# Neoclassical Model

Applies to long run, when prices are flexible and markets clear (supply=Demand)

"General equilibrium" model
- multiple markets
- price in each market adjusts to make supply equal demand

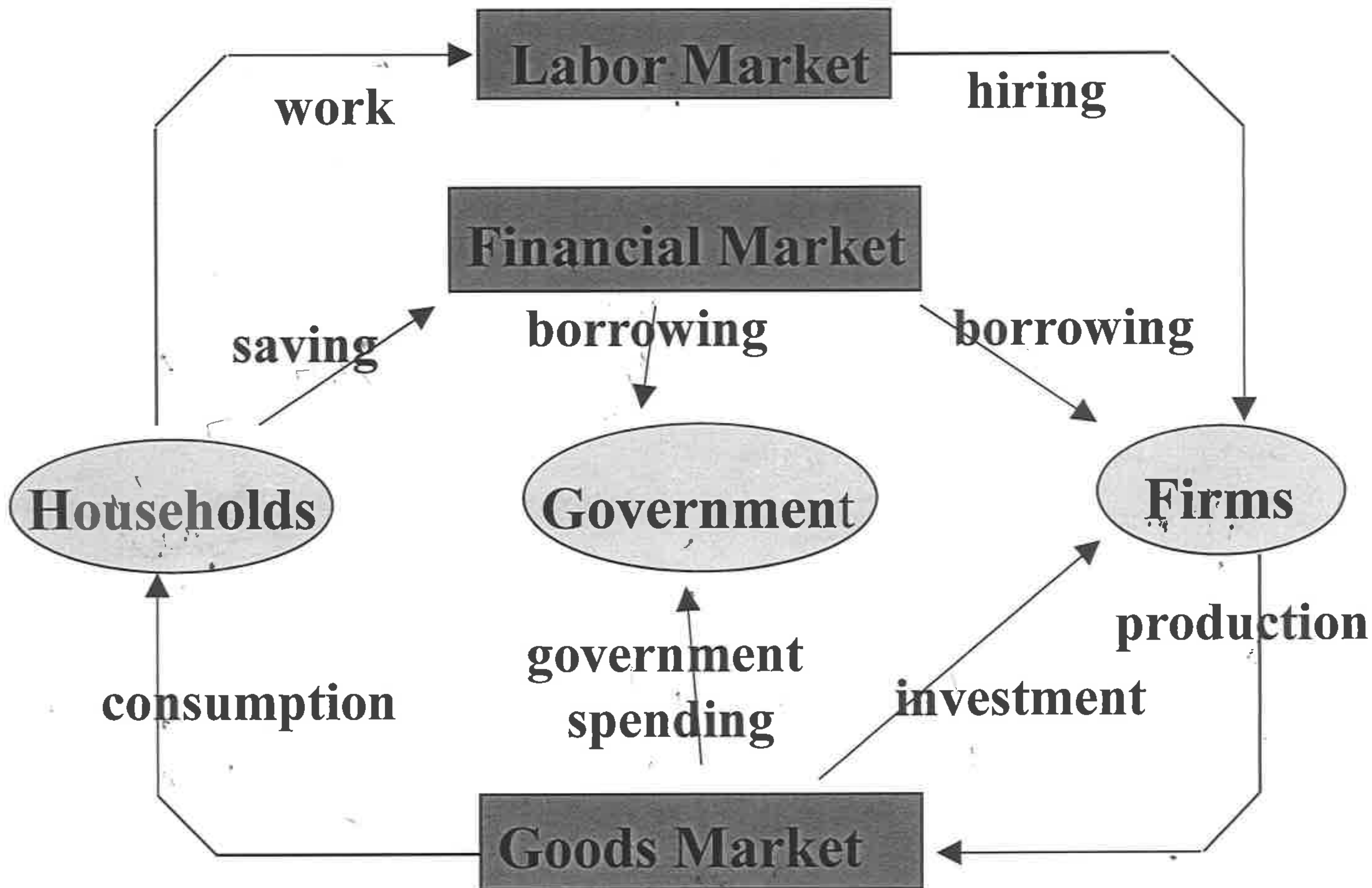
## Three types of markets

1) Goods market
2) factors markets -- labor, capital
3) financial market

## Three types of agents

1) households
2) firms
3) government

# Three Markets – Three agents



Supply in goods market

Depends on a production function,
denoted $Y = F(K, L)$
where $K$ = capital
$L$ = labor

