Solow Model of Economic Growth

how this model differs from previous chapter:

1) K and L no longer fixed.
2) consumption function somewhat simpler
3) No G or T

Production function in 'per person' terms

$Y = F(K, L)$

define $y = Y/L$ output per person

$k = K/L$ capital per person

assume constant returns to scale

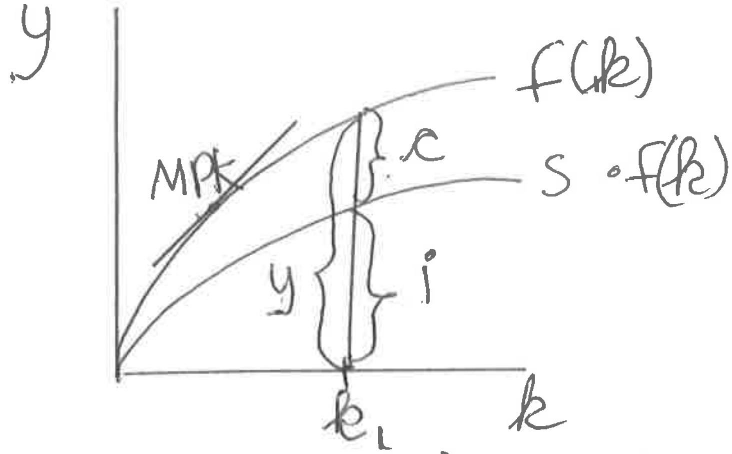
$zY = F(zK, zL)$

pick $z = 1/L$

$\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right)$

$y = F(k, 1)$

$\boxed{y = f(k)}$



note: MPK diminishes in k

National income identity in 'per person' terms

$$Y = C + I + \cancel{G}$$

or $$\frac{Y}{L} = \frac{C}{L} + \frac{I}{L}$$

$$y = c + i$$

where $c = C/L$, $i = I/L$

Define s = saving rate
(an exogenous parameter)

Define consumption function:

$$c = (1-s)y$$

so saving per person $= y - c$

$$= y - (1-s)y$$

$$= sy$$

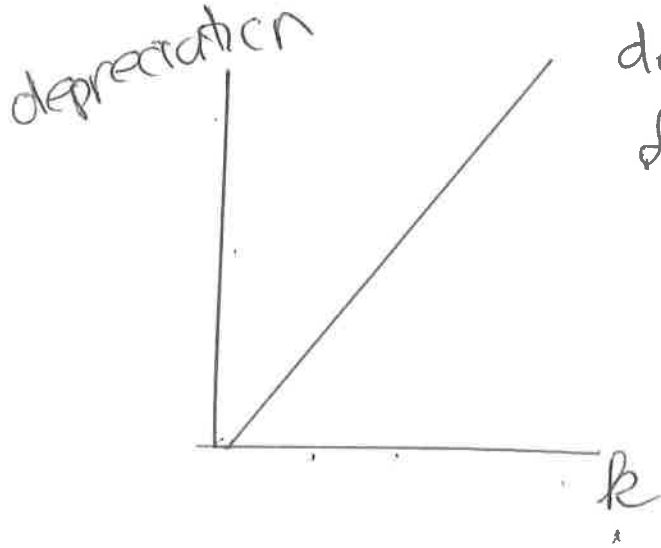
investment

investment per person = saving per person

$$i = sy = sf(k)$$

Depreciation rate (δ)

= fraction of the capital stock that wears out each period.



main idea: investment makes capital bigger; depreciation makes it smaller.

Change in capital stock = investment − depreciation

$$\Delta k = i - \delta k$$

since $i = sf(k)$

$$\Delta k = sf(k) - \delta k$$

If investment is just enough to cover depreciation ($sf(k) = \delta k$) then capital per person will remain constant

$$\Delta k = 0$$

This constan value, k^*, is called the "steady state."

