Monetary stabilization of sectoral tariffs

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September 15, 2025

Abstract

This paper studies the optimal monetary stabilization of tariffs using a two-country, two-traded sector New Keynesian model, where tariffs imply inefficient sectoral reallocation as well as changes in aggregate output and inflation. We find that the optimal response to a tariff on imports of differentiated goods includes domestic currency appreciation to offset the tariff's distortion on international relative prices. In our two-country environment it is efficient for this exchange rate appreciation to be implemented primarily through monetary expansion in the foreign country; the monetary response in the home country is typically small, and can be either mildly expansionary or contractionary. We identify two cases where robust home monetary expansion may be appropriate: one case is price stickiness in the local currency (LCP); the other is home tariffs that impact the competitive non-differentiated goods sector rather than the differentiated goods sector.

Keywords: macroeconomic effects of tariffs, optimal monetary policy, sectoral reallocation
JEL classification: F4

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We thank Javier Bianchi, Louphou Coulibaly, Charles Horioka, Timothy Kehoe, Jinill Kim, Jaewoo Lee, Tommaso Monacelli, and Christopher Waller for helpful comments, as well as participants of the Society for Economic Dynamics meetings in Copenhagen, American Economic Association meetings, Recent Advances in Macroeconomics conference at Korea University, Trade Disputes and their Consequences on the Global Economy conference at Korea University Business School, and seminar participates and Kyungpook University Korea.

1. Introduction

As the world faces large and volatile U.S. tariffs, central banks around the world are grappling with the question of how to respond to the macroeconomic implications. The question is complicated by the fact that central bank objectives of stabilizing inflation and stabilizing employment and output may conflict in the presence of tariff shocks, as these shocks can simultaneously produce inflationary and recessionary pressures. This question is complicated further by the fact that recent tariffs have targeted different types of U.S. imports, including a broad range of final consumption goods as well as less differentiated materials such as aluminum and steel that were the main targets in the first Trump administration.

This paper studies the Ramsey optimal monetary stabilization of tariff shocks using a two-country New Keynesian model enriched with elements from the trade literature, including simple global value chains in production and multiple traded sectors that differ in terms of market structure and price rigidity. The model builds on Bergin and Corsetti (2023), to our knowledge the first paper in the academic literature to study the question of optimal monetary policy response to tariff shocks, showing that an expansion is desirable in a symmetric currency war even though it may worsen headline inflation. This point has been pushed farther by Monacelli (2025) and Bianchi and Coulibaly (2025) in small open economy environments, arguing that monetary expansion is optimal even in the case of a unilateral tariff on a country's imports.

This paper contributes to this debate in three ways. First, it focuses on the asymmetric case of unilateral tariffs rather than the case of retaliation in a symmetric tariff war, which was the main focus in Bergin and Corsetti (2023). This shift in focus reflects the evolution of events in which unilateral tariffs appear to the be more relevant case, as many countries accept Trump tariffs without retaliation. Second, relative to the small open economy environments used in the related literature listed above, our two-country environment allows us to study the optimal monetary response of the tariff victim as well as the tariff imposer, a question of keen interest around the world. Third, we add a multi-sector dimension, which traditionally has been ignored in the macro treatment of tariffs.[1] In the trade literature, by contrast, the study of tariffs has systematically focused on their

[1] For another recent multi-sector treatment of tariffs, see Kalemli-Özcan, et al. (2025).

implications for the reallocation of production across different sectors. In fact, tariff policies often are motivated by the goal of favoring such reallocation -- say, promoting domestic manufacturing. In our two-sector environment, differentiated goods, often associated with manufacturing, are characterized by monopolistic competition, entry costs, supply chains, and price stickiness. The non-differentiated sector, associated with commodities and agriculture, are associated with competitive markets and flexible prices.

Our two-country environment, in contrast with recent macro literature using small open economies, indicates that it is not typically optimal for a unilateral imposer of tariffs to employ expansionary monetary policy. The main reason is that a two-country model highlights the role of international relative prices that are endogenous, where the exchange rate potentially may be a tool for optimal policy to redress the distortion of tariffs on the relative price of imports. When tariffs raise the relative price of imports, an exchange rate appreciation of the country can counteract this effect under certain conditions. This appreciation requires a monetary policy in the country targeted by the tariff to be expansionary *relative* to the policy of the tariff imposer. Whether the policy of the tariff imposer is expansionary or contractionary depends on particulars of the model calibration, including trade elasticity, but the policy action of this country tends to be small compared to that of the tariff victim. Nonetheless, we emphasize that both home and foreign countries benefit from this home exchange rate appreciation in terms of higher welfare.

Analytical solution of a simplified version of the model provides more intuition for the mechanism. The simplified model assumes one-period price stickiness and no round-about production, as well as a unit trade elasticity and log preferences. Analytical solution with a second sector is facilitated by letting the trade elasticity for this sector approach infinity, thereby replicating the "homogenous world good" often used in trade models. In this environment, the role of optimal monetary policy consists of stabilizing uncertain demand for a country's differentiated exports. The risk of a foreign tariff dampening foreign demand for home exports induces home firms to set a higher fixed price. Under producer currency pricing, this risk premium applies to sales of home goods both at home and abroad, which raises home price index and lowers home welfare. A home currency depreciation can in principle offset the effect of this tariff and stabilize export demand, thus lowering average prices. Arbitrage in the second sector prevents the exchange rate from adjusting in the absence of monetary policy intervention. Optimal monetary policy intervention implies a partial exchange rate offset of the tariff, as it balances the benefit of stabilizing export demand with the side effects of monetary expansion inducing fluctuations in domestic demand. In this environment there is no role for optimal monetary policy to stabilize domestic tariffs on the differentiated sector, since they do not affect the foreign demand for home exports either directly, or indirectly through affecting the exchange rate. While this result follows in part from the simplifying assumptions required for analytical solution, numerical simulations confirm that it holds approximately also in the reasonably calibrated version of the full model.

The optimal policy response to a tariff on the second sector, characterized by non-differentiated goods and price flexibility, is the opposite. In this case, the endogenous exchange rate response to the tariff is the source of the problem: a home tariff on the non-differentiated sector induces a home currency appreciation through arbitrage in this sector and the resulting rise in home wage. This currency appreciation raises the price foreign households pay for home exports and creates volatility in the foreign demand for home exports, raising the home price index. In this case optimal monetary policy is expansionary, to fully offset the home currency appreciation and hold exchange rate fixed.

We also find that optimal policy changes in an environment where prices are sticky in the currency of the buyer rather than producer, and where one currency is dominant in the invoicing of international trade. Drawing on a combination of analytical results and numerical simulations, we find that optimal home policy expands in response to the home tariff on differentiated goods rather than the foreign tariff, when import prices are priced in local currency. This results from the fact that the price set by home firms in the home market no longer is affected by uncertainty in export demand in the foreign market. However, the home price index is affected by the price set by foreign exporters, and home tariffs create uncertainty with an associated rise in price. Home monetary expansion can stabilize the drop in home demand facing foreign exporters, thus lowering the implied risk premium in the price they charge home customers, and lowering home price index.

The lack of exchange rate pass-through also alters the monetary response to a tariff on the non-differentiated sector. Since the exchange rate appreciation no longer poses a problem for export demand of the differentiated sector, there is no monetary response to the non-differentiated tariff mandated in this case.

We conclude that whether the optimal monetary policy of a tariff imposer expands or contracts depends upon certain factors, such as sector targeted by the tariff and the nature of price stickiness. This conclusion differs from the related literature. Bianchi and Coulibaly (2025) find monetary expansion is always optimal for a small open economy under an import tariff shock. This works counter to the exchange rate channel our model highlights, as it makes the home currency depreciate and exacerbates the distortion on relative import prices coming from the tariff. The logic of their result is that if tariffs make the quantity of imports inefficiently small, a way to redress this is to expand overall consumption to raise the level of imports. The specifics of their model serve to highlight the useful role of their labor wedge channel, while our two-country model tends to highlight the exchange rate channel.

Monacelli (2025) also finds that it is explicitly optimal for policy in a small open economy to expand, intestinally make the home currency depreciate. This result appears to arise from particular specifications of the model, such as the assumption that tariffs generate no revenue. In this case a fall in imports is actually efficient in the presence of tariffs.

Most of the rest of the growing literature studying the macroeconomic effects of tariffs relies on either real models or monetary models with a stylized monetary side.[2] We place the monetary dimensions of a trade war and the design of an efficient stabilization policy centerstage in our analysis. Bergin and Corsetti (2023) was the first paper to study the optimal monetary policy response to tariff shocks, comparing Ramsey policies with monetary rules targeting PPI, CPI and money growth, as well as detailing the nature of tariff shocks in relation to markup and productivity shocks -- but in the context of a single sector model that did not consider sectoral reallocation. Because of the tariff distortion, it is not efficient to pursue strict price stability. Werning et al. (2025) emphasizes that the

[2] Some leading contributions study macro dynamics in the context of standard monetary policy rules, e.g., Barattieri et al. (2021), Erceg et al. (2018), Jeanne and Son (2024), Kalemli-Özcan et al. (2025), and Auclert, et al. (2025). Caldara et al. (2018) investigates the macroeconomic implications of trade policy uncertainty. Linde and Pescatori (2019) reconsiders the Lerner symmetry. For recent studies of the macroeconomic effects of tariffs abstracting from monetary policy see Baqaee and Malmberg (2025), and Costinot and Werning (2025).

optimal response to tariff shocks is akin to the optimal monetary stabilization of cost-push shocks. Auray, et al. (2025) considers the optimal mix of CPI and PPI inflation targeting in the class of standard monetary policy rules. Auray et al. (2024) study how alternative monetary policies affect an endogenous, strategic tariff policy, turning our question, the choice of optimal monetary policy in the face of an exogenous tariff policy, around. Our present work is distinct from all these papers in considering implications for sectoral allocation.

The paper proceeds as follows. The next section describes the model environment and calibration. Section 3 develops intuition from analytical solution for a simplified model environment. Sections 4 and 5 use impulse responses to a calibrated version of the full model to illustrate results and demonstrate robustness to a more general setting. Section 6 considers tariff retaliation. Section 7 summarizes conclusions and policy implications.

2. Model

The theoretical framework builds upon the framework of Bergin and Corsetti (2023), augmented with two traded sectors as in the model of Bergin and Corsetti (2020). The model features two countries, home and foreign, each of which produce two types of tradable goods. The first type of good comes in differentiated varieties produced under monopolistic competition, where firm entry requires a sunk investment, and prices are subject to nominal rigidities. The second type of good is modeled according to the standard specification in real business cycle models, assuming perfect substitutability among producers within a country, but imperfect substitutability across countries. In the text to follow, we present the households' and firms' problems as well as the monetary and fiscal policy rules from the vantage point of the home economy, with the understanding that similar expressions and considerations apply to the foreign economy—foreign variables are denoted with a "*".

2.1. Goods consumption demand and price indexes

In the benchmark version of the model, households consume goods produced in both sectors, and of both domestic and foreign origin. The differentiated goods come in many varieties, produced by a time-varying number of monopolistically competitive firms in the home and foreign country, n_t and n_t^* respectively, each producing a single variety. Each variety is an imperfect substitute for any other variety in this sector, either of home or foreign origin, with elasticity ϕ. The non-differentiated goods come in a home and foreign version, which are imperfect substitutes with elasticity η. However, within each country, all goods in this sector are perfectly substitutable with each other, and are produced in a perfectly competitive environment. We will refer to the differentiated sector as "manufacturing," and denote this sector with a *D*; we will denote the non-differentiated sector with a *N*.

Tariffs are specified as ad-valorem duties imposed at the dock. They directly enter the relative prices observed by consumers, and which enter the demand equations. Tariff revenue is collected by the government of the importing country and rebated to domestic consumers, thus canceling out in the consolidated national budget constraint.

The overall consumption index is specified as follows:

$$C_t \equiv \left(\theta^{\frac{1}{\xi}} C_{D,t}^{\frac{\xi-1}{\xi}} + (1-\theta)^{\frac{1}{\xi}} C_{N,t}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}},$$

where $$C_{D,t} \equiv \left(\int_0^{n_t} c_t(h)^{\frac{\phi-1}{\phi}} dh + \int_0^{n_t^*} c_t(f)^{\frac{\phi-1}{\phi}} df \right)^{\frac{\phi}{\phi-1}}$$

is the index over the endogenous number of home and foreign varieties of the differentiated manufacturing good, $c_t(h)$ and $c_t(f)$, and

$$C_{N,t} \equiv \left(\nu^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

is the index over goods differentiated only by country of origin, $C_{H,t}$ and $C_{F,t}$ with $\nu \in [0,1]$ accounting for the weight on domestic goods. The corresponding welfare-based consumption price index is

$$P_t = \left(\theta P_{D,t}^{1-\xi} + (1-\theta)(P_{N,t})^{1-\xi} \right)^{\frac{1}{1-\xi}}, \quad (1)$$

where

$$P_{D,t} = \left(n_t p_t(h)^{1-\phi} + n_t^* \left(p_t(f) T_{D,t} \right)^{1-\phi} \right)^{\frac{1}{1-\phi}} \quad (2)$$

is the index over the prices of all varieties of home and foreign manufacturing goods, $p_t(h)$ and $p_t(f)$, and

$$P_{N,t} = \left(\nu P_{H,t}^{\;1-\eta} + (1-\nu)\left(P_{F,t}T_{N,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3}$$

is the index over the prices of home and foreign non-differentiated goods. In these indexes, $T_{D,t}$ represents the quantity of 1 plus the ad valorem tariff rate imposed by the home country on imports of foreign differentiated goods, and $T_{N,t}$ represents the quantity of 1 plus the ad-valorem tariff rate imposed by the home country on imports of foreign non-differentiated goods. In reporting results, we will distinguish between the "ex-tariff" price determined by an exporter, $p_t(f)$, and the "tariff-inclusive" price, $p_t(f)T_{D,t}$, paid by an importer.

The relative demand functions for domestic residents implied from our specification of preferences are listed below:

$$C_{D,t} = \theta\left(P_{D,t} / P_t\right)^{-\xi} C_t \tag{4}$$

$$C_{N,t} = (1-\theta)\left(P_{N,t} / P_t\right)^{-\xi} C_t \tag{5}$$

$$c_t(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} C_{D,t} \tag{6}$$

$$c_t(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} C_{D,t} \tag{7}$$

$$C_{H,t} = \nu\left(P_{H,t} / P_{N,t}\right)^{-\eta} C_{N,t} \tag{8}$$

$$C_{F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} C_{N,t} \tag{9}$$

Note that demand functions for imports (Eqs. (7) and (9)) depend upon the tariff-inclusive price.

2.2 Home households' problem

The representative home household derives utility from consumption (C_t), and from holding real money balances (M_t/P_t); it suffers disutility from labor (l_t). The household budget consists of labor income from working at the nominal wage rate W_t; profits rebated from home firms denoted with (Π_t) in real terms and defined below, as well as interest income on bonds in home currency ($i_{t-1}B_{H,t-1}$) and foreign currency ($i_{t-1}^{*}B_{F,t-1}$), where e_t is the nominal exchange rate in units of home currency per foreign. Income is net of lump-sum taxes (T_t), used for monetary transfers and to rebate tariff payments on imports. It is assumed that consumers do not internalize the effects of their consumption decisions on government tariff rebates.