Assume it exhibits constant returns to scale

Let $Y_1 = F(K_1, L_1)$

Scale all inputs by same multiple, $z$

$$K_2 = zK_1 \quad L_2 = zL_1 \quad \text{for } z > 1$$

What happens to output?

if $Y_2 = zY_1$

Then $F$ exhibits constant returns to scale.

example: suppose $F(K, L) = 2K + 15L$

$$F(zK, zL) = 2(zK) + 15(zL)$$
$$= z(2K + 15L)$$
$$= zF(K, L) \checkmark$$

yes, C.R.S.

## Assumptions

1) technology is fixed
2) supplies of capital and labor are fixed at:

$$K = \bar{K} \qquad L = \bar{L}$$

So we have a simple initial theory of supply in the goods market

$$\bar{Y} = F(\bar{K}, \bar{L})$$

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## Demand in Factors market:

Decision of a typical firm:

- hire labor in labor market where price is wage, $W$.
- it rents capital in factor market at rate $R$.
- It uses labor and capital to produce the good, which it sells in the goods market, at price $P$.

Assume the markets are "competitive"

- each firm is small relative to the market
- so it takes market prices as given: $W$, $R$, $P$.

Firm hires labor to maximize its profits.

profit = revenue − labor costs − capital costs

$$= P \cdot Y - W \cdot L - R \cdot K$$

$$= P \cdot F(K,L) - WL - RK$$

Question: how much does an extra unit of L raise output?

Use marginal product of Labor

version in text: $MPL = F(K, L+1) - F(K,L)$

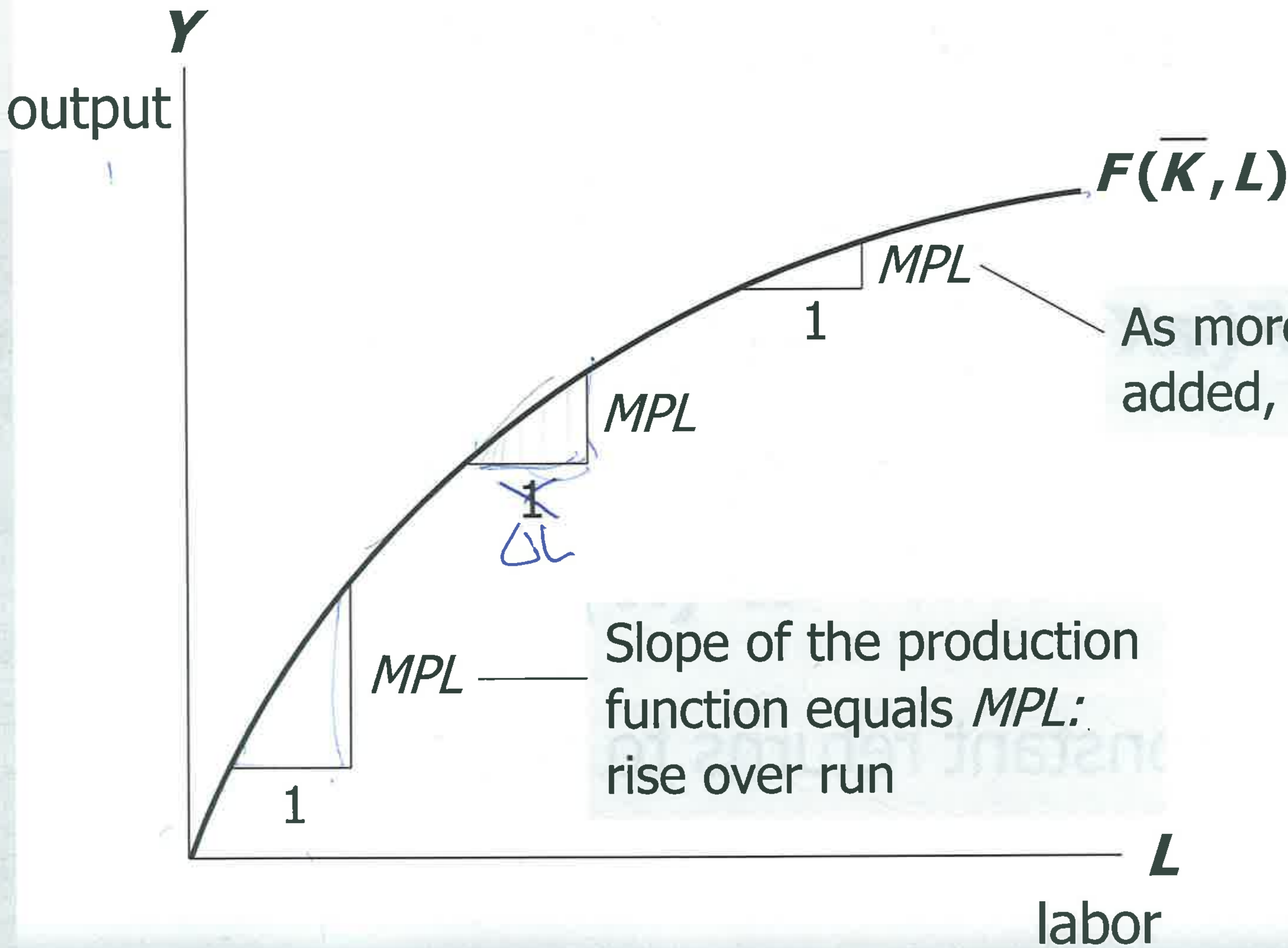
more precise definition: "$\Delta$: change"