<u>example</u>

suppose production functions

$Y = K^{\frac{1}{2}} L^{\frac{1}{2}}$

in per persion terms:

$$\frac{Y}{L} = \frac{K^{\frac{1}{2}} L^{\frac{1}{2}}}{L} = \left(\frac{K^{\frac{1}{2}}}{L^{\frac{1}{2}}}\right)\left(\frac{L^{\frac{1}{2}}}{L^{\frac{1}{2}}}\right) = \left(\frac{K}{L}\right)^{\frac{1}{2}}\left(\frac{L}{L}\right)^{\frac{1}{2}}$$

$$= k^{\frac{1}{2}} \cdot 1 = k^{\frac{1}{2}}$$

so $y = k^{\frac{1}{2}}$

suppose $s = 0.3$ (saving rate)

$\delta = 0.1$ (depreciation rate)

compute steady state:

$\Delta k = sf(k) - \delta k$

and set $\Delta k = 0$

so $\boxed{sf(k^*) = \delta k^*}$ at steady state k^*

$$0.3\sqrt{k^*} = 0.1 k^*$$

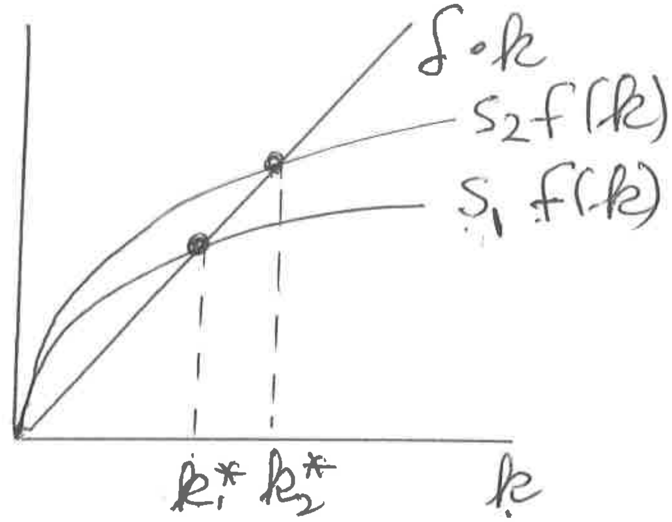
$$\frac{0.3}{0.1} = \frac{k^*}{\sqrt{k^*}}$$

$$3 = \sqrt{k^*} \rightarrow k^* = 9$$

and $y^* = \sqrt{k^*} = \sqrt{9} = 3$

$c^* = (1-s)y^* = 0.7 * 3 = 2.1$ } steady state

An increase in saving rate:



Shifts up the saving curve

→ higher steady state k^*

→ higher y^*

Golden rule

The steady state level of capital per person that maximizes consumption per person (k^*_{gold})

To find it, express steady state consumption (c^*) in terms of k^*:

$$c^* = y^* - i^*$$
$$= f(k^*) - i^*$$
$$= f(k^*) - \delta k^*$$

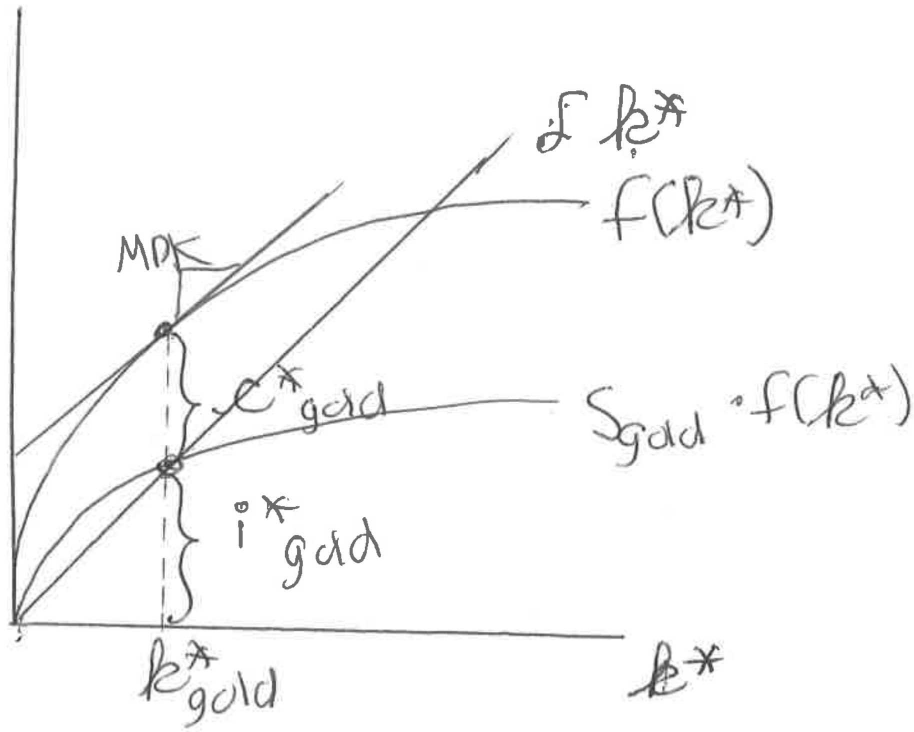
note: $i^* = \delta k^*$ when $\Delta k^* = 0$ which applies to all steady states