Household optimization for the home country may be written:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, l_t, \frac{M_t}{P_t} \right)$$

where utility is defined by

$$U_t = \frac{1}{1-\sigma} C_t^{1-\sigma} + \ln \frac{M_t}{P_t} - \frac{1}{1+\psi} l_t^{1+\psi} ,$$

subject to the budget constraint:

$$P_t C_t + \left(M_t - M_{t-1}\right) + \left(B_{Ht} - B_{Ht-1}\right) + e_t \left(B_{Ft} - B_{Ft-1}\right) = W_t l_t + \Pi_t + i_{t-1} B_{Ht-1} + i_{t-1}^{*} B_{Ft-1} - P_t AC_{Bt} - T_t .$$

In the utility function, the parameter σ denotes risk aversion and ψ is the inverse of the Frisch elasticity. The constraint includes a small cost to holding foreign bonds

$$AC_{Bt} = \frac{\psi_B \left(e_t B_{Ft}\right)^2}{2 P_t p_{Ht} y_{Ht}} ,$$

scaled by ψ_B, which is a common device to assure long run stationarity in the net foreign asset position, and resolve indeterminacy in the composition of the home bond portfolio. The bond adjustment cost is a composite of goods that mirrors the consumption index, with analogous demand conditions to Eqs. (4)-(9).

Defining $\mu_t = P_t C_t^{\sigma}$, household optimization implies an intertemporal Euler equation:

$$\frac{1}{\mu_t} = \beta\left(1+i_t\right) E_t\left[\frac{1}{\mu_{t+1}}\right] \tag{1}$$

a labor supply condition:

$$W_t = l_t^{\psi} \mu_t \tag{11}$$

a money demand condition:

$$M_t = \mu_t \left(\frac{1+i_t}{i_t}\right), \tag{12}$$

and a home interest rate parity condition:

$$\mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}} \frac{e_{t+1}}{e_t}\left(1+i_t^{*}\right)\left(1+\psi_B\left(\frac{e_t B_{ft}}{p_{Ht} y_{Ht}}\right)\right)\right] = \mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}}\left(1+i_t\right)\right]. \tag{13}$$

The problem and first order conditions for the foreign household are analogous.

2.3 Home firm problem and entry condition in the differentiated goods sector

In the manufacturing sector, the production of each differentiated variety follows

$$y_t(h) = \alpha_D \left[G_t(h)\right]^{\zeta} \left[l_t(h)\right]^{1-\zeta}, \tag{14}$$

where α_D is productivity specific to the production of differentiated goods but common to all firms within that sector, $l_t(h)$ is the labor employed by firm h, and $G_t(h)$ is a composite of differentiated goods used by firm h as an intermediate input. $G_t(h)$ is specified as an index of home and foreign differentiated varieties that mirrors the consumption index specific to differentiated goods ($C_{D,t}$). If we sum across firms, $G_t = n_t G_t(h)$ represents economy-wide demand for differentiated goods as intermediate inputs. Given that the index is the same as for consumption, this implies demands for differentiated goods varieties, $d_{G,t}(h)$ and $d_{G,t}(f)$, analogous to Eqs. (6)–(7).[3]

Differentiated goods firms set prices $p_t(h)$ subject to an adjustment cost:

$$AC_{P,t}(h) = \frac{\psi_P}{2}\left(\frac{p_t(h)}{p_{t-1}(h)} - 1\right)^2 \frac{p_t(h)\, y_t(h)}{P_t}, \tag{15}$$

where ψ_P is a calibrated parameter governing the degree of price stickiness. For the sake of tractability, we follow Bilbiie et al. (2008) in assuming that new entrants inherit from the price history of incumbents the same price adjustment cost, and so make the same price setting decision.[4]

There is free entry in the sector, but, once active, firms are subject to an exogenous death shock. Since all differentiated goods producers operating at any given time face the same exogenous probability of exit δ, a fraction δ of them exogenously stop operating each period. The number of firms active in the differentiated sector, n_t, at the beginning of each period evolves according to:

$$n_{t+1} = (1-\delta)(n_t + ne_t), \tag{16}$$

[3] See section 1 of the online appendix for the demand equations not listed here.

[4] The price index for adjustment cost is identical to the overall consumption price index, implying demands analogous to those for consumption in Eqs. (4)-(9). See section 1 of the online appendix for the demand equations not listed here.

where ne_t denotes new entrants.

To set up a firm, managers incur a one-time sunk cost, K_t, and production starts with a one-period lag. This cost is not constant but varies reflecting an entry congestion externality, represented as an adjustment cost that is a function of the number of new firms:

$$K_t = \left(\frac{ne_t}{ne_{t-1}} \right)^{\lambda} \overline{K}, \tag{17}$$

where $\overline{K}$ indicates the steady state level of entry cost, and the parameter λ indicates how much the entry cost rises with an increase in entry activity. The congestion externality plays a similar role as the adjustment cost for capital standard in business cycle models, which moderates the response of investment to match dynamics in data. In a similar vein, we calibrate the adjustment cost parameter, λ, to match data on the dynamics of new firm entry.[5] The demands for varieties for use as entry investment, $d_{K,t}(h)$ and $d_{K,t}(f)$, are determined analogously to demands for consumption of differentiated goods.

We now can specify total demand facing a domestic differentiated goods firm:

$$d_t(h) = c_t(h) + d_{G,t}(h) + d_{K,t}(h) + d_{AC,P,t}(h) + d_{AC,B,t}(h) \tag{18}$$

which includes the demand for consumption ($c_t(h)$) by households, and the demand by firms for intermediate inputs ($d_{G,t}(h)$), investment (the sunk entry costs) ($d_{K,t}(h)$), and goods absorbed as adjustment costs for prices ($d_{AC,P,t}(h)$) and bonds holding costs ($d_{AC,B,t}(h)$). There is an analogous demand from abroad $d_t^*(h)$. We assume iceberg trade costs τ_D for exports, so that market clearing for a firm's variety is:

$$y_t(h) = d_t(h) + (1+\tau_D) d_t^*(h), \tag{19}$$

Firm profits are computed as:

$$\pi_t(h) = p_t(h) d_t(h) + e_t p_t^*(h) d_t^*(h) - mc_t y_t(h) - P_t AC_{p,t}(h). \tag{20}$$

where $mc_t = \zeta^{-\zeta}(1-\zeta)^{\zeta-1} P_{D,t}{}^{\zeta} W_t^{1-\zeta} / \alpha_D$ is marginal cost.

Thus the value function of firms that enter the market in period t may be represented as the discounted sum of profits of domestic sales and export sales:

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[5] The value of steady state entry cost $\overline{K}$ has no effect on the dynamics of the model, and so will be normalized to unity.

$$v_t(h) = E_t\left\{\sum_{s=0}^{\infty}\left(\beta(1-\delta)\right)^s \frac{\mu_{t+s}}{\mu_t}\pi_{t+s}(h)\right\},$$

where we assume firms use the discount factor of the representative household, who owns the firm, to value future profits. With free entry, new producers will invest until the point that a firm's value equals the entry sunk cost:

$$v_t(h) = P_{D,t}K_t\ . \tag{21}$$

By solving for cost minimization we can express the relative demand for labor and intermediates as a function of their relative costs:

$$\frac{P_{D,t}G_t(h)}{W_t l_t(h)} = \frac{\zeta}{1-\zeta}\ . \tag{22}$$

Managers optimally set prices by maximizing the firm value subject to all the constraints specified above. The price setting equation:

$$\begin{aligned} p_t(h) = {} & \frac{\phi}{\phi-1}mc_t + \frac{\psi_P}{2}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)^2 p_t(h) - \psi_P\frac{1}{\phi-1}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)\frac{p_t(h)^2}{p_{t-1}(h)} \\ & + \frac{\psi_P}{\phi-1}E_t\left[\beta\frac{\Omega_{t+1}}{\Omega_t}\left(\frac{p_{t+1}(h)}{p_t(h)}-1\right)\frac{p_{t+1}(h)^2}{p_t(h)}\right] \end{aligned} \tag{23}$$

expresses the optimal pricing as a function of the stochastically discounted demand faced by producers of domestic differentiated goods,

$$\begin{aligned} \Omega_t = {} & \left[\left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi}\left(C_{D,t} + G_t + ne_t(1-\theta_K)K_t + AC_{P,D,t} + AC_{B,D,t}\right)\right. \\ & \left. + \left(\frac{(1+\tau_D)T^*_{D,t}p_t(h)}{e_t P^*_{D,t}}\right)^{-\phi}(1+\tau_D)\left(C^*_{D,t} + G^*_t + ne^*_t(1-\theta_K)K^*_t + AC^*_{P,D,t} + AC^*_{B,D,t}\right)\right] \Big/ \mu_t \end{aligned}$$

This sums the demand arising from consumption, use as intermediate inputs, sunk entry cost, price adjustment costs, and bond holding costs.

Under the assumption that firms preset prices in own currency, i.e., assuming producer currency pricing, the good price in foreign currency moves one-to-one with the exchange rate, net of trade costs:

$$p^*_t(h) = (1+\tau_D)\,p_t(h)/e_t, \tag{24}$$

where recall the nominal exchange rate, *e*, measures home currency units per foreign.

Note that, since households own firms, they receive firm profits but also finance the creation of new firms. In the household budget, the net income from firms may be written:

$$\Pi_t = n_t \pi_t(h) - ne_t v_t(h).$$

In reporting our quantitative results, we will refer to the overall home gross production of differentiated goods defined as: $y_{D,t} = n_t y_t(h)$.

2.4 Home firm problem in the undifferentiated goods sector

In the second sector firms are assumed to be perfectly competitive in producing a good differentiated only by country of origin. The production function for the home non-differentiated good is linear in labor:

$$y_{H,t} = \alpha_N l_{H,t}, \tag{25}$$

where α_N is productivity specific to this country and sector. It follows that the price of the homogeneous goods in the home market is equal to marginal costs:

$$p_{H,t} = W_t / \alpha_N. \tag{26}$$

An iceberg trade cost specific to the non-differentiated sector implies prices of the home good abroad are

$$p^*_{H,t} = p_{H,t}(1+\tau_N)/e_t. \tag{27}$$

Analogous conditions apply to the foreign non-differentiated sector.

2.5 Monetary policy

To compute the cooperative Ramsey allocation, we posit that the monetary authority maximizes aggregate welfare of both countries:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}\left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\psi} l_t^{1+\psi} \right) + \frac{1}{2}\left(\frac{1}{1-\sigma} C_t^{*1-\sigma} - \frac{1}{1+\psi} l_t^{*1+\psi} \right) \right)$$

under the constraints of the economy defined above. As common in the literature, we write the Ramsey problem by introducing additional co-state variables, which track the value of the planner committing to a policy plan.

For comparison, we also study two alternative nominal specifications. In the first one, we assume a monetary policy of perfect producer price inflation targeting:

$$\frac{p_t(h)}{p_{t-1}(h)} = 1. \quad (28)$$

In the second one, we assume a standard Taylor rule of the form

$$1 + i_t = (1 + i_{t-1})^{\gamma_i} \left[(1 + \bar{i}) \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_P} \left(\frac{Y_t}{\bar{Y}} \right)^{\gamma_Y} \right]^{1-\gamma_i}, \quad (29)$$

where terms with overbars are steady-state values. In this rule, inflation is defined in terms of the CPI, while Y_t is a measure of GDP defined net of intermediates as:[6]

$$Y_t = \left(n_{t-1} p_t(h) y_t(h) - P_{D,t} G_t + p_{H,t} y_{H,t} \right) / P_t . \quad (30)$$

Across these different specifications of monetary policy, we will abstract from public consumption expenditure, so that the government uses seigniorage revenues and taxes to finance transfers, assumed to be lump sum. Government transfers are also used to rebate to consumers the tariff duties paid to the government by consumers and firms on imported goods. The government budget constraint thus is specified as follows:

$$T_t = (M_{t-1} - M_t) + (T_{D,t} - 1) n_{t-1}^* d_t(f) + (T_{N,t} - 1)(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}). \quad (31)$$

2.6 Shocks process and equilibrium definition

Shocks are assumed to follow joint log normal distributions:

$$\begin{bmatrix} \log T_{D,t} - \log \overline{T_D} \\ \log T_{D,t}^* - \log \overline{T_D^*} \\ \log T_{N,t} - \log \overline{T_N} \\ \log T_{N,t}^* - \log \overline{T_N^*} \end{bmatrix} = \rho_T \begin{bmatrix} \log T_{D,t-1} - \log \overline{T_D} \\ \log T_{D,t-1}^* - \log \overline{T_D^*} \\ \log T_{N,t-1} - \log \overline{T_N} \\ \log T_{N,t-1}^* - \log \overline{T_N^*} \end{bmatrix} + \varepsilon_{Tt}$$

with autoregressive coefficient matrix ρ_T, and the covariance matrix $E\left[\varepsilon_{Tt}\varepsilon_{Tt}^{'}\right]$.

To conserve space, the market clearing conditions to close the model are reported in section 2 of the appendix. A competitive equilibrium in our world economy is defined along the usual lines, as a set of processes for quantities and prices in the home and foreign country satisfying: (i) the household and firms optimality conditions; (ii) the market clearing conditions for each good and asset, including money; (iii) the resource

[6] For computational simplicity, the Taylor rule is specified in terms of deviations of GDP from its steady state value, which is distinct from the output gap.

constraints—whose specification can be easily derived from the above and is omitted to save space.

2.7 Welfare computation

We report the effects on welfare of a given policy regime configuration relative to the Ramsey allocation. The change in welfare customarily is computed in terms of consumption units that households would be willing to forgo to continue under the Ramsey policy regime. We posit identical initial conditions across different monetary policy regimes using the Ramsey allocation, and we include transition dynamics in the computation to avoid spurious welfare reversals.[7]

2.8 Calibration

Where possible, parameter values are taken from standard values in the literature. Risk aversion is set at $\sigma = 2$; labor supply elasticity is set at $1/\psi = 1.9$ following Hall (2009). Consistent with a quarterly frequency, $\beta = 0.99$.

The price stickiness parameter is set at ψ_p =49, a value which implies in simulations of a productivity shock that approximately half the firms resetting price during the first year.[8] The firm death rate is set at δ =0.025. The mean sunk cost of entry is normalized to the value $\overline{K}$ =1, and the adjustment cost parameter for new firm entry, λ, is taken from Bergin and Corsetti (2020). The share of intermediates in differentiated goods production follows Bergin and Corsetti (2020) in setting ζ =1/3.

To choose parameters for the differentiated and non-differentiated sectors we draw on Rauch (1999). We choose θ so that differentiated goods represent 55 percent of U.S. trade in value: ($\theta = 0.45$). We assume the two countries are of equal size with no

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[7] We adopt the methodology created by Giovanni Lombardo and used in Coenen et al. (2010), available from https://www.dropbox.com/s/q0e9i0fw6uziz8b/OPDSGE.zip?dl=0.