$$MPL = \lim_{\Delta L \to 0} \frac{F(K, L + \Delta L) - F(K,L)}{\Delta L}$$

$$= f_L(K,L)$$

MPL is the derivative of the production function with respect to L.

# The MPL and the production function




As more labor is added, $MPL \downarrow$

Slope of the production function equals *MPL*: rise over run

# Derivative as marginal product

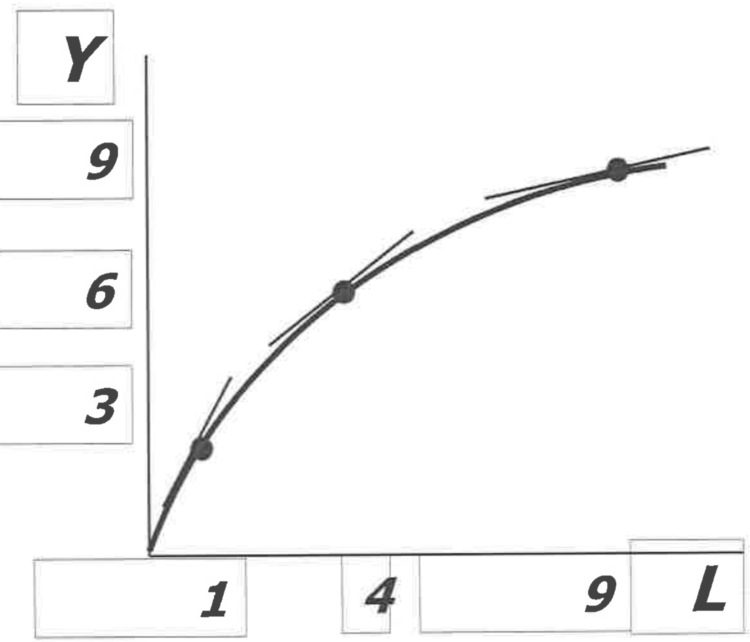
$$Y = F(K,L) = 3K^{\frac{1}{2}}L^{\frac{1}{2}}$$

*and let* $K = 1$

*so* $Y = 3\sqrt{L}$

$$\frac{\partial Y}{\partial L} = f_L = 3K^{\frac{1}{2}} \cdot \frac{1}{2}L^{\frac{1}{2}-1}$$

$$= \frac{3}{2}L^{-\frac{1}{2}} = \frac{3}{2\sqrt{L}}$$




| | | | |
|---|---|---|---|
| $L$: | 1 | 4 | 9 |
| $F(L)$: | 3 | 6 | 9 |
| $f_L$: | 1.5 | 0.75 | 0.5 |

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recall,

Marginal product of labor (MPL) is the derivative of the production function with respect to Labor (L).

Diminishing marginal product:
As the factor input is increased, all other things constant, its marginal product falls.