To maximize c^*, take derivative and set to zero

$$\frac{\partial c^*}{\partial k^*} = MPK - \delta = 0$$

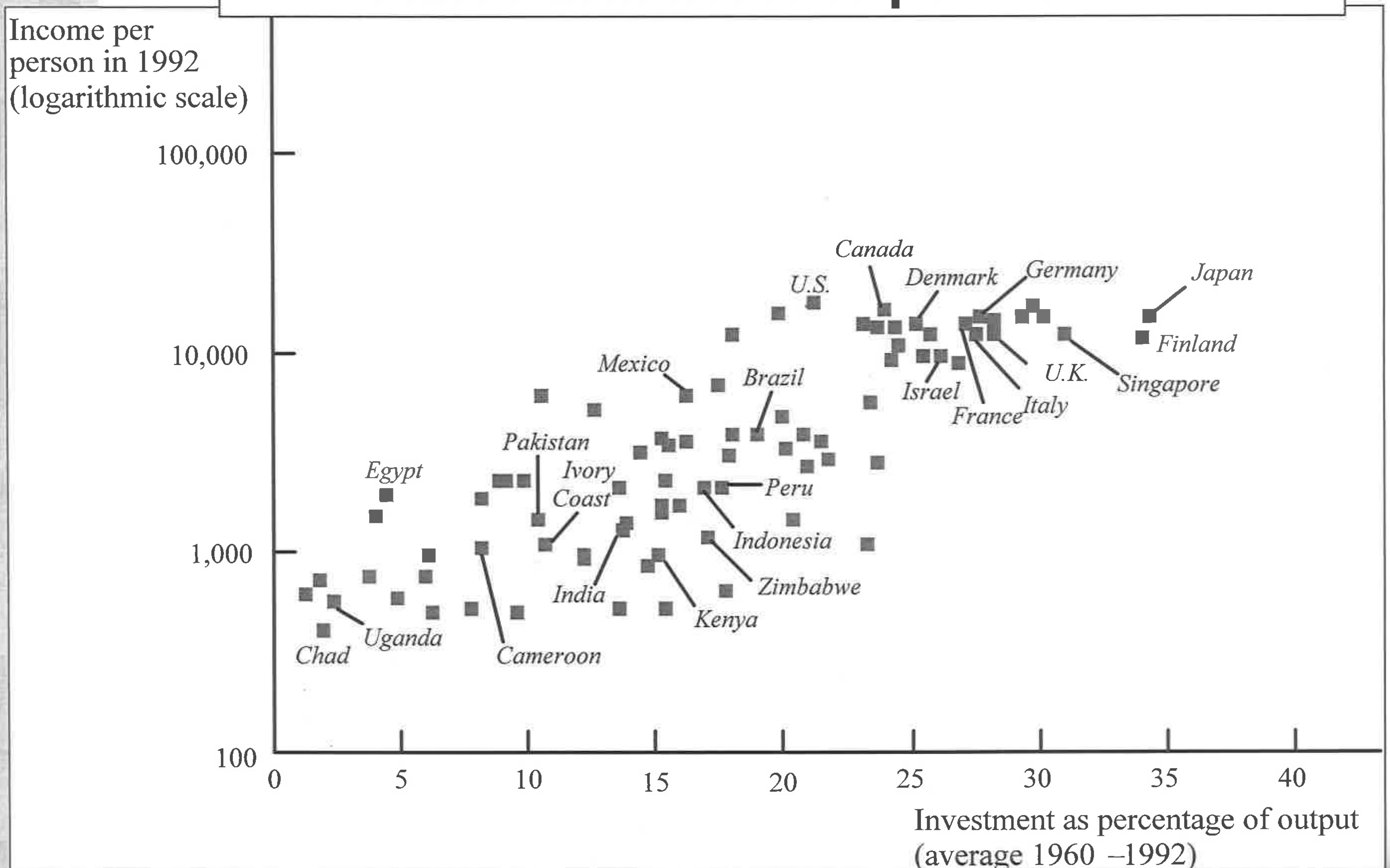
so $\boxed{MPK = \delta}$

So golden rule capital stock here is where marginal product of capital equals the depreciation rate.



