

Solution Key: Homework 2

Economics 101 - Chapters 8 and 9

- 1a) Defining $y = Y/L$, and $k = K/L$, the production function can be written: $y = k^{1/2}$.
- b) At steady state k^* : $sf(k^*) = (\delta+n)k^*$ so $s k^{*1/2} = (\delta+n)k^*$ so $k^* = (s/(\delta+n))^2$
 country A: $k_A^* = (0.2 / 0.1)^2 = 4$ country B: $k_B^* = (0.3 / 0.1)^2 = 9$
- c) steady-state output per worker:
 country A: $y_A^* = (k_A^*)^{1/2} = (4)^{1/2} = 2$ country B: $y_B^* = (k_B^*)^{1/2} = (9)^{1/2} = 3$
 steady-state consumption per worker:
 country A: $c_A^* = (1 - s_A) y_A^* = (1 - 0.2)(2) = 1.6$ country B: $c_B^* = (1 - 0.3)(3) = 2.1$
 So country B has a higher steady state level of consumption, and is therefore closer to the golden rule. Because country B saves more of its output, it can maintain a higher level of capital stock in steady state; the extra output permits higher levels of *both* saving and consumption
- d) Golden rule condition: $MPK = \delta+n$ so $(1/2) k_{gold}^{*-1/2} = 0.1$ so $k_{gold}^* = 25$
 find the required saving rate: $s f(k_{gold}^*) = (\delta+n)k_{gold}^*$ so $s 25^{1/2} = 0.1 * 25$ so $s = 0.50$
 Countries with this production function should have a saving rate of 50 percent, if they want to enjoy the maximum level of consumption in steady state.

- 2 a) According to the convergence hypothesis, both the U.S. and China should converge to the same steady state level of output per person and then both stop growing (in the case of no technological progress). So both disparity in levels and growth rates should disappear in time. (In the case of technological progress, both countries will reach the same steady state, which implies growth in capital and income per person at the rate of the technological progress.)
- b) If China has a higher saving rate than the U.S., this would tend to make China's steady state levels of capital and output per worker higher than the U.S. levels. But growth rates still will be the same (zero growth rate for the case of no technological progress).
- c) If China has a higher population growth rate, it would tend to make China's steady state levels of capital and output per worker lower than the U.S. levels. Growth rates still will be the same (zero growth rate for the case of no technological progress).

- 3) As the optimal point, the solution must satisfy the golden rule condition: $MPK = \delta$. (1)
 Second, as a steady state, it must satisfy: $sy^* = \delta k^*$. (2)
 Also, using Euler's theorem we know the share of income going to capital is $1 - 0.70 = 0.30$
 This is useful, because this share may be written as: $MPK * K / Y$
 So we know a third condition here: $MPK(k^*/y) = 0.30$ (3)
 Combining conditions (1) and (2): $s = MPK k^*/y^*$
 And then using condition (3): $s = 0.30$