Demand in factors market for labor

$\Delta$ Profit = $\Delta$ revenue $-$ $\Delta$ cost

↑ "change" $= P \cdot MPL - W$

Firm choosing Labor (L) to maximize its profit

if this is: $>0$ should hire more

$<0$ should hire less

$=0$ hiring right amount

So firm's demand for labor is determined by:

$P \cdot MPL - W = 0$

or $P \cdot MPL = W$

or $\boxed{MPL = W/P}$

where $W/P$ is "real wage"

$W = \$/\text{hour}$

$P = \$/\text{good}$

$\frac{W}{P} = (\cancel{\$}/\text{hour}) / (\cancel{\$}/\text{good}) = \frac{\text{goods}}{\text{hour}}$

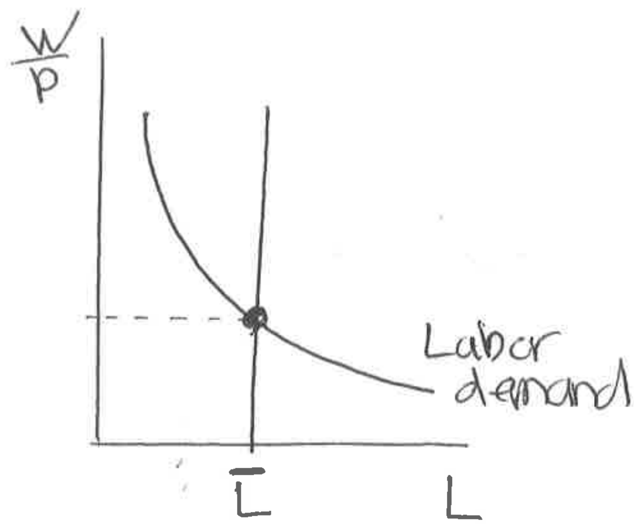
## Example

Suppose production function: $Y = 3K^{\frac{1}{2}} L^{\frac{1}{2}}$

Labor demand is:

$$MPL = \frac{W}{P}$$

$$\frac{3}{2} K^{\frac{1}{2}} L^{-\frac{1}{2}} = \frac{W}{P}$$




Supply in labor market:

$$L^{supply} = \bar{L}$$

Combine supply and demand:

$$\frac{W}{P} = \frac{3}{2} K^{\frac{1}{2}} \bar{L}^{-\frac{1}{2}}$$

to determine equilibrium real wage.