Achieving the golden rule requires policy makers adjust saving rate, s, through policies.

International Evidence on Investment Rates and Income per Person



Consider population growth

Assume population (and labor force) grow at exogenous rate, n.

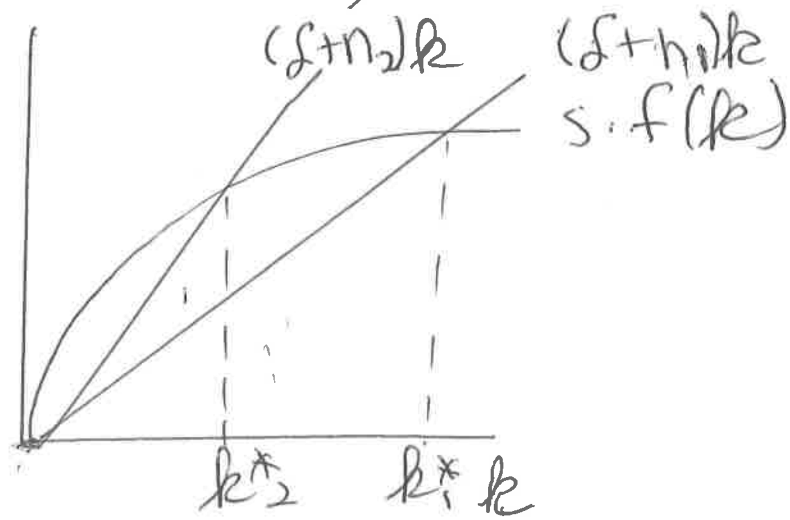
Break-even level of investment: amount of investment needed to keep k constant;

includes:

δk : replace worn out machines

nk : equip new workers with machines

$$k = \frac{K \leftarrow \text{shrinks at rate } \delta}{L \leftarrow \text{grows at rate } n}$$



So a higher population growth rate implies a lower steady state level of capital and income per person so now:

$$\Delta k = s f(k) - (\delta + n) k$$

so steady state now is

$$s f(k) = (\delta + n) k$$

Effect of population growth on Golden rule

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n) k^*$$

derivative

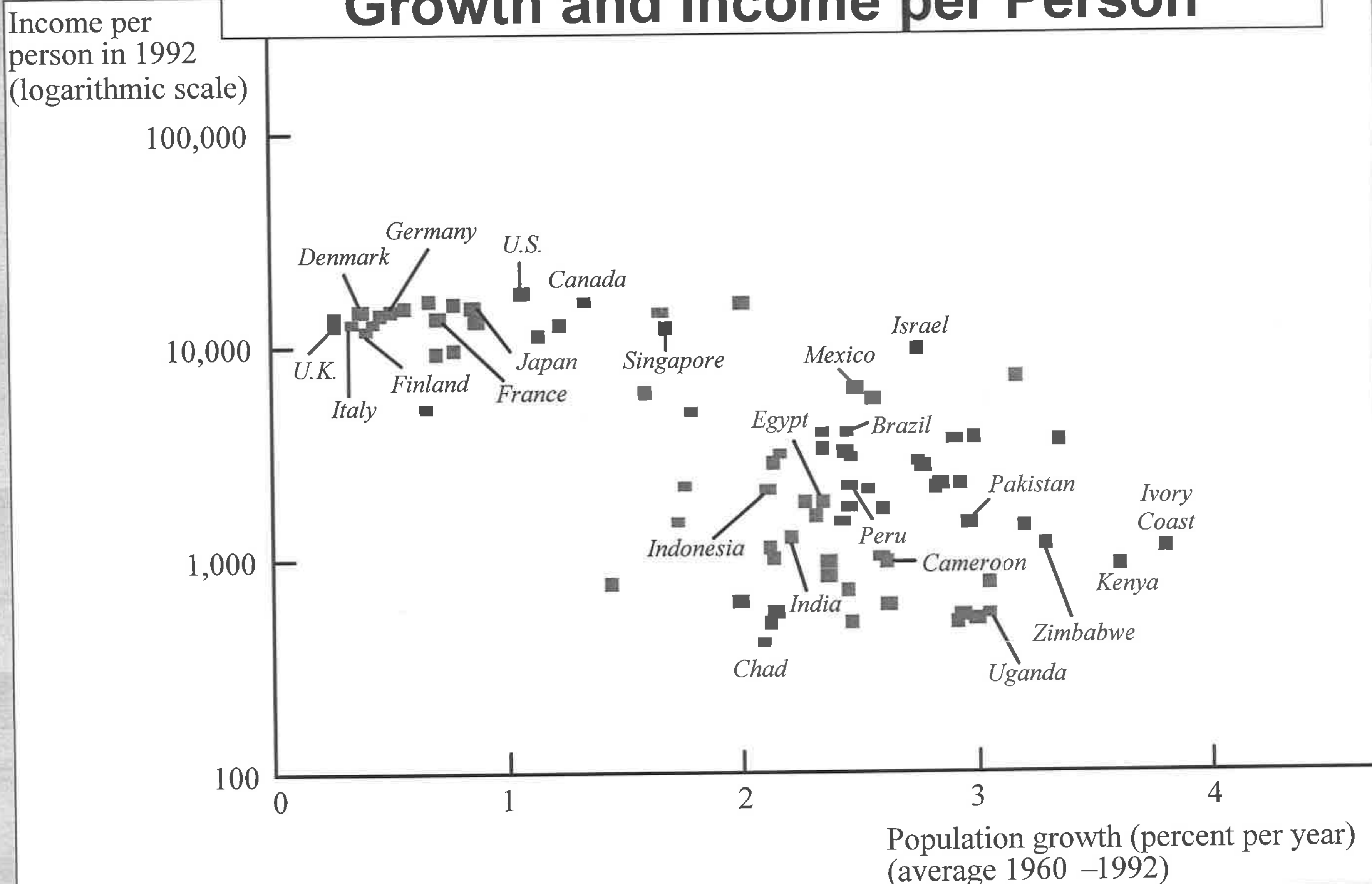
$$0 = \cancel{MPK} \; MPK - (\delta + n)$$

$$\boxed{MPK = \delta + n}$$

new Golden rule condition

So higher population growth implies lower optimal level of capital stock per person.

International Evidence on Population Growth and Income per Person



technological progress in Solow Model

new production function:

$Y = F(K, L \cdot E)$

where E is "labor efficiency"

assume E grows at constant rate, g.

$$g = \frac{\Delta E}{E}$$

$L \cdot E$ is the number of "effective workers"

redefine our notation

$y = \frac{Y}{LE}$: output per effective worker

$k = \frac{K}{LE}$: capital " " "

production function per effective worker: $y = f(k)$

saving per effective worker:

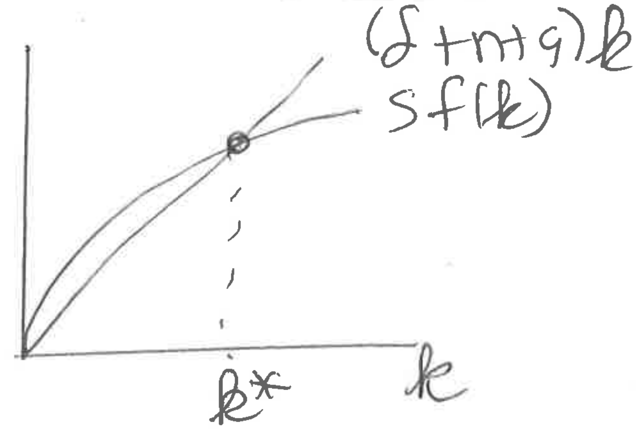
$s \cdot y = s f(k)$

break-even investment now:

$(\delta+n+g)k$

so $\Delta k = sf(k) - (\delta+n+g)k$

steady state: $\boxed{sf(k) = (\delta+n+g)k}$



<u>Golden rule:</u>

$c^* = y^* - i^*$

$= f(k^*) - (\delta+n+g)k^*$

set derivative equal to zero

$$\boxed{MPK = \delta+n+g}$$

Steady-State Growth Rates in the Solow Model with Tech. Progress

Variable	Symbol	Steady-state growth rate
Capital per effective worker	$k = K/(L \times E)$	0
Output per effective worker	$y = Y/(L \times E)$	0
Output per worker	$(Y/L) = y \times E$	g
Total output	$Y = y \times E \times L$	$n + g$

+

conditional convergence

Solow theory says if countries have the same underlying characteristics (saving rate, production function, population growth), then poor countries should grow faster than rich countries.