[8] As is well understood, a log-linearized Calvo price-setting model implies a stochastic difference equation for inflation of the form $\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$, where *mc* is the firm's real marginal cost of production, and where $\lambda = (1-q)(1-\beta q)/q$, with q is the constant probability that a firm must keep its price unchanged in any given period. The Rotemberg adjustment cost model used here gives a similar log-linearized difference equation for inflation, but with $\lambda = (\phi - 1)/\kappa$. Under our parameterization, a Calvo probability of $q = 0.5$ implies an adjustment cost parameter of $\psi_P = 49$.

exogenous home bias, $\nu = 0.5$, but allow trade costs to determine home bias ratios. The Broda and Weinstein (2006) estimate of the elasticity of substitution between differentiated goods varieties is ϕ=5.2 (the sample period is 1972-1988), though alternative values will be considered in sensitivity analysis. We adopt the same elasticity between home and foreign goods in the non-differentiated sector. We initially adopt a Cobb-Douglas specification for the aggregator function combining the two sectors ($\xi \to 1$), but sensitivity analysis will report results for alternative calibrations of this parameter. To set trade costs, we calibrate τ_D so that exports represent 26% of GDP, as is the average in World Bank national accounts data for OECD countries from 2000-2017.[9] This requires a value of $\tau_D = \tau_N$ =0.44.[10]

Calibration of policy parameters for the historical monetary policy Taylor rule are taken from Coenen, et al. (2010): γ_i=0.7, γ_p=1.7, γ_Y=0.1.

The process for tariff shocks is calibrated with a mean value of 1.02 (2 percentage point mean tariff rate) to match U.S. tariff data in Barattieri et al. (2021). The autoregressive parameter is set to 0.56, estimated from Barattieri et al. (2021). The standard deviation of the shock is 0.08 (raising tariff rate from a steady stat of 2% to 10%).

3. Intuition from analytical results

This section derives analytical results for a simplified version of the model, to highlight intuition for some of the key simulation results to follow.

3.1 Simplified model environment

To facilitate analytical solution, suppose differentiated goods come in distinct country-specific types, with a Cobb-Douglas aggregator over the home and foreign types. For the non-differentiated sector we instead employ an approach from the trade literature to facilitate analytical solution, by letting the elasticity of substitution between home and foreign varieties approach infinity, thereby approximating the case of world homogeneous good. In order to study a tariff in such an environment, we assume that the home country is both a producer and also an importer of the non-differentiated good (we show below

[9] See https://data.worldbank.org/indicator/NE.EXP.GNFS.ZS?locations=OE.
[10] To coincide with standard accounting definitions, differentiated goods used as intermediates are included in the measure of exports, and excluded in the measure of GDP, as is appropriate.

permissible ranges for productivity levels and tariffs to ensure this condition holds in equilibrium). This assumption arguably suits an interpretation of the non-differentiated goods as steel; U.S. tariff policy may promote domestic production of this sector, but an underlying foreign comparative advantage means domestic production will not likely fully replace foreign sources. The consumption aggregator is: $C_t \equiv \left(C_{DHt}^{1/2} C_{DFt}^{1/2}\right)^{\theta} \left(C_{Nt}^{1/2}\right)^{1-\theta}$,

with associated price index: $P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right] P_{DHt}{}^{\theta/2}\left(T_{Dt} e_t P_{DFt}^{*}\right)^{\theta/2} P_{Nt}{}^{1-\theta}$.

The demand condition for home imports of foreign differentiated goods (7) becomes:

$$c_t(f) = \left(e_t p_t^{*}(f) T_{D,t} / P_{D,t}\right)^{-1} C_{D,t}$$

where $p_t^{*}(f)$ is the foreign currency price of foreign exports, which is fixed in the initial period under producer currency pricing. This equation highlights how, under price stickiness, the effect on relative prices of a rise in tariff, T_D, can in principle be offset by an equal rise in nominal exchange rate, *e*. If a tariff distorts prices to make imports more expensive for a home consumer, a home currency appreciation (foreign deprecation) can make them cheaper.

Household utility is simplified as log in consumption and linear in labor $U_t = \ln C_t - \kappa l_t$ .We follow Corsetti and Pesenti (2009) in defining a variable, $\mu_t = P_t C_t$, to denote nominal expenditure, and in using it to summarize the effect of monetary policy stance on aggregate nominal spending. Utility maximization implies the usual labor supply condition $W_t = \kappa \mu_t$.

The assumption that both home and foreign-produced homogeneous goods are consumed in home requires that the prices of all homogeneous goods facing home consumers must be equal: $P_{NH,t} = e_t T_{N,t} P_{NF,t}^{*}$. Price setting in this perfectly competitive sector follows: $P_{Nt} = W_t / \alpha_N$. In combination with the labor supply condition above, this relative price equation pins down the exchange rate:

$$e_t = \frac{P_{NH,t}}{P_{NF,t}^{*} T_{N,t}} = \frac{\left(W_t / \alpha_N\right)}{\left(W_t^{*} / \alpha_N^{*}\right) T_{N,t}} = \frac{\mu_t \alpha_N^{*}}{\mu_t^{*} \alpha_N T_{N,t}}. \qquad (32)$$

Intuitively, a home monetary expansion (rise in μ_t) induces a proportional home currency depreciation (rise in e_t). A home tariff on the non-differentiated sector causes home currency appreciation, as arbitrage induces a rise in home wages and production costs. A tariff on the differentiated sector has a fully neutral effect on the exchange rate in this environment.

Production in the differentiated goods sector is simplified to use just labor, with no production chains $y_t(h) = \alpha_t l_t(h)$. Firms are assumed to set prices one period ahead, in producer currency units. Tariffs are assumed to be i.i.d. shocks. As shown in the appendix, the optimal price setting of the home differentiated goods firm is

$$P_{DHt} = \frac{\phi}{\phi - 1} E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right] \Big/ E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]. \tag{33}$$

Firm price setting is affected by uncertainty in the demand for the firm's exports, which can arise from several sources. First, a rise in foreign differentiated tariff, T_{Dt}^*, raises the price foreign households pay for home exports and lowers the demand. Second, a rise in home non-differentiated tariff, $T_{N,t}$, causes currency appreciation as noted above, which also raises the price of home goods for foreign consumers. Third, a monetary contraction, fall in μ_t, implies an appreciation of home currency, which also raises the price foreign consumers pay under prices sticky in the producer currency. If the tariff shocks are negatively correlated with μ_t, the expectation term implies a rise in average prices of home differentiated goods, as they to imply more variable fluctuations in foreign demand. However, since μ_t can be regarded as a policy variable, this equation also implies that a rise in μ_t in response to a rise in tariffs can generate a smaller average fluctuation in demand, which lowers average prices. Lastly we note that home tariffs on differentiated goods do not enter this equation, since they do not affect the price foreign consumers pay for home exports either directly, or indirectly through affecting the exchange rate (as discussed above in equation 32).

3.2 Optimal policy under producer currency pricing

We compute the optimal Ramsey policy by choosing the policy stance, μ_t^{OP}, to maximize expected utility subject to the equilibrium conditions listed in the preceding section. Appendix B derives this optimal policy in terms of the following rules:[11]

$$\mu_t^{OP} = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1} \tag{34}$$

and

$$\mu_t^{*OP} = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}, \tag{35}$$

where a is a constant of proportionality.

Observe that this policy fully eliminates all fluctuations in the numerator of the firm price setting rule in equation (33). As noted above, this lowers risk associated with exports and induces home firms to set lower prices. This in turn lowers the home overall price index, which raises welfare.

Drawing on equations (32)-(35) we can tell a more complete story of how policy can use the exchange rate to improve welfare in the face of tariffs. Substitute the optimal policies (34-35) into the exchange rate solution (32):

$$e_t = \left(\frac{1}{T_{N,t}}+\frac{\alpha_N}{\alpha_N^*}\frac{1}{T_{Dt}}\right)\Bigg/\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right). \tag{36}$$

In the case of a home tariff on differentiated goods, the impact on foreign demand for home exports could in principle be offset by a proportional home currency depreciation. In the absence of monetary policy, there is no automatic exchange rate response in this environment, as shown in equation (32). The optimal policy chooses to redress this fall in demand with a home exchange rate deprecation that is less than fully proportional to the tariff. Apply (36) to the case where the only shock is the foreign differentiated tariff: $e_t = \left(1+\frac{\alpha_N}{\alpha_N^*}\right)\Big/\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{Dt}^*}\right)$. In the case where countries are nearly symmetric in productivity of the non-differenated sector ($\alpha_N \to \alpha_N^*$), this equation implies that the home exchange rate depreciates enough to wipe out approximately half of the price rise induced by the tariff, with this share reflecting the fact that imports are assumed to comprise half of the differentiated consumption aggregate. The monetary expansion

[11] The Nash and cooperative solutions coincide for this simple environment. As usual, the Nash solution chooses μ_t, to maximize $E_{t-1}\left[U_t\right]$ and foreign chooses μ_t^*, to maximize $E_{t-1}\left[U_t^*\right]$; the cooperative solution chooses μ_t and μ_t^* jointly to maximize $E_{t-1}\left[U_t+U_t^*\right]/2$. See appendix B for details.

needed to induce exchange rate adjustment also has an effect on the domestic demand for differentiated, so athe monetery expansion required for a full exchange rate offset of the tariff is not optimal.

Next, consider the case of a home tariff on non-differentiatevd goods. As noted above, this tariff shock induces an appreciation of the home currency, which then lowers demand for home differentiated exports. The optimal policy imlies a monetary expansion sufficient to fully offset this currency apprection: applying (36) in the context of only home non-differentiated tariffs: $e_t = \left(\frac{1}{T_{N,t}} + \frac{\alpha_N}{\alpha_N^*}\right) \Big/ \left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}}\right) \to 1$ as productivities across countries become more symmetric ($\alpha_N \to \alpha_N^*$).

Lastly, we reiterate that there is no role for monetary policy to deal with home tariffs on differentiated goods in this environment, as this tariff does not enter the home price setting equation. It thus does not affect the price foreign consumers pay for home exports, either directly, or indirectly through affecting the exchange rate (as discussed above in equation 32). This result follows in part from the simplifying assumptions required for analytical solution, but numerical simulations to follow confirm that it holds approximately also in the reasonably calibrated version of the full model.

3.3 Admissible range for analytical results

We also can derive the limits on admissible productivity levels under which equation (32) holds, which allows home to both produce some the non-differentiated good and at the same time to have positive net imports of it:

$$1 < \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta} - 1 . \qquad (36)$$

On one hand, to ensure home imports, foreign must have higher productivity implying an absolute advantage over home in producing non-differentiated goods, even after accounting for tariffs. On the other hand, to ensure domestic production, this foreign advantage cannot be too large. For example, if θ = ½, and in absence of tariffs, foreign productivity cannot be more than three times that of home productivity.

3.4 Optimal policy under local currency pricing

Appendix C studies optimal policy in the case of prices that are sticky in the currency of buyer. Optimal policy must satisfy the following condition for home:

$$\frac{1}{\mu_t} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} + \frac{1}{2}\frac{1}{T_{Dt}} \bigg/ E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right], \tag{37}$$

and for foreign

$$\frac{1}{\mu_t^*} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t^*\right]} + \frac{1}{2}\frac{1}{T_{Dt}^*} \bigg/ E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]. \tag{38}$$

While we cannot present a closed form solution, we can conclude that the optimal home policy now responds to the home tariff on differentiated goods rather than the foreign tariff, inverting the result from the case with prices sticky in the currency of the producer. Derivations in Appendix C show that this results from the fact that the price set by home firms in the home market no longer is affected by uncertainty in export demand in the foreign market. However, the home price index is affected by the price set by foreign exporters, and home tariffs create uncertainty with an associated rise in price. Home monetary expansion can stabilize the drop in home demand facing foreign exporters, thus lowering the implied risk premium in the price they charge home customers, and lowering home price index.

The condition above also shows that home policy no longer responds to tariffs on the non-differentiated sector. This results from the fact the exchange rate, which was how the non-differentiated tariff affected the demand for differentiated goods above, cancels out of the expectation term for price setting under local currency pricing. Numerical simulations to follow will illustrate this case further.

4. Baseline Simulation: unilateral home tariff on foreign differentiated exports

This section uses numerical simulations of the calibrated full model to demonstrate implications of key results and explore robustness in a more general setting. Consider first a tariff shock imposed by the home country on its imports of differentiated goods exported from the foreign country. Figure 1 reports impulse responses showing the macroeconomic effects on selected variables under different policy regimes, contrasting the Ramsey optimal policy (solid line), PPI-targeting rule (dashed line), and CPI-based Taylor rule (dotted line).

4.1 Transmission under suboptimal policy

Consider first as a benchmark the case of monetary policy that perfectly targets producer prices, which under the assumptions of our model correspond to the flexible price allocation. Under this policy, the tariff generates a modest but sustained fall in home GDP, and a spike of CPI inflation. This stagflationary outcome presents a classic challenge to policy makers. Stagflation also characterizes the impact on the foreign country, though the drop in output is steeper, as the tariff reduces demand for foreign exports.

The effect on activity at the aggregate level, however, masks a large sectoral reallocation. The percentage fall in the production of differentiated goods in the foreign country is three times the percentage fall in GDP. This sectoral contraction is matched by a rise in foreign production of non-differentiated goods of a similarly large magnitude. Sectoral reallocation in the home country inversely mirrors the foreign country. The modest fall in home GDP is the net effect of a larger percentage fall in non-differentiated goods production and smaller offsetting rise in non-differentiated production.

PPI targeting implies a rise in home interest rate and fall in foreign, and the home currency appreciates. Observe that the rate of appreciation is not large enough to offset the impact of the tariff on the relative price of home exports to home imports.[12] The home tariff-inclusive terms of trade (defined here as the relative price of home differentiated exports to differentiated imports) improves, reflecting the rise of import prices due the tariff. The home country experiences a temporary trade surplus.[13]

A standard Taylor rule (dotted line in Figure 1) dampens the interest rate adjustment compared to the strict PPI target, due to the substantial interest rate smoothing term in this rule. The smaller rise in home interest rate means home GDP now rises and inflation now is positive; the slightly smaller fall in foreign interest rate means foreign GDP falls slightly more and inflation is negative. The sectoral reallocation isa also amplified relative to the PPI targeting rule.

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[12] Linde and Pescatori (2019) have pointed out that, in its stronger form, Lerner symmetry fails in many macroeconomic contexts, depending on the structure of financial markets and nominal rigidities.

[13] Appendix Figure 1 reports responses for additional variables. Home consumption falls and foreign rises, reflecting interest rates in each country. Firm entry rises at home and falls in foreign, reflecting movements in differentiated goods output. Employment moves likewise; wages fall in both countries reflecting the fall in GDP.

4.2 Transmission under optimal policy

In Figure 1, economic dynamics under the optimal (cooperative) policy are depicted with a solid (red) line. The policy response implies a strong monetary expansion in the foreign country relative to the PPI target, implying a fall in PPI and a lower interest rate compared the PPI targeting rule. In the home country, the optimal policy is quite close to the PPI targeting rule, with a moderate contraction. This simulation result broadly corroborates the analytical result from the simplified model in the preceding section, in which optimal policy called for a large monetary expansion in the foreign country, with no response in the home country. As in the analytical model a primary outcome of this optimal policy is a foreign currency depreciation, which works to offset the tariff's effect on relative prices of imports in the home country. Note in the figure that this exchange rate change offsets approximately half of the terms of trade effect of the tariff, which again broadly corresponds to the result in the analytical model. This policy serves to substantially dampen the sector reallocation observed under the PPI targeting rule. It thus substantially undercuts the distortions in prices and sectoral allocation of the tariff.