So a rise in labor supply → fall in equilibrium wage

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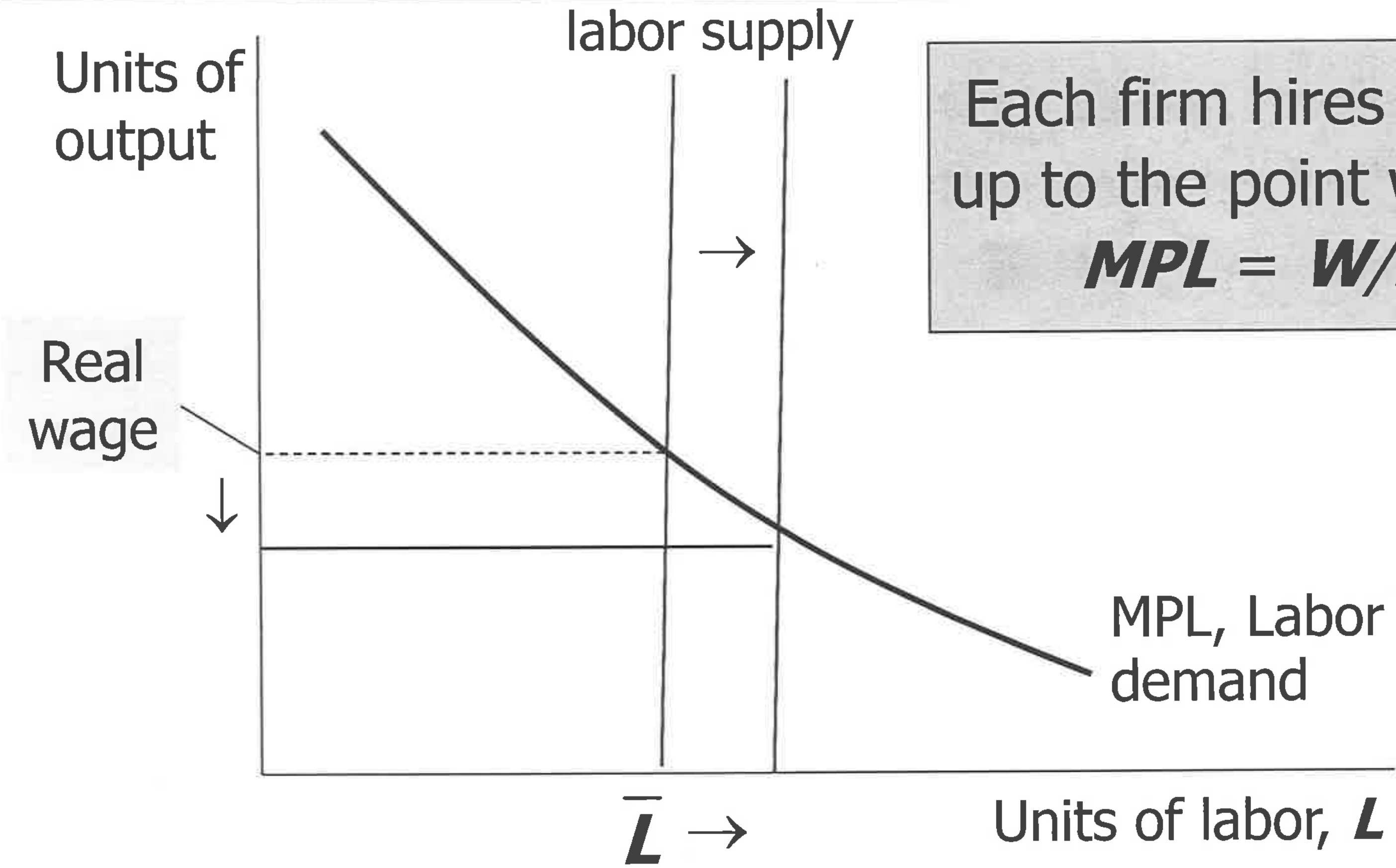
Same logic applies to factor market for capital:

$$\underbrace{MPK}_{} = R/P$$

"real rental rate" on capital

marginal product of capital = derivative of production function with respect to K.

# *MPL* and the demand for labor




Each firm hires labor up to the point where
***MPL* = *W/P***

How is income distributed:

total labor income:

$$= \frac{W}{P} \cdot \overline{L} = MPL \cdot \overline{L}$$

total capital income:

$$= \frac{R}{P} \overline{K} = MPK \cdot \overline{K}$$

Euler's Theorem:

Under our assumptions (constant returns to scale, profit maximization, and competitive markets) ...

total output is divided between the payments to capital and labor, depending on their marginal productivities, with no extra profit left over.

$$\overline{Y} = \underbrace{MPL \cdot \overline{L}}_{\text{labor income}} + \underbrace{MPK \cdot \overline{K}}_{\text{capital income}}$$

national income

Demonstration of Euler's Theorem (4/8/15)

production: $Y = AK^{\alpha}L^{1-\alpha}$

use $MPL = (1-\alpha)AK^{\alpha}L^{-\alpha}$

$MPK = \alpha A K^{\alpha-1} L^{1-\alpha}$

So factor payments are:

$MPL \cdot L + MPK \cdot K$

$= (1-\alpha)AK^{\alpha}L^{-\alpha} \cdot L + \alpha AK^{\alpha-1}L^{1-\alpha} \cdot K$

$= (1-\alpha)AK^{\alpha}L^{1-\alpha} + \alpha AK^{\alpha}L^{1-\alpha}$

$= AK^{\alpha}L^{1-\alpha} = Y$

So yes, total factor payments equals total production

101-B 4/10/15 ①

Demand in goods market = C + I + G

1) Consumption (C)

consumption function

$$C = C(Y-T)$$

where Y−T is "disposable income" income minus taxes (T)