Convergence: U.S. regions

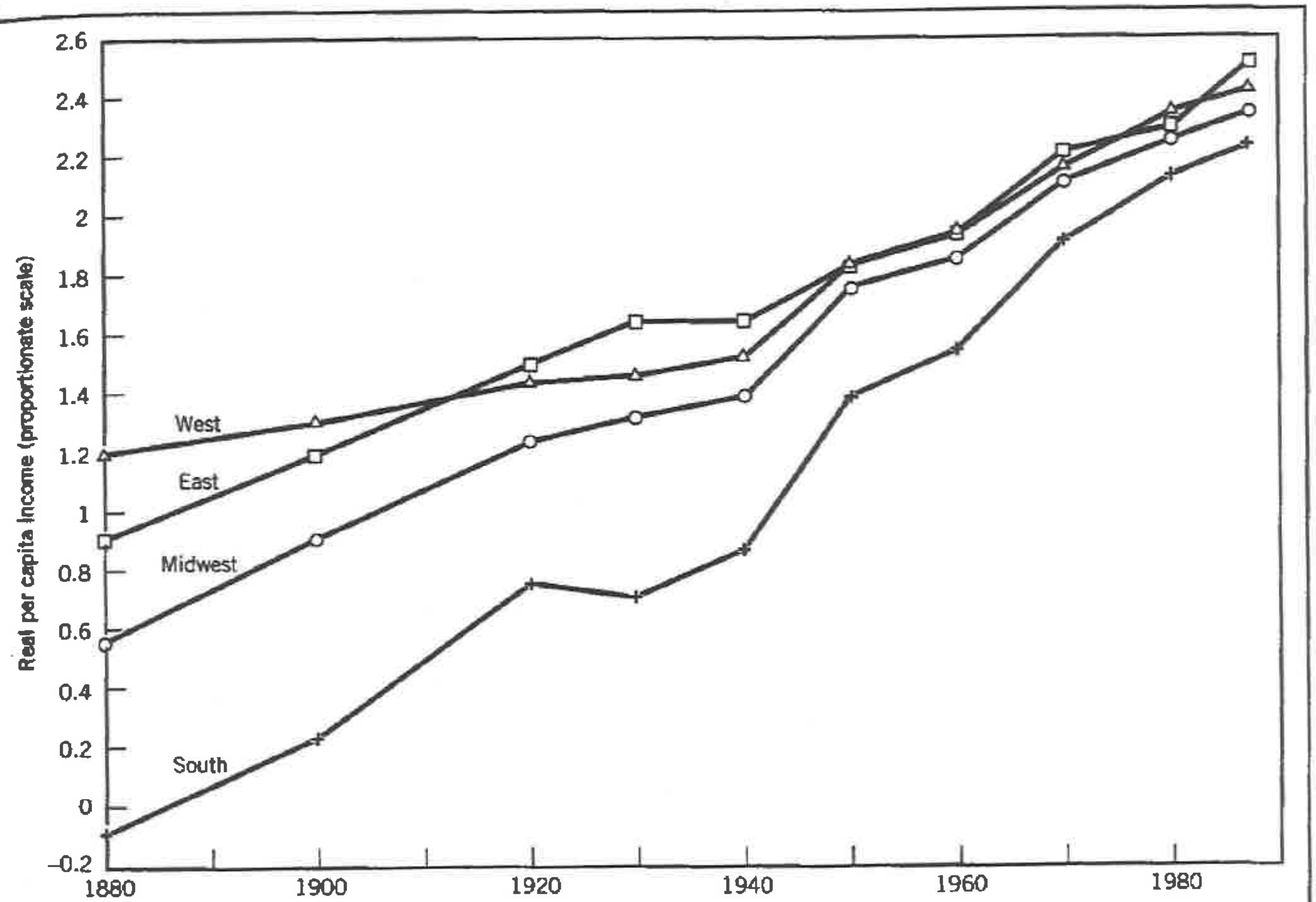


FIGURE 11.7 *Per Capita Income for Four U.S. Regions*
The figure shows the average of per capita personal income for the four regions from 1880 to 1988. The spread across the regions has narrowed substantially over time.

