") to 5 ("distinguished"), Size is the number of undergraduate students, and Dpriv, Dlibart, and Dreligion are binary variables indicating whether the institution is private, a liberal arts college, and has a religious affiliation. The numbers in parentheses are heteroskedasticity–robust standard errors.

(a) Interpret the results and indicate whether or not the coefficients are significantly different from zero. Do the coefficients have the expected sign?

(b) What is the forecasted cost for a liberal arts college, which has no religious affiliation, a size of 1,500 students and a reputation level of 4.5? (All liberal arts colleges are private.)

(c) To save money, you are willing to switch from a private university to a public university, which has a ranking of 0.5 less and 10,000 more students. What is the effect on your cost? Is it substantial?

(d) What is the p-value for the null hypothesis that the coefficient on Size is equal to zero? Based on this, should you eliminate the variable from the regression? Why or why not?

(e) You want to test simultaneously the hypotheses that \( \beta_{\text{Size}} = 0 \) and \( \beta_{\text{Dlibart}} = 0 \). Your regression package returns the F-statistic of 1.23. Can you reject the null hypothesis?

(f) Eliminating the Size and Dlibart variables from your regression, the estimation regression becomes

\[
\widehat{Cost} = 5,450.35 + 3,538.84 \times \text{Reputation} + 10,935.70 \times \text{Dpriv} - 2,783.31 \times \text{Dreligion};
\]

\[
(1,772.35) \quad (590.49) \quad (875.51) \quad (1,180.57)
\]
Why do you think that the effect of attending a private institution has increased now?

(g) You give a final attempt to bring the effect of Size back into the equation by forcing the assumption of homoskedasticity onto your estimation. The results are as follows:

\[
\tilde{\text{Cost}} = 7,311.17 + 3,985.20 \times \text{Reputation} - 0.20 \times \text{Size} \\
+ 8,406.79 \times D\text{priv} - 416.38 \times D\text{libart} - 2,376.51 \times D\text{religion} \\
\]

\[
\begin{align*}
(1,985.17) & & (593.65) & & (0.07) \\
(1,423.59) & & (1,096.49) & & (989.23)
\end{align*}
\]

\[R^2 = 0.72, \ SER = 3,682.02\]

Calculate the \( t \)-statistic on the Size coefficient and perform the hypothesis test that its coefficient is zero. Is this test reliable? Explain.

(h) What can you say about causation in the above relationship? Is it possible that Cost affects Reputation rather than the other way around?

2) (Requires Appendix Material) Consider the following multiple regression model

\[
Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i
\]

You want to consider certain hypotheses involving more than one parameter, and you know that the regression error is homoskedastic. You decide to test the joint hypotheses using the rule-of-thumb \( F \)-statistics. For each of the cases below specify a restricted model and indicate how you would compute the \( F \)-statistic to test for the validity of the restrictions.

(a) \( \beta_1 = -\beta_2; \beta_3 = 0 \)

(b) \( \beta_1 + \beta_2 + \beta_3 = 1 \)

(c) \( \beta_1 = \frac{\beta_2}{\beta_3} \)

(d) \( \beta_1 = -\beta_2; \beta_3 = 0 \)
3) Write the following four restrictions in the form $R\beta = r$, where the hypotheses are to be tested simultaneously.

$\beta_3 = 2\beta_5$,  
$\beta_1 + \beta_2 = 1$,  
$\beta_4 = 0$,  
$\beta_2 = -\beta_6$.

Can you write the following restriction $\beta_2 = -\frac{\beta_2}{\beta_1}$ in the same format? Why not?