Homework #3 – Answers

1. (Mankiw, Q. 2, p. 189) From the quantity theory, expressed in percentage change form, we have:
\[ g_m + g_v = \pi + g_y \]
where \( g_m, g_v, g_y \) are the growth rates of money, velocity and output respectively and \( \pi \) is the growth rate of prices – i.e. the rate of inflation. For the numbers given in the problem, the implication is that \( \pi = 9\% \). From the Fisher relation we have: \( i = r + \pi \) where \( i \) is the nominal interest rate and \( r \) is the real interest rate. We are told that the nominal interest rate is 11\%, so this implies that \( r = 2\% \).

2. (Mankiw, Q. 5, p. 189) The attempt was to create a high inflation rate. An increase in the inflation rate causes an increase in the tax on holding money – hence individuals will change their shopping patterns and portfolio holdings to escape this tax. If the inflation rate is high, the resulting change in behavior can be disruptive of the economic system.

3. (Mankiw, Q. 8, p. 189) This problem uses several components: the model of the real interest rate developed in Chapter 3, the Fisher relation, the demand for real balances, and the quantity theory. To these is added the assumption that real balances affect consumption.

**Step 1:** The first step is to use the quantity theory to make the link between money growth and inflation. We have

\[ g_m + g_v = \pi + g_y \]

The growth rate of velocity is 0 while the growth rate of output is pinned down by the rates of population growth and technological progress. This implies that

\[ \Delta g_m = \Delta \pi \]

So an increase in inflation causes a one-for-one change in inflation.

**Step 2:** From the Fisher relation, we have \( i = r + \pi \). Hence, the increase in inflation causes *ceteris paribus* an increase in the nominal interest rate.

**Step 3:** From the demand for real balances:

\[ \frac{M}{P} = L(Y, i) \]

(where \( L \) is used to denote the demand for liquidity), an increase in the nominal interest rate will result in a fall in the holdings of real balances.

**Step 4:** From the assumption made in the question, the fall of real balances will result in a fall in consumption. But note this means that household savings will increase.

**Step 5:** The increase in savings will cause the national savings curve to shift rightward. To restore equilibrium \( I = S \), the equilibrium real interest rate must fall. (This is the opposite of what is depicted in Figure 3-8 in your text.)

The question asks if the nominal interest rate changes one-for-one with inflation. From the Fisher relation, we see that while the inflation rate has increased, this has caused a fall in the equilibrium real interest rate. Hence, the nominal interest rate changes less than one-for-one.

4. a. By the quantity theory, we have: \( k = \frac{M}{PY} \). This implies the following values:

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b. Since the $k$ is given by the ratio of dollars ($M$) to dollars per year ($PY$), the units are years – the fraction of a year’s GDP that the current money holdings represent.
c. Yes, $k$ was relatively constant over this period.

5. The neutrality of money refers to the concept that changes in the money supply do not affect real economic activity - only the price level. This is often referred to as the classical dichotomy: real factors (tastes and technology) affect output, the money supply determines the price level.

6. The quantity theory of money is derived from the quantity equation (or equation of exchange) by assuming that velocity is constant. That is $M\bar{V} = PY$ or $M = kPY$ where $k = \frac{1}{\bar{V}}$. This can be interpreted as a money demand function in which the demand for money is proportional to the level of transactions (i.e. GDP). In equilibrium, demand and supply of money are equal -- this turns the quantity theory into a model of aggregate demand. That is $\bar{M} = kPY$ where $\bar{M}$ denotes the supply of money. The supply of money pins down nominal GDP.

7. The aggregate demand curve is defined by equilibrium in the money market. The money supply is being held constant and the curve slopes down because money demand is proportional to GDP ($PY$). Hence, if the price level is lower, output must be higher so that money demand is constant. The aggregate demand curve is not a typical demand curve.

8. This question involves two steps: first constructing the aggregate supply curve - which will be vertical under the assumption of perfectly flexible prices and then constructing the aggregate demand curve using the quantity theory.

**Aggregate Supply**

Output is determined solely by labor as represented by the production function: $Y = 3L^{\frac{1}{3}}$. So to determine the level of output we must first determine the equilibrium level of labor. This is at the intersection of labor supply and labor demand. Labor supply is given:

$$L^s = \frac{w}{P}$$

-- this says that labor increases one-for-one with the real wage.

To find labor demand, we use the optimality condition that firms will hire labor until the MPL is equal to the real wage. Taking the derivative of the production function:

$$MPL = L^{\frac{2}{3}} = \frac{w}{P}$$

Solving for $L$ (raise both sides to the power $-\frac{3}{2}$) yields: $L^D = \left(\frac{w}{P}\right)^{\frac{3}{2}}$.

What does the labor demand function look like. Note that the demand for labor goes to infinity as the real wage goes to zero. To solve for equilibrium, set labor supply equal to labor demand:

$$L^s = L^D \Rightarrow \frac{w}{P} = \left(\frac{w}{P}\right)^{\frac{3}{2}}$$
Solving this for the real wage (multiply both sides by $\left(\frac{w}{P}\right)^{\frac{3}{2}}$) yields:

$$\left(\frac{w}{P}\right)^* = 1.$$  

Substituting this into either labor demand or labor supply yields $L^* = 1$. Then substituting this equilibrium quantity of labor into the production function yields: $Y^* = 3$. Then the aggregate supply curve is a vertical line at the output level of 3.

**Aggregate Demand**

From the quantity theory, the model of aggregate demand is: $\bar{M} = kPY$ or, expressing this relationship consistent with a graph in which $P$ is on the vertical axis:

$$P = \frac{\bar{M}}{k} \left(\frac{1}{Y}\right)$$

Note that the aggregate demand curve will have a negative slope and the location will be determined by the money stock. We know that in equilibrium $Y = 3$; also $\bar{M} = 600$ and $k = 2$. This implies that $P^* = 100$.

Since the real wage is equal to 1, this implies that the nominal wage is 100.

9. Increases in money growth cause a one-for-one increase in inflation. From the Fisher relation, this implies an increase in nominal interest rates.