Since home currency appreciation simply requires a "relative" contraction of home compared to foreign, this mechanism for offsetting a tariff can be consistent with a home expansion, provided it is smaller than that for foreign. Appendix Figure 2 shows such a case, due to a smaller trade elasticity that serves to dampen the effectiveness of the exchange rate as a mechanism of adjustment. The market structure for the differentiated sector is taken from the trade literature and implies that the parameter governing the elasticity of substitution between home and foreign varieties is the same parameter governing the elasticity between different home varieties and hence firm markup. So we are limited in the range of trade elasticities admissible. We are able to lower this parameter from our benchmark value of ϕ =5.2 to 3.8, a value common in some parts of the firm dynamics literature (see Ghironi and Melitz 2003).[14] This change in parametrization is sufficient to flip the sign of the optimal home policy from a small contraction to small

[14] This experiment also lowered the trade elasticity for non-differentiated goods (η=1.5), to satisfy Blanchard-Kahn condition of dynamic stability.

expansion (with positive PPI inflation and lower interest rate relative to the PPI targeting rule), and this implies a small rise in home GDP relative to the PPI targeting benchmark,

4.3 Sensitivity to Dominant Currency Price Stickiness

A common alternative specification of price stickiness is to assume export prices are set in a dominant currency, the dollar. The analytical results in the preceding section suggested that the nature of price stickiness has fundamental implications for the optimal monetary policy response to tariffs. In Figure 2, we present impulse responses to a home tariff shock under the assumption that the prices of both home imports and exports of differentiated goods are set in home currency units, reflecting the role of the dollar as a dominant currency. Simulations may be interpreted as a hybrid of the two analytical cases in the previous section, where the home market is characterized by local currency pricing and the foreign market by producer currency pricing. (The case of local currency price stickiness applied symmetrically to both countries is reported in Appendix Figure 3.)

Note first that the effects of the tariff under PPI targeting are very similar to the benchmark model with stickiness in the currency of the producer. The dynamics of sectoral output and inflation are nearly the same as in Figure 1. The reason is that since tariffs are added on to import prices at the dock, after prices have been set by firms, the pass through of tariffs is unaffected by the specification of price stickiness. Home GDP is somewhat different in this case, since the alternative price stickiness specification means tariffs raise the relative price of differentiated goods relative to non-differentiated more, hence giving more weight in the aggregation of overall GDP to the rise in home production of differentiated goods than the fall in home production of non-differentiated goods. Regarding the exchange rate, in contrast, it is well understood in the literature that stickiness in the local currency of the buyer dampens pass-through to import prices paid by consumers. This means that the home currency appreciation in the figure does not lower home import prices, and the terms of trade is more strongly affected by the tariff.

Since the exchange rate appreciation no longer can be used as an effective means of countering the relative import price distortion of the tariffs, the optimal policy no longer employs this channel. The exchange rate under optimal policy actually appreciates less than under the PPI-target, and the movement of the terms of trade is unchanged. The home monetary policy now becomes strongly expansionary, with PPI inflation positive, and interest rate rising less than the PPI inflation target would require. This home policy reflects the prediction of the simplified analytical model, that in the case of local currency stickiness home policy should begin to respond to a home tariff, with the logic that stabilizing domestic demand for imports can induce foreign exporters to lower average prices for home imports. The simulation extends the analytical result to make clear that the direction of this home optimal policy respond is expansionary. The policy focuses on stabilizing overall demand rather than the relative price of exports, and the policy does nothing to dampen the sectoral reallocation induced by the tariff. The policy implies higher CPI inflation in both countries. The effect on GDP again is complicated by the change in sectoral relative prices used in aggregating output over the two sectors.

Appendix Figure 3 shows the additional case when local currency pricing (LCP) is applied symmetrically to both home and foreign countries. The observations from Figure 2 are amplified: the optimal home policy is even more expansionary, and optimal foreign policy now flips fully from expansionary to contractionary. Appendix Figure 4 shows the additional case where the foreign currency is dominant, rather than the home currency. Optimal policy in this case is an amplified version of that in the benchmark case, in that home policy becomes distinctly contractionary, and the home currency appreciation is even stronger than in the benchmark case. The logic is that this case eliminates a side effect of the exchange rate appreciation. While appreciation serves to offset the distortionary effect of the tariff on home import prices, it introduces a distortion in the relative price of imports in the foreign country. This adverse side effect is dampened under foreign import prices sticky in their local currency.

4.4 Welfare implications and sensitivity

Table 2 reports the welfare gain of optimal policy relative to a suboptimal Taylor Rule, measured in units of steady state consumption. Table 3 reports welfare gains relative to the PPI targeting rule. To compute these implications, we conduct a stochastic simulation of the model in which unilateral home tariff shocks are mean zero (shocks include both hikes and cuts in tariff rates).[15]

[15] Perturbation solution methods require that shocks be mean zero.

For the benchmark case of the model, depicted in Figure 1, optimal policy raises world welfare by 0.096 percent relative to the Taylor Rule. We note that welfare levels of both counties rise, though by different amounts, home by 0.162 percent and foreign by 0.031 percent (first line of Table 2). Even though the cooperative policy imposes a monetary contraction on the home country raising the tariff, this policy benefits both countries in terms of conditional welfare. In fact, home benefits somewhat more, since a path of tariff shocks that start and stop randomly is more harmful in our model to home welfare than foreign, so a policy that remediates such shocks is particularly beneficial to the home country. The gains from optimal policy relative to the PPI targeting rule in Table 3 are smaller: 0.020 percent for the world welfare, 0.024 for home, and 0.016 for foreign.

Under the assumption of home dominant currency, welfare gain in line 2 of the tables are smaller (0.046 percent and 0.042 percent relative to Taylor and PPI targeting, respectively). This is due to the limited ability of monetary policy to remediate the distortion of the tariff on home relative prices.[16] Welfare gains are fairly similar under the specification of symmetric LCP price stickiness.

We also conducted sensitivity analysis for welfare gains to alternative parametrization of the model, especially with regard to key elasticities, with results also reported in Tables 2 and 3. In summary, welfare gains are somewhat greater if the two sectors are modeled as complements (ξ=0.5). This is meant to reflect the possibility that non-differentiated goods like steel and commodities are combined with differentiated goods as complementary inputs in producing the final consumption good. Welfare gains are somewhat lower if the two sectors are substitutes (ξ=1.4).

The market structure for the non-differentiated sector in our model can nest the case of a homogeneous good common in the trade literature (η large), or a country specific good calibrated as in the macro real business cycle literature (η=1.5). Welfare gains from optimal policy are slightly larger in the form case and slightly smaller in the latter (see lines 6 and 7 of Tables 2 and 3). Welfare gains are substantially larger in the case of persistent tariffs (ρ_T = 0.95)), with gains of 0.44% percent. Though dynamics of variables exhibit longer lasting effects of tariffs (see Appendix Figure 5), the optimal policy is similar to that in the benchmark case. We also contrast results in our model with the standard assumption in the macro literature

[16] Relative to the PPI targeting rule, the welfare gains of the cooperative optimal policy are negative for the home country, suggesting that a cooperative policy might be harder to sustain for this case. Relative to the standard Taylor rule, the welfare gains remain positive for both countries.

that the second sector is nontraded. While overall welfare gains in table 2 and 3 are similar to the benchmark case (with smaller welfare gains specific to the home country), Appendix Figure 6 shows that the sectoral reallocation is a fraction of that in our benchmark case with two traded sectors. Without a second traded sector to offer alternative specialization of the foreign country, the home country cannot achieve as much specialization in the differentiated sector.

5. Contrasting the effects of tariffs on differentiated and non-differentiated goods

In Figure 3, we bring our model to bear on the case of a home tariff on imports of the foreign non-differentiated good, in contrast to the differentiated good. As shown in this figure, under a PPI-targeting rule, the sectoral reallocation induced by the tariff is precisely the opposite of the benchmark case in Figure 1, shifting home demand and production away from differentiated goods toward non-differentiated goods, with offsetting sectoral reallocation in the foreign country. As predicted by the analytical results, this reallocation is facilitated by a home currency appreciation induced by the home tariff, which makes home exports of differentiated goods less competitive. Also in contrast with Figure 1, home overall GDP rises for this tariff. The rise in home non-differentiated production is larger in percentage terms than the fall in differentiated production. Different from Figure 1, also, the dynamics of differentiated production are positive *and* smoother over time, due to the slow adjustment of the sticky prices in the sector.

The direction of optimal policy also differs for the non-differentiated sectoral tariff compared to the benchmark case of a differentiated sectoral tariff, coinciding with the prediction of the simple analytical model. Home monetary policy is now more expansionary than the PPI-targeting rule, with interest rate rising less and positive PPI inflation. Foreign monetary policy, in contrast, is now more contractionary than the PPI-targeting rule, with interest rate falling less, and negative PPI inflation. As a result, the home exchange rate now appreciates less due to the policy and the terms of trade movement induced by the tariff is larger than under PPI-targeting. This policy serves to significantly moderate the fluctuation in differentiated goods production in the two countries, though there is essentially no impact of the policy to moderate sectoral reallocation in the foreign country. The reason is that the latter sector has flexible prices, and this is insulated from the effect of monetary policy, which can do little to remediate the effects of the tariff on this sector. But sticky prices in the differentiated goods sector mean that monetary policy can remediate the side effects of this policy on the differentiated sector.

In sum, we can add this case, a home tariff on non-differentiated goods, to the list of cases calling for optimal home monetary expansion, along with home dominant currency and low trade elasticity.

6. Tariff with retaliation

While the focus of this paper is on a unilateral home (U.S.) tariff, it remains possibility that central banks will be called upon to deal with a situation in which the foreign country reciprocates U.S. tariffs with tariffs of its own in a tariff war.

6.1 Symmetric Tariff war

Appendix Figure 7 shows the case of symmetric tariff hikes imposed on the differentiated goods exports of both countries. The fall in home aggregate GDP now applies to both countries. In addition, the contraction in activity is largely driven by the fall in differentiated goods production. The production of non-differentiated goods actually rises somewhat, but not enough to compensate for the fall in overall output coming from the differentiated sector. In a symmetric tariff war, there is no shift in sectoral specialization across countries---rather, the tariff distortions result in a shift in the sectoral composition of output at a global level.

The optimal monetary policy stance is expansionary in both countries, despite the inflationary impact of the tariff. Given that a symmetric tariff war cannot be remedied by a currency depreciation, the optimal policy aims at resolving the distortion created by the tariff between differentiated and non-differentiated prices within each country. An expansionary monetary stance mitigates the contraction in the differentiated good sector, driving up overall aggregate demand as well as the prices of non-differentiated goods, which are flexible.[17]

[17] Appendix Figure 2 show the case of a symmetric trade war where firms in both countries set export prices in local currency (LCP stickiness). Monetary policy has minimal impact on trade flows.

Tables 2 and 3 (row 10) suggest that the welfare loss of a symmetric tariff war is lower than for tariffs that are not coincident. This can be attributed to the fact that a symmetric tariff does not imply a large asymmetric sectoral reallocation across countries associated with a shift in comparative advantage, as found in the case of a unilateral tariff in the analysis above. Row 11 shows that under home dominant currency pricing, optimal policy favors the foreign country, but welfare changes remain small. (See Appendix Figure 8 for dynamics).

A similarly diminished welfare loss is associated with a symmetric tariff war launched on exports of non-differentiated goods of both countries. (See row 13 of Tables 2 and 3 for welfare, and Appendix Figure 9 for impulse responses.)

6.2 Asymmetric Tariff war

We next consider a hybrid case where home tariffs on differentiated goods imports are met by foreign retaliation in the form of tariffs of an equal percentage on imports of less differentiated goods. This scenario could be interpreted as representing U.S. tariffs on manufacturing imports, while foreign countries retaliate with tariffs on US agricultural commodities (such as Chinese limits on imports of U.S. soybeans). Figure 4 shows that under PPI targeting, the sectoral reallocation is larger than in the case of a unilateral foreign tariff in Figure 1, as the foreign tariff further shifts production of non-differentiated goods to foreign, reinforcing the reallocation induced by home tariffs promoting home manufacturing (differentiated goods). In contrast with Figure 1, the fall in overall GDP in Figure 4 now is larger in the home country than the foreign.

Nonetheless, the optimal policy is qualitatively similar to that in the case of the unilateral foreign tariff in Figure 1, calling for a foreign expansion and home contraction. Since the non-differentiated sector is characterized by flexible prices, monetary policy has little power to affect demand in that sector. So optimal policy is driven by the goal of offsetting the home tariffs on differentiated goods.

Tables 2 and 3 (see row 14) show that welfare gains from optimal policy are asymmetric, favoring the home country more than foreign, and world welfare is somewhat higher than the benchmark case of the unilateral differentiated goods tariff.

7. Conclusion

Central banks around the world have grappled with the question of how to respond to the mix of inflation and output implications of tariffs, with the potential of igniting a trade war across and within geopolitical regions. The question is compounded by the fact that the recent rounds of tariffs may fall on a very broad range of goods, from final consumption goods to materials such as aluminum and steel, and tariff rates may be set quite high.

Using a New Keynesian model enriched with elements from the trade literature, including global value chains in production, and comparative advantage between multiple traded sectors that differ in terms of market structure and price rigidity, we find that the optimal monetary response can be either expansionary or contractionary, depending in particular on the sector targeted by tariffs and on the nature of price stickiness.

In the case of tariffs targeting differentiated final consumption goods, characterized with monopolistic competition and price stickiness at the producer level, a specific objective of monetary policy is to redress the distortionary effects of tariffs on relative prices between home and foreign goods, at least in part, via foreign currency depreciation. A foreign expansion with depreciation counteracts the impact of tariffs on both aggregate GDP and sectoral reallocation. In the case of non-differentiated goods, their price flexibility implies that a currency depreciation cannot significantly redress the distortionary effects of the tariff on relative prices, and an expansion will induce a strong price rather than a quantity response by firms in the sector. However, monetary stabilization can compensate the loss of production due to the fall in the foreign demand for non-differentiated goods (due to the tariff) by raising the demand and production of differentiated goods. Indeed, by pursuing an optimal expansion cum depreciation to hasten the reallocation toward the differentiated goods, monetary policy promotes welfare by amplifying the reallocation induced by the tariff rather than offsetting it. As a note of caution, however, we find that the role for optimal policy is limited when the country imposing the tariffs has the advantage of a currency dominant in global trade. Sticky dollar prices in exports reduce the ability of domestic central bank to affect the international demand for domestic output via currency depreciation.