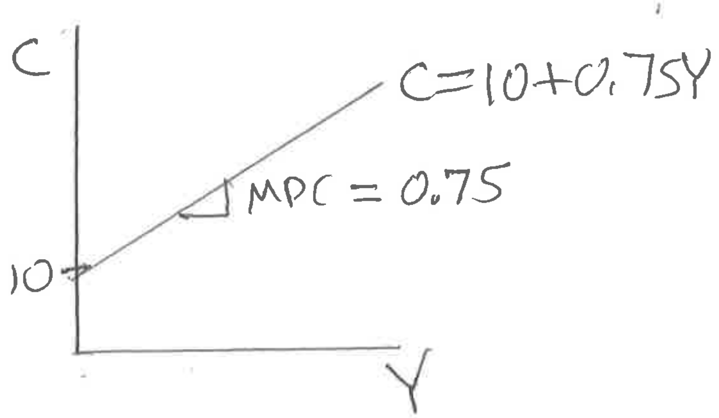
def: marginal propensity to consume (MPC) -- the increase in C caused by an increase in disposable income.

Assume MPC must be between 0 and 1.

MPC will be the derivative of the consumption function

example: $C = 10 + 0.75Y$

so $MPC = 0.75$




Def: Marginal propensity to save = 1 − MPC

## Investment

Investment function: $I = I(r)$
where $r$ denotes "real interest rate", the nominal interest rate adjusted for inflation.

The real interest rate is the cost of borrowing to get the funds to carry out an investment project

So $\uparrow r \Rightarrow \downarrow I$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## Government spending

$G = \overline{G}$ $\qquad$ $T = \overline{T}$

Government spending and taxes are taken as exogenous (determined outside the model)

## Equilibrium in Goods Market:

Demand: $C(Y-\overline{T}) + I(r) + \overline{G}$

Supply: $\overline{Y} = F(\overline{K}, \overline{L})$

Equilibrium condition:

$$\overline{Y} = C(\overline{Y} - \overline{T}) + I(r) + \overline{G}$$

The real interest rate ($r$) adjusts to equate demand with supply.

Note: output level is determined here by supply alone, not demand.

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A simple model of the financial market

Assume one asset: "loanable funds"

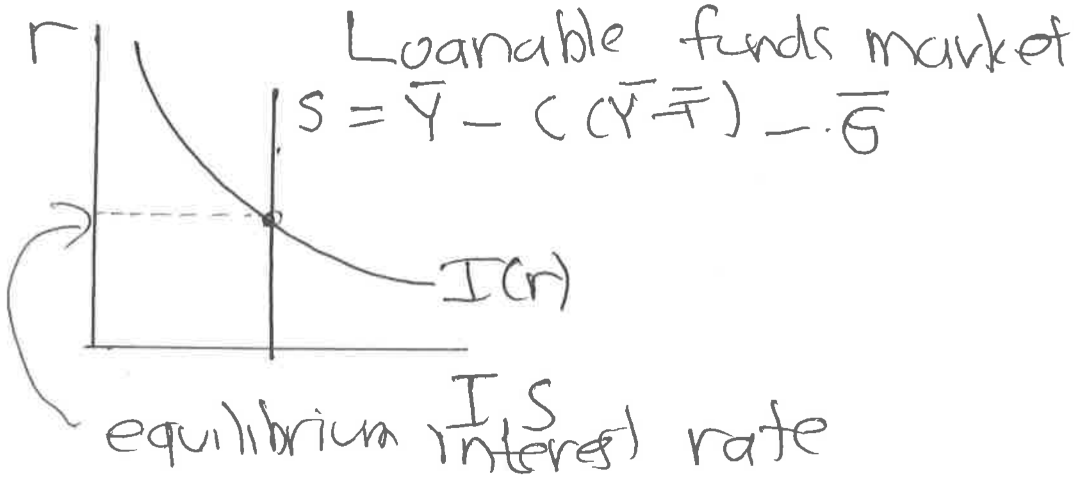
- supply: saving
- demand: investment (get loans)
- "price": real interest rate

Supply of loanable funds comes from saving of

- households: if not spend their income, they save it in a bank
- government: government saving = taxes − spending

Demand for loanable funds comes from investment -- depends negatively on the real interest rate ($r$).