Convergence: U.S. States

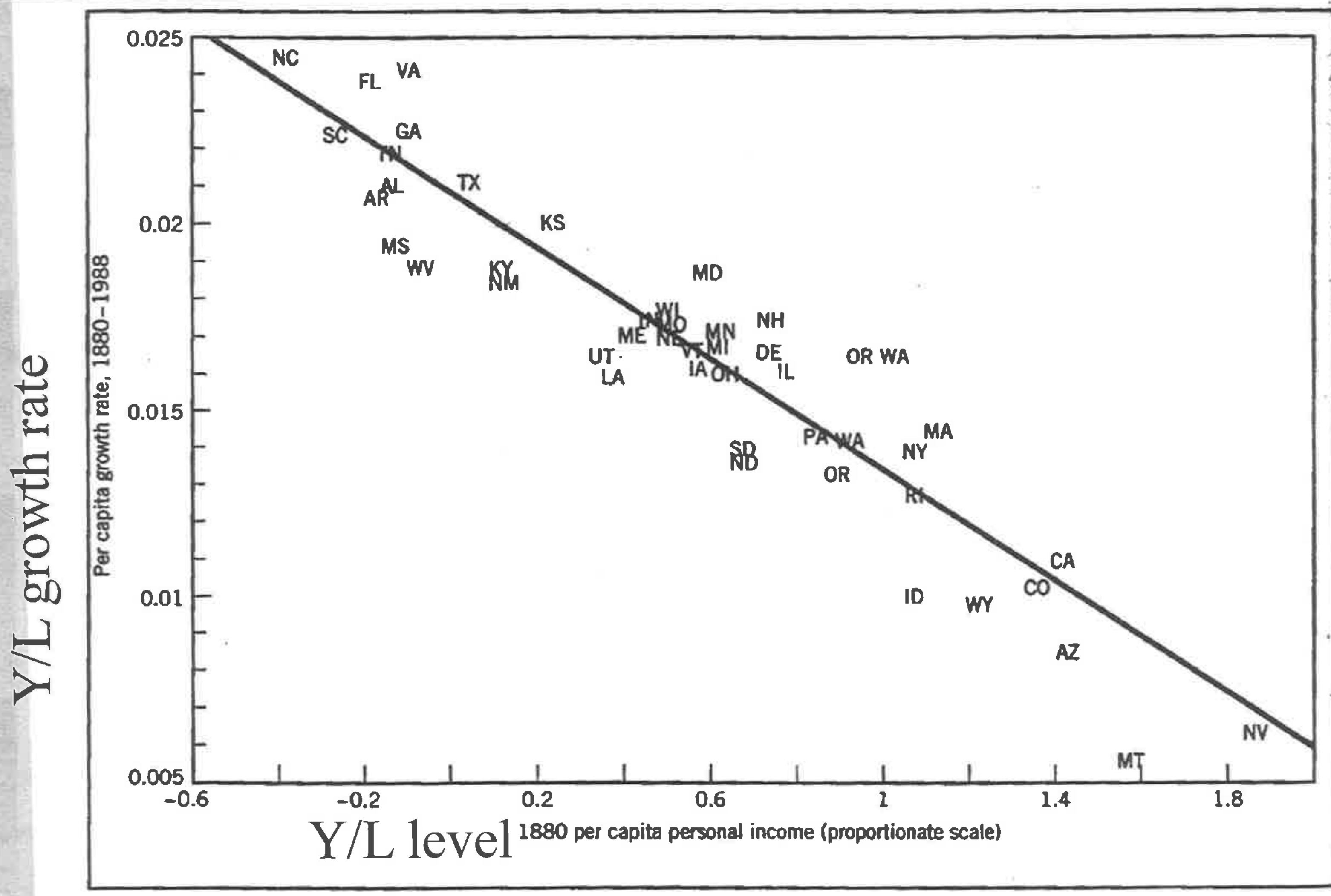


FIGURE 11.6 *Growth of Per Capita Income for U.S. States, 1880–1988*
The average growth rate of per capita personal income, shown on the vertical axis, is inversely related to the initial level of per capita income, shown on the horizontal. (The horizontal axis uses a proportionate scale.)