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Table 1. Parameter Values

<u>Preferences</u>	
Risk aversion	$\sigma = 2$
Time preference	$\beta = 0.99$
Labor supply elasticity	$1/\psi = 1.9$
Differentiated goods share	$\theta = 0.45$
Non-differentiated goods home bias	$\nu = 0.5$
Differentiated goods elasticity	$\phi = 5.2$ (also 3.8)
Non-differentiated goods elasticity	$\eta = 5.2$ (also 1.5, 15)
Substitution between sectors	$\xi = 1$ (also 0.5, 1.5)
<u>Technology</u>	
Firm death rate	$\delta = 0.025$
Price stickiness	$\psi_P = 49$
Intermediate input share	$\varsigma = 1/3$
Differentiated goods trade cost	$\tau_D = 0.44$
Non-differentiated goods trade cost	$\tau_N = 0.44$
Mean sunk entry cost	$\overline{K} = 1$
Firm entry adjustment cost	$\lambda = 0.10$
Bond holding cost	$\psi_B = 10^{-6}$
<u>Monetary Policy (for Taylor Rule)</u>	
Interest rate smoothing	$\gamma_i = 0.7$
Inflation response	$\gamma_p = 1.7$
GDP response	$\gamma_Y = 0.1$
<u>Tariff shock</u>	
Mean	$\overline{T_D} = \overline{T_N} = 1.02$
Standard deviation	$\sigma_{\varepsilon_T} = 0.08$
Autoregressive parameter	$\rho_T = 0.56$

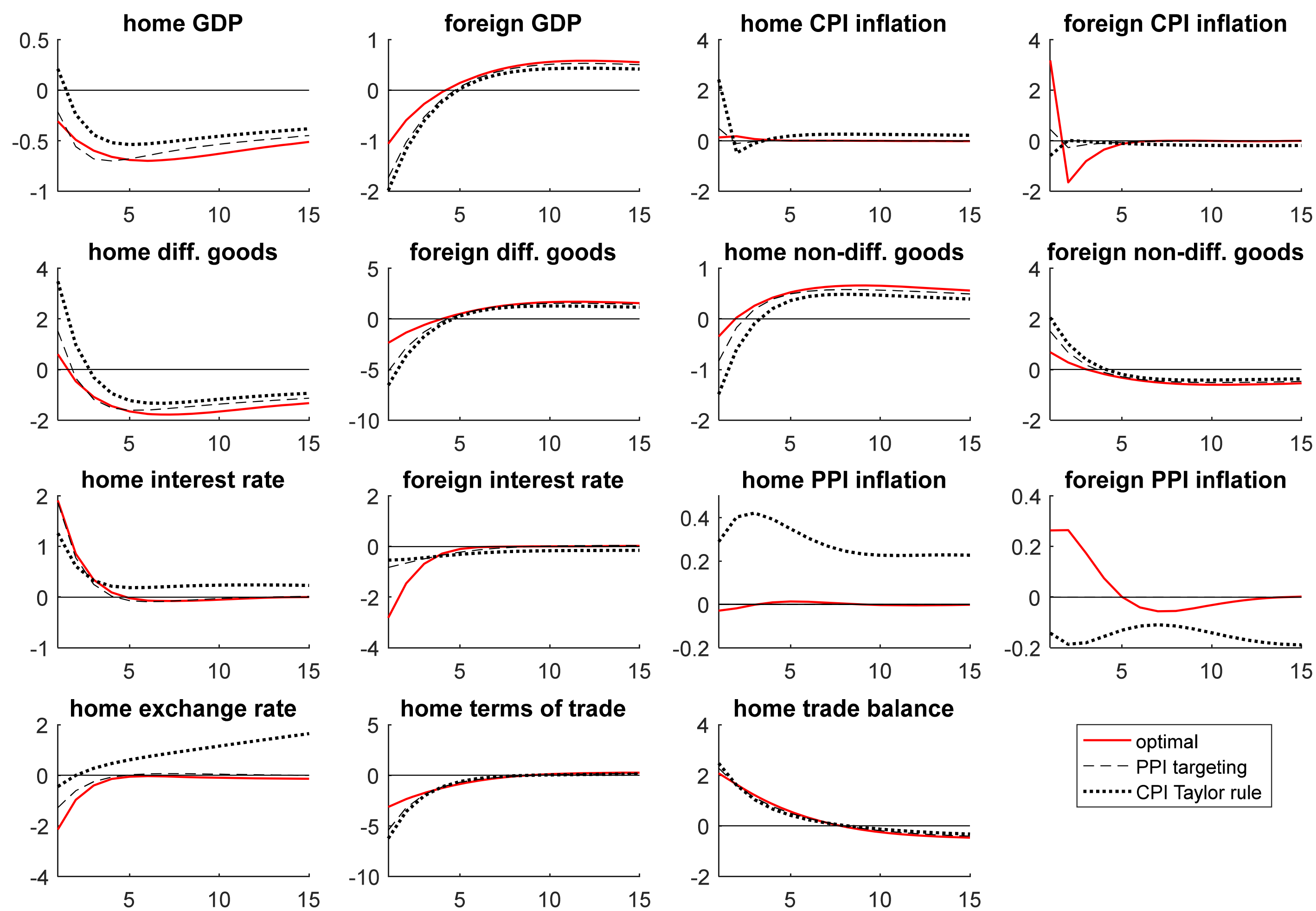
Table 2. Welfare gains from Ramsey Optimal policy relative to Taylor Rule
(percent change, in consumption units):

	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.096	0.162	0.031
2.	home DCP	0.046	0.066	0.026
3.	both countries LCP	0.042	0.037	0.047
4.	sectors complements (ξ=0.5)	0.113	0.185	0.040
5.	sectors substitutes (ξ=1.4)	0.093	0.151	0.034
6.	nondifferentiated goods BKK (η=1.5)	0.092	0.148	0.036
7.	nondiff. goods homogeneous (η=15)	0.108	0.118	0.099
8.	persistent tariff ($\rho_T = 0.95$)	0.444	0.194	0.694
9.	nontraded non-differentiated good	0.110	0.116	0.103
10.	symmetric tariff on differentiated goods	0.027	0.027	0.027
11.	home DCP	0.057	-0.100	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	0.035	-0.006
13.	symmetric tariff on non-differentiated goods	0.013	0.013	0.013
14.	home diff. tariff; foreign non-diff. tariff	0.121	0.174	0.067

Table 3. Welfare gains from Ramsey Optimal policy relative to PPI targeting Rule
(percent change, in consumption units):

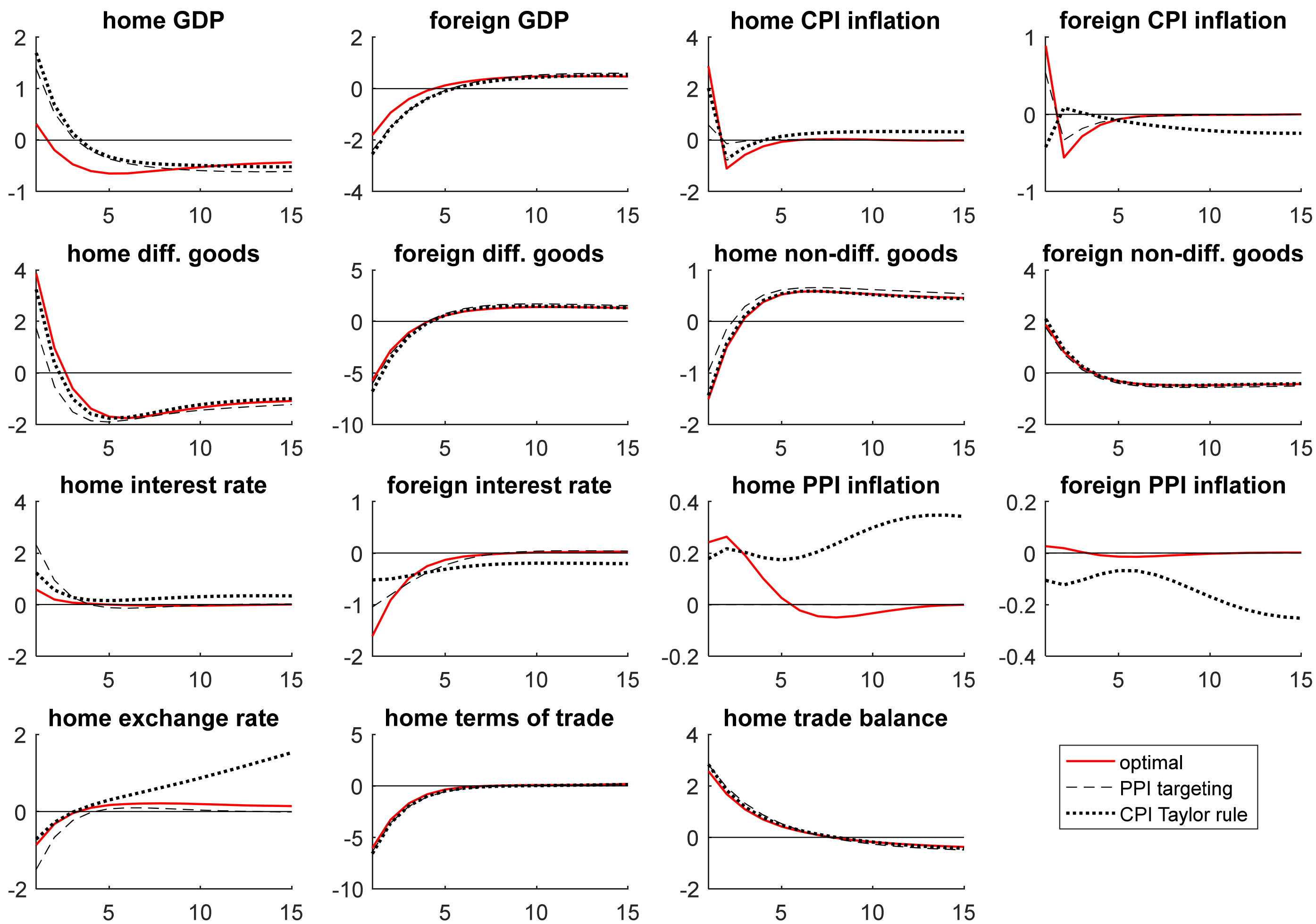
	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.020	0.024	0.016
2.	home DCP	0.017	-0.083	0.117
3.	both countries LCP	0.023	-0.127	0.172
4.	sectors complements (ξ=0.5)	0.023	0.032	0.014
5.	sectors substitutes (ξ=1.4)	0.016	0.016	0.017
6.	nondifferentiated goods BKK (η=1.5)	0.019	0.024	0.013
7.	nondiff. goods homogeneous (η=15)	0.018	0.010	0.025
8.	persistent tariff ($\rho_T = 0.95$)	0.053	0.034	0.071
9.	nontraded non-differentiated good	0.017	0.010	0.024
10.	symmetric tariff on differentiated goods	0.071	0.028	0.028
11.	home DCP	0.034	-0.096	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	-0.015	0.025
13.	symmetric tariff on non-differentiated goods	0.001	0.001	0.001
14.	home diff. tariff; foreign non-diff. tariff	0.034	0.068	0.000

Figure 1. Impulse responses to a rise in home tariff on differentiated imports



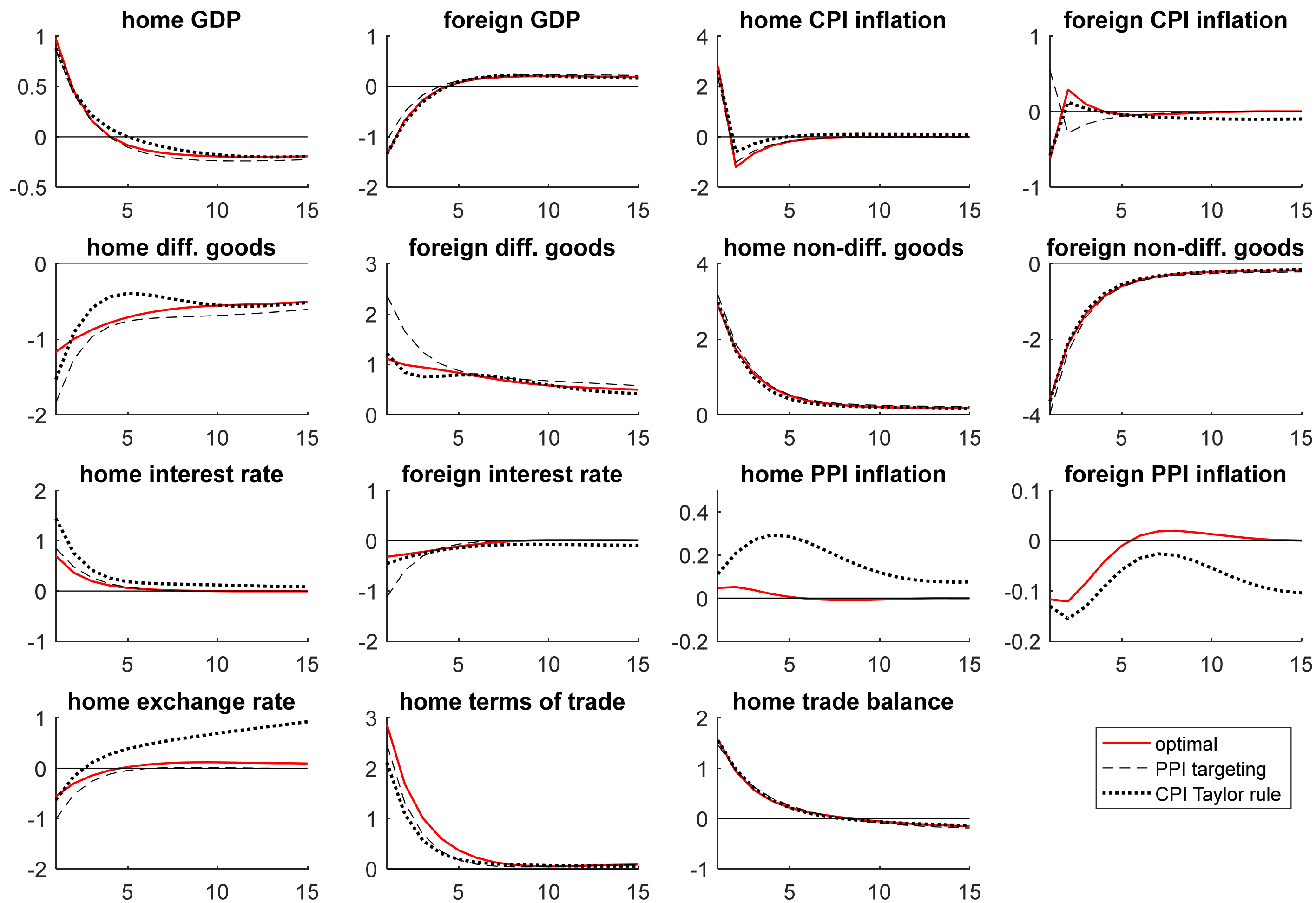
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 2. Impulse responses to a rise in home tariff on differentiated imports; Home currency dominant