Loanable funds market



equilibrium condition in <u>financial market</u>

supply of loanable funds = demand for loanable funds

national saving = Investment

$$\boxed{S = I(r)}$$

<u>types of saving:</u>

private saving: $S^p = (Y-T) - C$

government saving: $S^g = T - G$

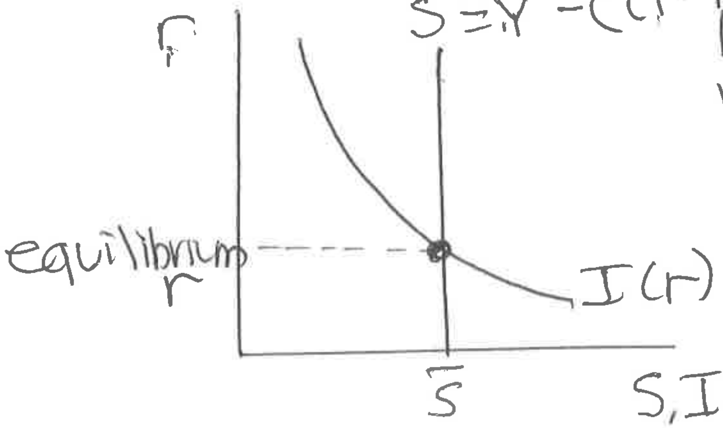
national saving: $S = S^p + S^g$

$$= (Y - \cancel{T}) - C + \cancel{T} - G$$

$$S = Y - C - G$$

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$

loanable funds market



rearrange financial equilibrium condition

$$S = I(r)$$

or $Y - C - G = I(r)$

$\rightarrow Y = C + I(r) + G$

This shows that r adjusts to equilibrate both the good market and financial market.

# Algebra example

Suppose an economy characterized by:

- Factors market supply:
  - labor supply= ***1000***
  - Capital stock supply= ***1000***
- Goods market supply:
  - Production function: $Y = 3K + 2L$
- Goods market demand:
  - Consumption function: $C = 250 + 0.75(Y-T)$
  - Investment function: $I = 1000 - 5000r$
  - $G=1000$, $T = 1000$

# Algebra example continued

Given the exogenous variables ($Y, G, T$), find the equilibrium values of the endogenous variables ($r, C, I$)

Find r using the goods market equilibrium condition:

$$Y = C + I + G$$
$$5000 = 250 + 0.75(5000-1000) + 1000 - 5000r + 1000$$
$$5000 = 5250 - 5000r$$
$$-250 = -5000r \quad \text{so } r = 0.05$$

And $I = 1000 - 5000*(0.05) = 750$

$C = 250 + 0.75(5000 - 1000) = 3250$

<u>example</u>

<u>Reagan policies</u> during 1980s

big tax cuts: $\Delta T < 0$

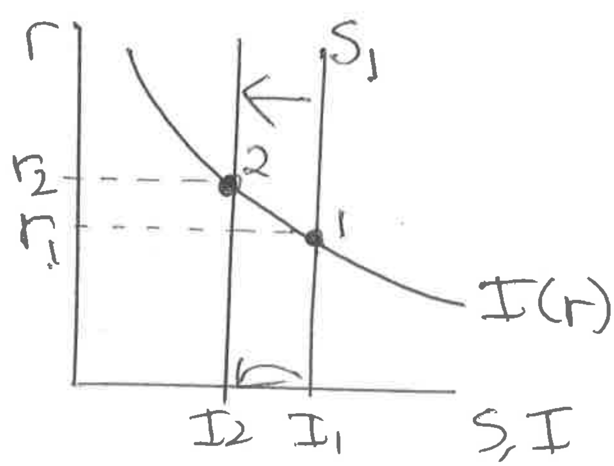
increase in government spending (on defense) $\Delta G > 0$

both policies reduce national saving

$$\underset{\downarrow}{S} = Y - \underbrace{C(Y - \overset{\downarrow}{T})}_{\uparrow} - \underset{\uparrow}{G}$$

$\uparrow G \rightarrow \downarrow S$

$\downarrow T \rightarrow \uparrow C \rightarrow \downarrow S$




The policy shifts the saving line left, which raises the equilibrium interest rate and lowers investment.

# *Are the data consistent with these results?*

| variable | 1970s | 1980s |
|---|---|---|
| $T - G$ | –2.2 | –3.9 |
| $S$ | 19.6 | 17.4 |
| $r$ | 1.1 | 6.3 |
| $I$ | 19.9 | 19.4 |

*$T$–$G$, $S$, and $I$ are expressed as a percent of GDP*

*All figures are averages over the decade shown.*