Convergence: 98 countries

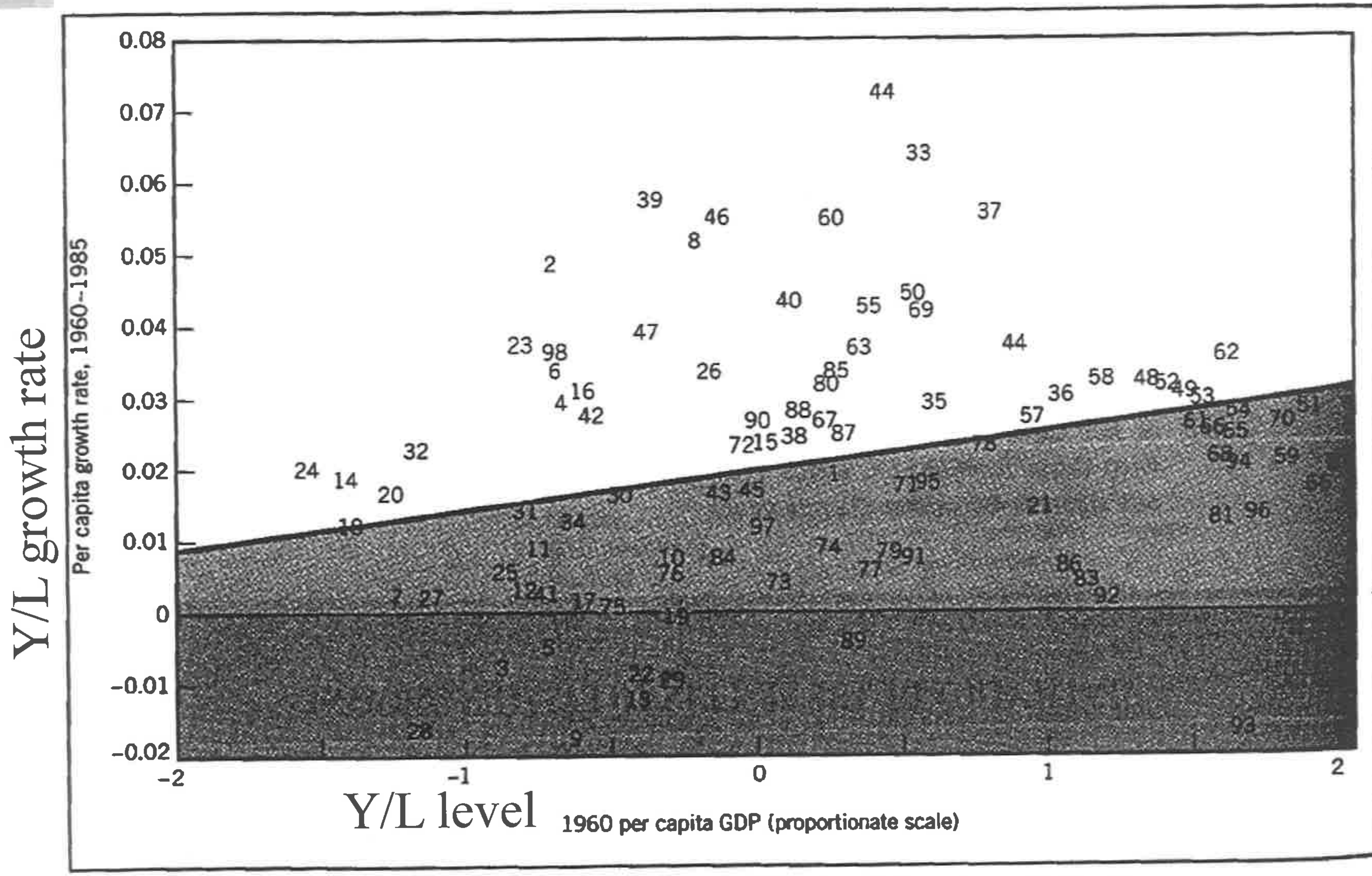


FIGURE 11.10 *Growth of Per Capita GDP for 98 Countries, 1960–85*
The growth rate of per capita GDP, shown on the vertical axis, bears little relation to the initial level of per capita GDP, shown on the horizontal. (The horizontal axis uses a proportionate scale.) Table