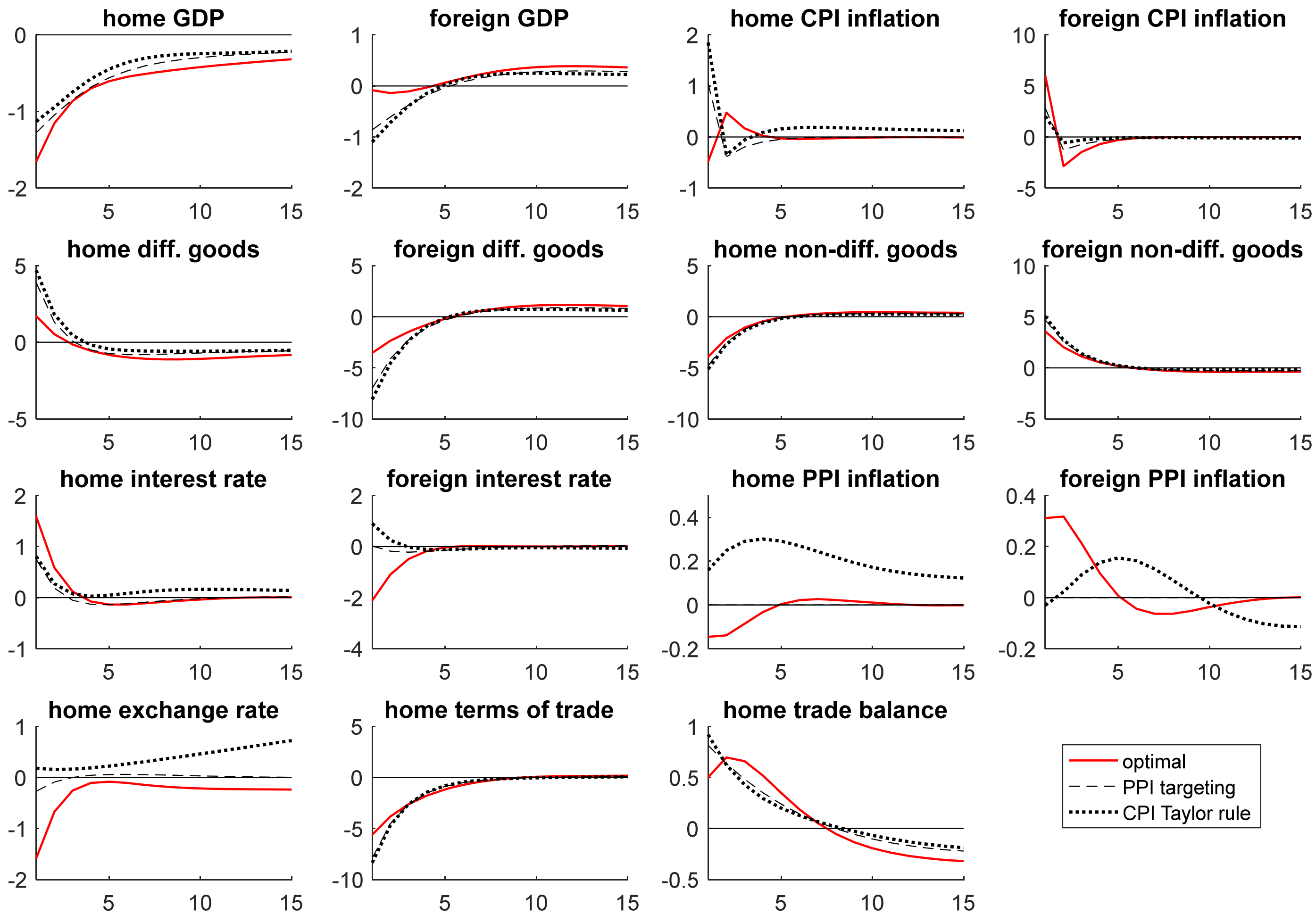
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 3. Impulse responses to a rise in home tariff on non-differentiated imports



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 4. Impulse responses to a hybrid case: home tariff on differentiated imports; foreign tariff non-differentiated imports



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix

For
"Monetary stabilization of sectoral tariffs"

by
Paul R. Bergin
and
Giancarlo Corsetti

Appendix A. Demand equations from the benchmark model not listed in text

The composition of expenditure on adjustment costs, both for prices and bond holding, follows the same preferences as for consumption, and the associated demands mirror Eqs. (4)-(9). Adjustment costs for bond holding are as follows:

$$AC_{B,D,t} = \theta P_t AC_{B,t} / P_{D,t}$$

$$AC_{B,N,t} = (1-\theta) P_t AC_{B,t} / P_{N,t}$$

$$d_{AC,B,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$d_{AC,B,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$AC_{B,H,t} = \nu \left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t}$$

$$AC_{B,F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t}\ .$$

The economy-wide demand for goods arising from price adjustment costs sums across the demand arising among *n* home firms: $AC_{P,t} = n_t AC_{P,t}(h)$. This is allocated as follows:

$$AC_{P,D,t} = \theta P_t AC_{P,t} / P_{D,t}$$

$$AC_{P,N,t} = (1-\theta) P_t AC_{P,t} / P_{N,t}$$

$$d_{AC,P,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$d_{AC,P,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$AC_{P,H,t} = \nu \left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t}$$

$$AC_{P,F,t} = (1-\nu)\left(P_{F,t}T_{N,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t} .$$

The demand for differentiated goods for use as intermediates in production mirrors Eqs. (6)-(7), as follows:

$$d_{G,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} G_t$$

$$d_{G,t}(f) = \left(p_t(f)T_{D,t} / P_{D,t}\right)^{-\phi} G_t .$$

The demand for differentiated goods for use in the sunk entry investment of new firms mirrors Eqs. (6)-(7), as follows:

$$d_{K,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} ne_t K_t$$

$$d_{K,t}(f) = \left(p_t(f)T_{D,t} / P_{D,t}\right)^{-\phi} ne_t K_t .$$

2. Market clearing conditions not listed in the text

Market clearing for the non-differentiated goods market requires:

$$y_{H,t} = C_{H,t} + AC_{P,H,t} + AC_{B,H,t} + (1+\tau_N)\left(C^*_{H,t} + AC^*_{P,H,t} + AC^*_{B,H,t}\right)$$

$$y_{F,t} = (1+\tau^*_N)\left(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}\right) + C^*_{F,t} + AC^*_{P,F,t} + AC^*_{B,F,t} .$$

The market clearing condition for the manufacturing goods market is given in Eq. (19) in the main text.

Labor market clearing requires:

$$\int_0^{n_t} l_t(h)dh + l_{H,t} = l_t .$$

Bond market clearing requires:

$$B_{Ht} + B^*_{Ht} = 0$$

$$B_{Ft} + B^*_{Ft} = 0 .$$

Balance of payments requires:

$$\int_0^{n_t} p^*_t(h)\left(d^*_t(h)\right)dh - \int_0^{n^*_t} p_t(f)\left(d_t(f)\right)df + P^*_{Ht}\left(C^*_{H,t} + AC^*_{P,H,t} + AC^*_{B,H,t}\right)$$

$$-P_{F,t}\left(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}\right) - i_{t-1}B^*_{H,t-1} + e_t i^*_{t-1}B_{F,t-1} = \left(B^*_{H,t} - B^*_{H,t-1}\right) + e_t\left(B_{F,t} - B_{F,t-1}\right).$$

Appendix B: Derivation of Analytical Results, Producer Currency Pricing

a) Demands

The modified consumption index implies the following demands:

$$C_{DHt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{P_{DHt}} = \frac{\theta}{2}\frac{P_t C_t}{P_{DHt}}$$

$$C_{DFt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{T_{Dt}e_t P^*_{DFt}} = \frac{\theta}{2}\frac{P_t C_t}{T_{Dt}e_t P^*_{DFt}}$$

$$C^*_{DHt} = \frac{\theta}{2}\frac{e_t P^*_t C^*_t}{T^*_{Dt}P_{DHt}} = \frac{\theta}{2}\frac{P_t C_t}{T^*_{Dt}P_{DHt}}$$

$$C_{Dt} = \theta\frac{P_t C_t}{P_{Dt}}$$

$$C_{Nt} = \left(1-\theta\right)\frac{P_t C_t}{P_{Nt}}$$

$$c_t(h) = \left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi} C_{DH,t}$$

b) Optimal price setting differentiated good

The home firm maximizes

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(\frac{T^*_t p_t\left(h\right)}{e_t}\Big/\left(\frac{T^*_t P_{DH,t}}{e_t}\right)\right)^{-\phi}C^*_{DH,t}\right]\right]$$

or $E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi}\left(C_{DH,t}+C^*_{DH,t}\right)\right]$,

implying the price setting rule

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*_{DH,t}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*_{DH,t}\right)\right]}.$$

Substitute in demands from above

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_t C_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_t P^*_t C^*_t}{T^*_{Dt}P_{DHt}}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_t C_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_t P^*_t C^*_t}{T^*_{Dt}P_{DHt}}\right)\right]},$$

and substitute in for μ_t and exchange rate

$$P_{DHt} = \frac{\phi}{\phi-1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\right]}.$$

Use $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$ from the main text:

$$P_{DHt} = \frac{\phi}{\phi-1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}.$$

The foreign firm counterpart is:

$$P_{DFt}^* = \frac{\phi}{\phi-1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt} + C_{DF,t}\right)\frac{W_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt} + C_{DF,t}\right)\right]}.$$

Substitute in for in μ_t^* mu and exchange rate:

$$P_{DFt}^* = \frac{\phi}{\phi-1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*}\right)\right]}$$

$$P_{DFt}^* = \frac{\phi}{\phi-1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}$$

So the home price index can be written:

$$P_t = 2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^\theta\right] P_{DHt}{}^{\theta/2}\left(T_{Dt}e_t P_{DFt}^*\right)^{\theta/2} P_{NHt}{}^{1-\theta}.$$

Use $P_{NHt} = \frac{W_t}{\alpha_N} = \frac{\kappa\mu_t}{\alpha_N}$ to write the price index in terms of exogenous variables:

$$P_t = 2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^\theta\right] P_{DHt}{}^{\theta/2}\left(T_{Dt}e_t p_t^*(f)\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}$$

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}e_t\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}$$

.

c) labor

Given the homogenous second sector, the easiest way to derive equilibrium labor is from the household budget constraint, which under balanced trade, implies labor income equals total nominal expenditure minus profits from the home differentiated sector.

Write the household budget constraint:

$$W_t l_t + \pi_t = P_t C_t ,$$

where π is profits of home differentiated goods firms, used in the firm maximization problem above to determine price setting. Use labor supply condition to substitute out wage: $\kappa\mu_t l_t + \pi_t = \mu_t$, and use this to compute the term in welfare including labor, $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home differentiated good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right)\right].$$

Use $C_{DHt} + C^*_{DHt} = \frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}$ from price setting derivation

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + P_{DHt}\frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

So $\kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$

Note that there is no μ_t left in this term, so the labor term in the welfare condition will have no bearing on the optimal monetary policy under our specifcation.

d) Home optimal policy, Nash

Write home welfare, and express as a function of exogenous variables.

$W_t = E_{t-1}\ln C_t - E_{t-1}\kappa l_t$

$W_t = E_{t-1}\left[\ln\mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$

$$W_t = E_{t-1}\left[\ln\mu_t\right] - E_{t-1}\left[\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

$$\begin{aligned}W_t = {} & E_{t-1}\left[\ln\mu_t\right] - \ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right) - \theta\ln\left(\frac{\phi}{\phi-1}\right) - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right) \\ & -\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}\right)+\ln\mu_t-\ln\mu_t^*+\ln\alpha_N^*-\ln\alpha_N-\ln T_{N,t}\right] - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right) - (1-\theta)E_{t-1}\left(\ln\kappa+\ln\mu_t-\ln\alpha_N\right) \\ & -1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\end{aligned}$$

Take the derivative of home welfare with respect to the home monetary policy variable μ:

$$\frac{1}{\mu} - \frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} - \left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu} = 0$$

$$\mu = \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)}$$

Conjecture the solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$,

which is easily verified by substituting this in the equation immediately above.

e) Foreign optimal policy, Nash

The foreign price index is:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}/e_t\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

Foreign labor:

$$\kappa E_{t-1}\left[l_t^*\right]=1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Welfare:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t^*\right] - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t^*\right] - E_{t-1}\left[\ln\left(2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right]-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

$$W_t^* = E_{t-1}\left[\ln \mu_t^*\right] - \ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right)-\theta\ln\left(\frac{\phi}{\phi-1}\right)-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right)$$

$$-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}^*\right)+\ln\mu_t^*-\ln\mu_t-\ln\alpha_N^*+\ln\alpha_N+\ln T_{Nt}\right]-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)-(1-\theta)E_{t-1}\left(\ln\kappa+\ln\mu_t^*-\ln\alpha_N^*\right)$$

$$1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Which is directly analogous to home, with solution $\mu_t^* = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$

f) Cooperative policy

Take derivative of sum of home and foreing welfare with respect to μ_t

$$\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}-\left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu}+\frac{\theta}{2}\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}=0$$

$$\mu=\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}.$$

Conjecture solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$, which is easily verified

We note that the cooperative solution is same as Nash in this case.

Now take the derivative with respect to μ_t^*:

$$\frac{1}{\mu_t^*}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}-\left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu}+\frac{\theta}{2}\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\theta\frac{1}{\mu_t^*}-\theta\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\mu_t^*=\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\mu_t^*\right]}{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)}$$

Conjecture same solution as before: $\mu_t^*=a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$, verified.

This is also same as Nash solution above.

g) Defining conditions under which home is both producer and net importer of homogeneous good

We can easily compute home consumption of the non-differentiated good:

$$C_{Nt}=(1-\theta)\frac{P_tC_t}{P_{Nt}},$$

where $P_{Nt}=e_tT_{N,t}P_{Nt}^*$ and $P_{Nt}=\frac{W_t}{\alpha_N}=\frac{\kappa\mu_t}{\alpha_N}$, $P_{Nt}^*=\frac{W_t^*}{\alpha_N^*}=\frac{\kappa\mu_t^*}{\alpha_N^*}$.

So $C_{Nt}=(1-\theta)\frac{P_tC_t}{\frac{\kappa\mu_t}{\alpha_N}}=(1-\theta)\frac{\mu_t}{\frac{\kappa\mu_t}{\alpha_N}}=(1-\theta)\frac{\alpha_N}{\kappa}$.

Note this is consant, unaffected by tariffs.

Now compute the level of home production in this sector based on labor allocation. Recall total labor allocation above:

$$l_t=\frac{1}{\kappa}-\frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt}+P_{DHt}C^*_{DHt}\right).$$

We next subtract labor for the differentiated goods sector: $\frac{C_{DHt}+C^*_{DHt}}{\alpha_D}$.

So the labor allocation for the non-differentiated sector becomes:

$$l_{Nt}=\frac{1}{\kappa}-\frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt}+P_{DHt}C^*_{DHt}\right)-\frac{C_{DHt}+C^*_{DHt}}{\alpha_D}$$

$$l_t = \frac{1}{\kappa} - \frac{1}{\phi\kappa}\frac{\theta}{2}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] - \frac{1}{\alpha_D}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}\right)$$

$$l_t = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right]$$

$$l_{Nt} = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right].$$

So output in the sector may be written:

$$y_{Nt} = \frac{\alpha_N}{\kappa} - \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right].$$

We conclude that the condition for where $C_{Nt} > y_{Nt}$ may be written:

$$(1-\theta)\frac{\alpha_N}{\kappa} - \frac{\alpha_N}{\kappa} + \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$(-1) + \left(\frac{1}{\phi}\frac{1}{2} + \frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$\left(\frac{1}{\phi}\frac{1}{2} + \frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 1.$$

Evaluate this condition under perfect foresight, abstracting from risk premium in pricing of the sticky price good:

$$\left(\frac{1}{\phi}\frac{1}{2} + \frac{1}{2}\frac{\phi-1}{\phi}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 1$$

$$1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} > 2$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} > 1 .$$

Derive the condition under which we also can guarantee positive home production of the non-differentiated good:

$$y_{Nt} = \frac{\alpha_N}{\kappa} - \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi} \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} \right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$y_{Nt} = 1 - \left(\frac{1}{\phi}\frac{\theta}{2} + \frac{\theta}{2}\frac{\phi-1}{\phi} \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]} \right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0 .$$

Again, evaluate in perfect foresight, abstracting from the risk premium in pricing of sticky-price goods:

$$y_{Nt} = 1 - \left(\frac{1}{\phi}\frac{\theta}{2} + \frac{\theta}{2}\frac{\phi-1}{\phi}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$y_{Nt} = 1 - \frac{\theta}{2}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta}$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta} - 1 .$$

Appendix C: Derivation of Analytical Results, Local Currency Pricing

a) Price setting:

Home good now has distinct prices in home and foreign market, P_{DHt} and P^*_{DHt}, with foreign price in foreign currency .

