Professor Salyer, Economics 101, Winter 01

Homework 2 Answers

1. To answer this question, I did the following steps: (1) using the Mankiw Data Plotter, I obtained the figures for real GDP and population for the years 1990 and 1998. (2) Using the transform option, I converted these numbers to logs. (3) The average annual growth rate was calculated by:

   \[ g_x = \frac{\ln x_{98} - \ln x_{90}}{8} \]

   where \( g_x \) denotes the growth rate of variable \( x \). Letting \( y \) denote real GDP and \( L \) denote population, the growth rates are \( g_y = 0.026 \) and \( g_L = 0.01 \). Since \( g_x^L = g_y - g_L \), then per-capita real growth averaged 1.6% over the years 1990-1998.

2. This is straight from the web site.

3. Recall that nominal GDP (NGDP) is the dollar value of final goods and services sold in a year - this can be thought of as the product of the price level, \( P \), and real output, \( Y \). That is, \( NGDP = PY \) (think of the simple case where there is just one good - then nominal GDP is simply the total revenue from sales). Consequently, using our growth rate relationships discussed in class, \( g_{NGDP} = g_P - g_y \). Since the growth rates of nominal GDP and the price level are given, the growth of real GDP is: \( g_y = g_{NGDP} - g_P \). Finally, the question asks for the growth of per-capita real GDP which, as in question 1, is \( g_x^L = g_y - g_L \). The numbers given in the question therefore imply:

<table>
<thead>
<tr>
<th>Annual Growth Rate</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-capita real GDP</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

   The answer to (a) is country C while the answer to (b) is country A.

4. To answer this question, recall the relationship that, if a variable, \( x \), is growing at rate \( g_x \), then the value of the variable \( T \) periods in the future is:

   \[ x_T = x_0 (1 + g_x)^T \]

   where \( x_0 \) denotes the initial value. For part (a), let \( Z \) denote
the common starting income in countries \(X\) and \(Y\). Since \(Y\) has a faster growth rate, it will have a greater income than country \(X\) after 20 years of growth. By taking the ratio of their incomes, we calculate how much greater \(Y\)'s income is as a percentage of country \(X\) income:

\[
\%\Delta Y_X = \frac{Z(1.04)^{20}}{Z(1.03)^{20}} = 1.21
\]

so that country \(Y\)'s income is 21\% greater than country \(X\). For part (b), we need to solve the following expression for \(T\):

\[
(2Y_k)(1.03)^T = Y_k(1.04)^T
\]

where I have used the fact that, initially, country \(H\) has twice the income of country \(K\). The above expression is, as stated, difficult to solve since the unknown, \(T\), is in the exponent. To make things easier, take logs and simplify the expression to get:

\[
\ln 2 = T (\ln 1.04 - \ln 1.03) \approx T (0.01)
\]

which implies that \(T \approx 69\) years. (note that this is just another take on the "Rule of 72" or, more accurately, "Rule of 70".)

4. (whoops - two #4's) The ratio, \(\frac{K}{K}\), has indeed been roughly constant and is equal to just a bit over 1. This understates the true ratio (which, as your text states, is about 2.5) since the capital stock on the Mankiw site does not include residential capital (housing).

5. (Don't worry too much about this question.) Investment demand is characterized by the equality: \(MPK = r\) (why?). If the production function is \(y = k^\alpha\) then we have \(MPK = \alpha k^{\alpha-1} = r\). Solve this expression for \(k\) as an explicit function of \(r\): \(k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}}\). Taking the derivative yields:

\[
\frac{dk}{dr} = \left(\frac{1}{\alpha - 1}\right) \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1} - 1} = \left(\frac{1}{\alpha - 1}\right) \frac{\alpha}{r}
\]

Note that the expression for \(k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}}\) has been used in the derivative. Rearranging terms yields:

\[
\frac{dk}{dr} k = \frac{dk}{dr/k} = \frac{\%\Delta k}{\%\Delta r} = \frac{\alpha}{1 - \alpha}
\]

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As $\alpha$ increases (which means the production function becomes more linear, i.e. has less curvature), the RHS becomes more negative. That is, the elasticity of capital with respect to interest rates increases.

6. The Solow growth model can not suggest any policies to promote long-run growth in per-capita income since growth in the model is exogenous. Of course, an increase in the savings rate or decrease in the population growth rate will produce growth in per-capita income in the transition to the new steady-state.

7. A greater depreciation rate causes the line defined by $(\delta + n)k$ to shift upward (just as an increase in $n$ does - see figure 4-12). Hence $A - \text{land}$ will have a lower per-capita capital stock and, consequently, a lower per-capita income level. The capital stock must fall to re-establish equality between the flow causing $k$ to increase (i.e. investment - $sf(k)$) with that causing $k$ to fall (i.e. depreciation - $\delta k$).

8. To answer this question, first express the production function in per-capita form:

$$\frac{Y}{L} = y = \frac{K^{\alpha}L^{1-\alpha}}{L} = \frac{K^{\alpha}}{L^{\alpha}} = k^{\alpha}$$

The steady state is defined by (letting lower case $s$ denote the savings rate)

$$sk^{\alpha} = (\delta + n)k$$

The question states $s = 0.16, \delta = 0.10, \alpha = 0.50$ (why?). Using these values and solving for $k$ produces $k = 2.56$.

9. a. $y = k^{0.25}$ where $y = \frac{Y}{LE}$ and $k = \frac{K}{LE}$.
   b. $Y = K^{0.25}(LE)^{0.75} \Rightarrow Y = (160,000)^{0.25}((405,000)(2))^{0.75} = 540,000$. So $\frac{Y}{L} = 1.33$.
   c. Steady-state requires: $sk^{0.25} = (\delta + n + g)k$ or $s = (\delta + n + g)k^{0.75}$. Using the values, $0.08 = (\delta + 0.17)\left(\frac{100,000}{810,000}\right)^{0.75} = (\delta + 0.17)(0.296)$. (Why do we know that $n + g = 0.17$?) Solving for $\delta = 0.10$.
   d. For the Golden Rule, we need $MPK = (\delta + n + g)$. From (c) we know $(\delta + n + g) = 0.27$. Also, since the production function is Cobb-Douglas, $MPK = \alpha \frac{Y}{K} = 0.25\left(\frac{540,000}{100,000}\right) = 0.84375$. So $MPK > (\delta + n + g)$ implying that the $k$ is too small. More savings is needed.

10. For the developed country, we need to solve $0.28k^{0.5} = (0.04 + 0.01 + 0.02)k$ which yields: $k = 16$ or $y = 4$. For the undeveloped country, we need to solve $0.10k^{0.5} = (0.04 + 0.04 + 0.02)k \Rightarrow k = 1 \Rightarrow y = 1$. 

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