The home firm maximizes :

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p_t^*(h)-\frac{W_t}{\alpha_t}\right)\left(\frac{T_t^* p_t^*(h)}{T_t^* P^*_{DH,t}}\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p_t^*(h)-\frac{W_t}{\alpha_t}\right)\left(p_t^*(h)/P^*_{DH,t}\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

With respect to $p_t(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{1}{P_{DHt}}\frac{\mu_{t-1}}{\mu_t}\left(P_{DHt}-\frac{W_t}{\alpha_D}\right)\left(C_{DH,t}\right)\right]$$

$$P_{dHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=P_{DHt}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]-E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$(\phi-1)P_{DHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]$$

Maximizing with respect to $p_t^*(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{1}{P^*_{DH,t}}\frac{\mu_{t-1}}{\mu_t}\left(e_t P^*_{DH,t}-\frac{W_t}{\alpha_D}\right)C^*{}_{DH,t}\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]-\frac{1}{P^*_{DH,t}}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$\left(\phi-1\right)P^*_{DH,t}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]}$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\right]}.$$

Use the property of a homogeneous sector above: $e_t=\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}$:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\right]}$$

and cancel terms:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu^*_t}{T^*_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha^*_N}{\alpha_N T_{Nt}T^*_{Dt}}\right]}.$$

Analogously for foreign differentiated good:

$$P^*_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}E_{t-1}\left[\mu^*_t\right]$$

$$P_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha^*_N T_{Dt}}\right]}.$$

Non-differentiated prices are the same as in PCP case above.

Substitute into the home price index:

$$P_t=2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha^*_N T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}.$$

Analogously for foreign price index:

$$P_t^* = 2^\theta \left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt} T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

<u>Home equilibrium labor:</u>
Use labor supply condition to substitute out wage:
$\kappa\mu_t l_t + \pi_t = \mu_t$.
Use this to compute term for labor required in the welfare fucntion: $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home D good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + e_t P_{DHt}^* C_{DHt}^*\right)\right].$$

Use demands from above: $C_{DH,t} = \frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}}$, $C_{DF,t} = \frac{\theta}{2}\frac{P_t C_t}{T_{Dt} P_{DF,t}}$, $C_{DHt}^* = \frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}} + e_t P_{DHt}^*\frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + e_t\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right].$$

Sub in for exchange rate:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + \frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right]$$

So $\kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right]$.

Note this is the same as under PCP pricing.

<u>Compute welfare:</u>
Home:

$$W_t = E_{t-1}\ln C_t - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln\mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right]$$

$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\mu_t\right]\right) - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]\right) - (1-\theta)E_{t-1}\left[\ln \mu_t\right]$$

$$-E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{1}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

Compute derivative of home welfare with respect to μ_t:

$$\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} - (1-\theta)\frac{1}{\mu_t} = 0$$

$$\theta\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} = 0$$

$$\frac{1}{\mu_t} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} + \frac{1}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}$$

Foreign:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t^*\right] - E_{t-1}\kappa l_t^*$$

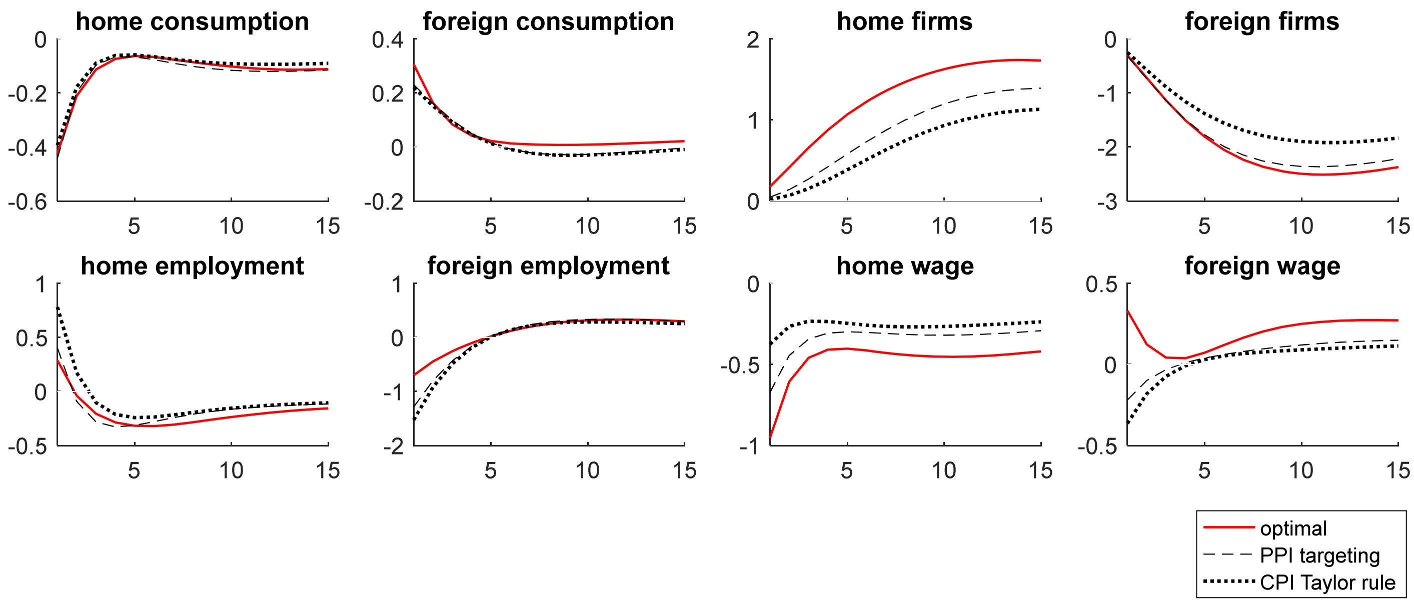
$$W_t^* = E_{t-1}\left[\ln \mu_t^*\right] - E_{t-1}\left[\ln\left(2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt}T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right]$$

$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

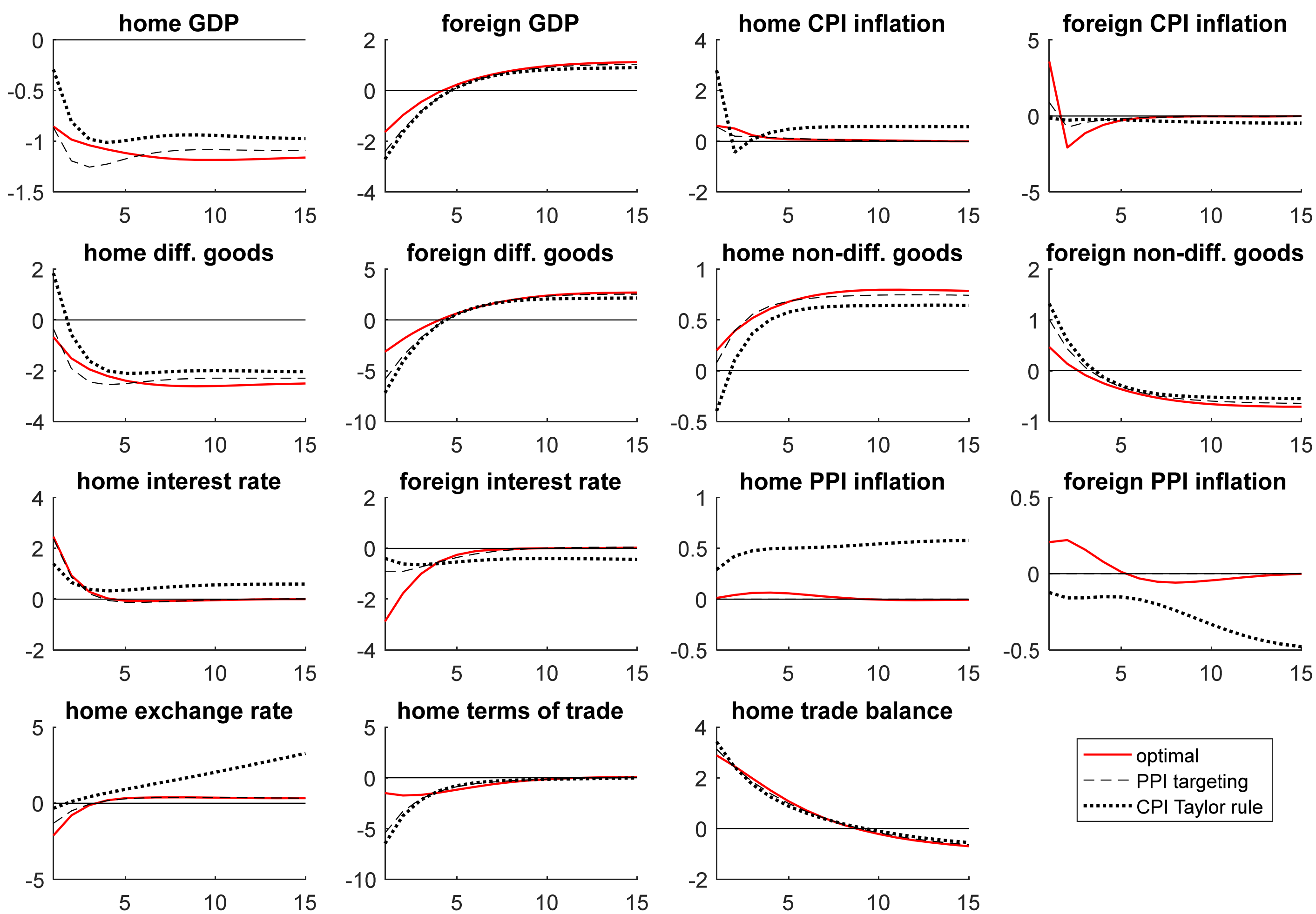
This is directly analogous to home. Now W* include only μ_t^* and no μ_t.
So directly analogous optimality condition:

$$\frac{1}{\mu_t^*}=\frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t^*\right]}+\frac{1}{2}\frac{\frac{1}{T_{Dt}^*}}{E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}.$$

Appendix Figure 1. Additional Impulse responses for benchmark case: home tariff on differentiated imports

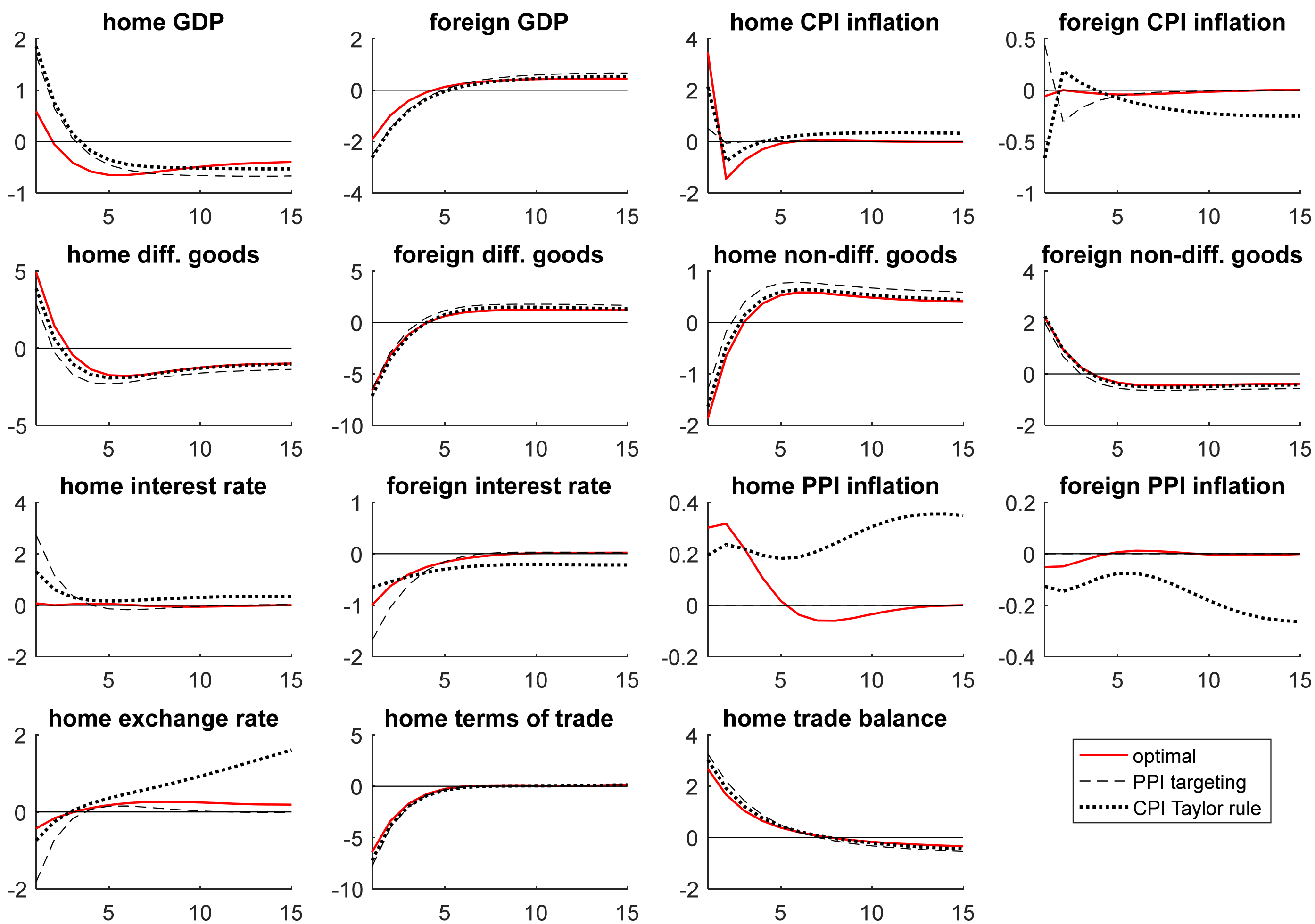


Appendix Figure 2. Impulse responses to a rise in home tariff on differentiated imports, lower trade elasticity (3.8)



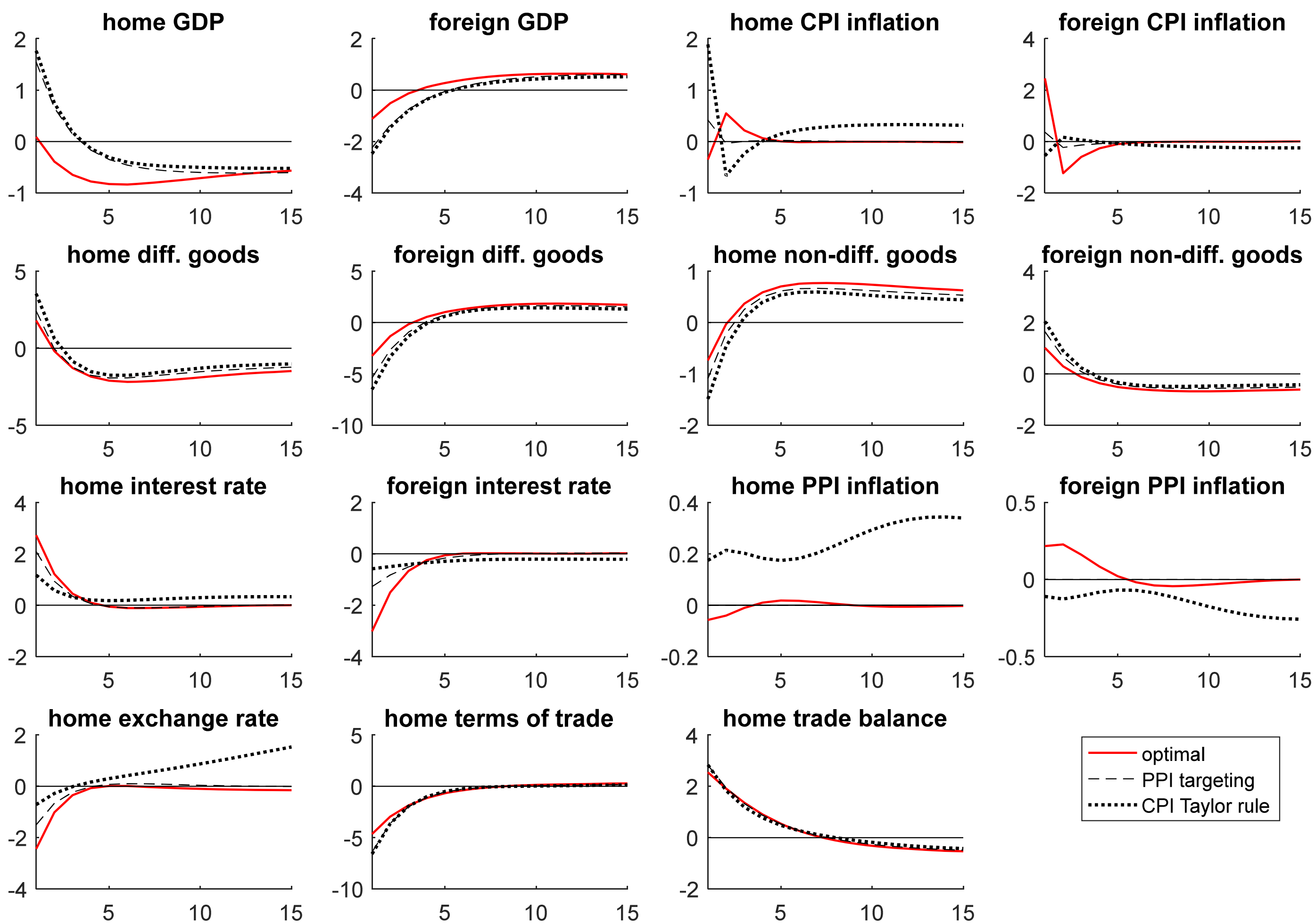
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 3. Impulse responses to a rise in home tariff on differentiated imports; LCP price stickiness in both countries



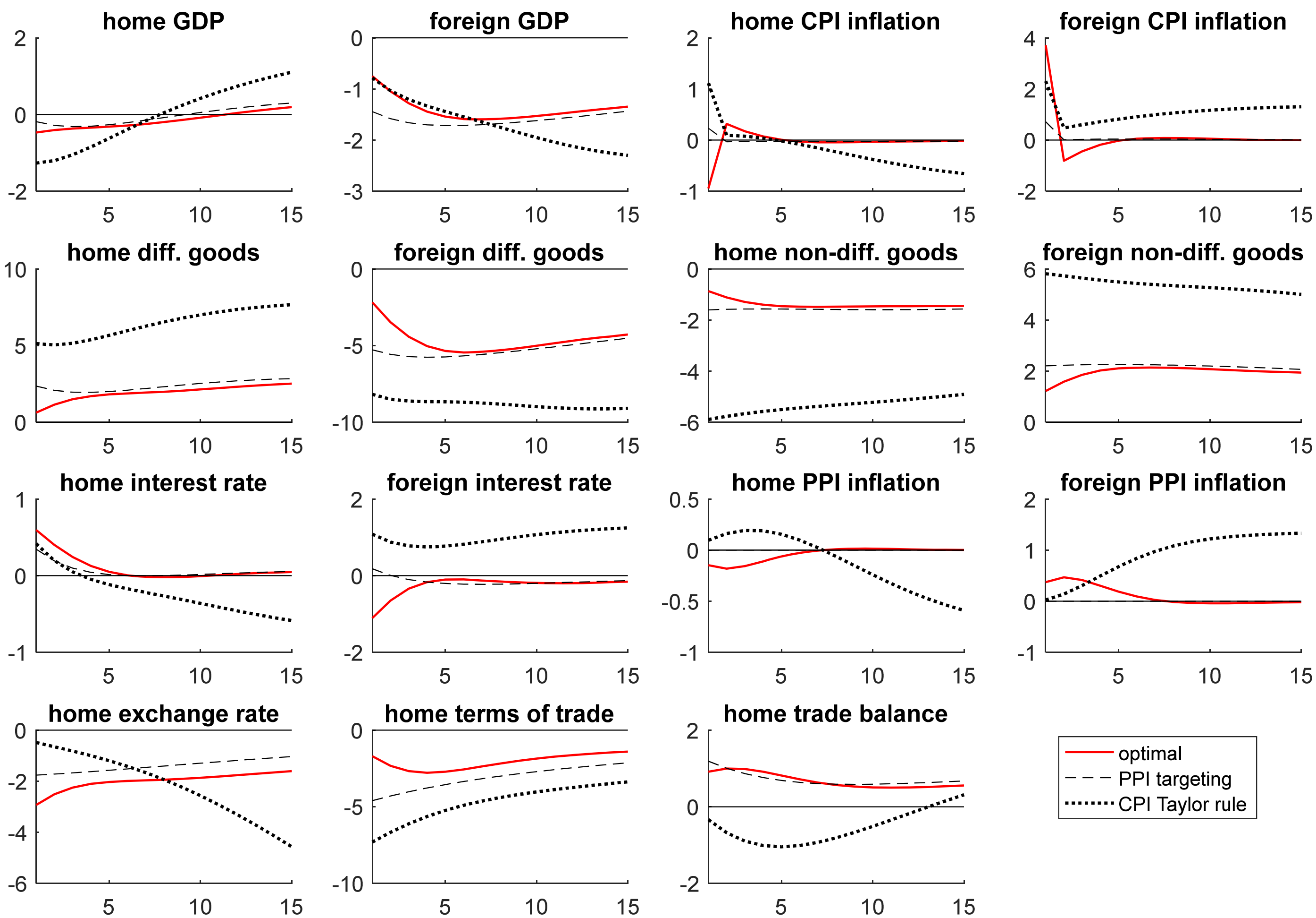
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 4. Impulse responses to a rise in home tariff on differentiated imports; foreign currency dominant



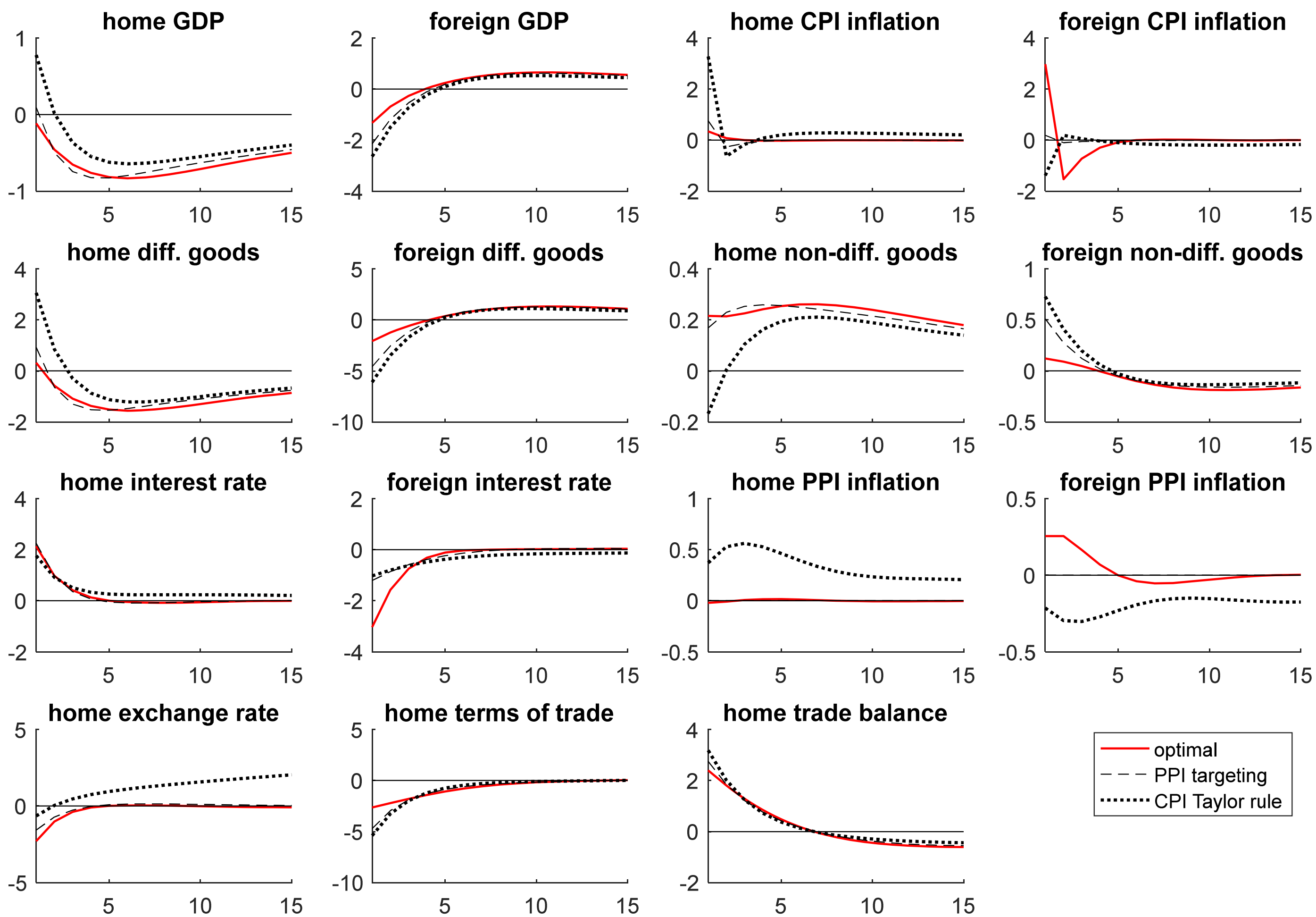
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 5. Impulse responses to a more persistent tariff on home differentiated imports



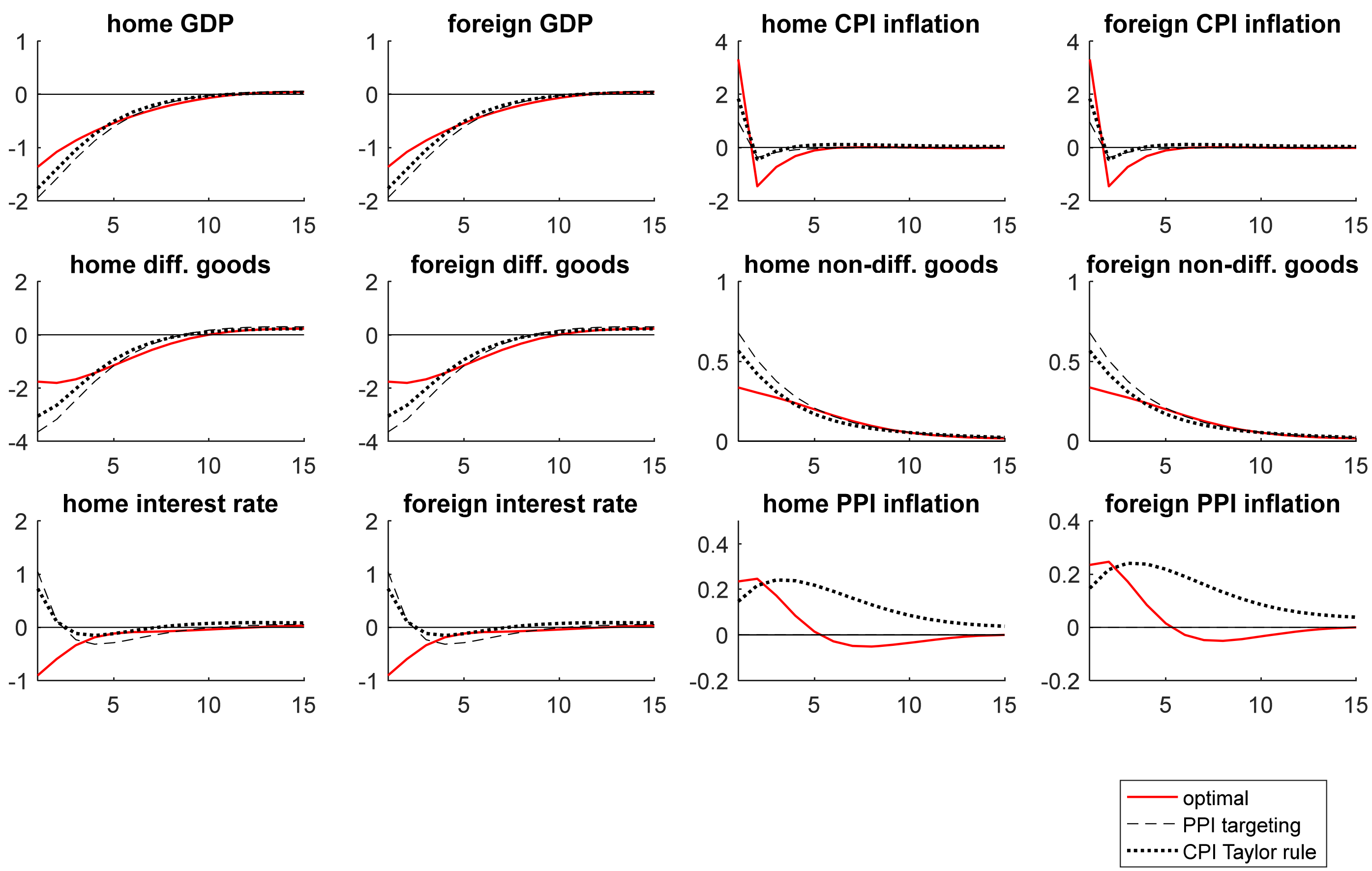
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 6. Impulse responses to home tariff on differentiated imports, nontraded non-diff. goods



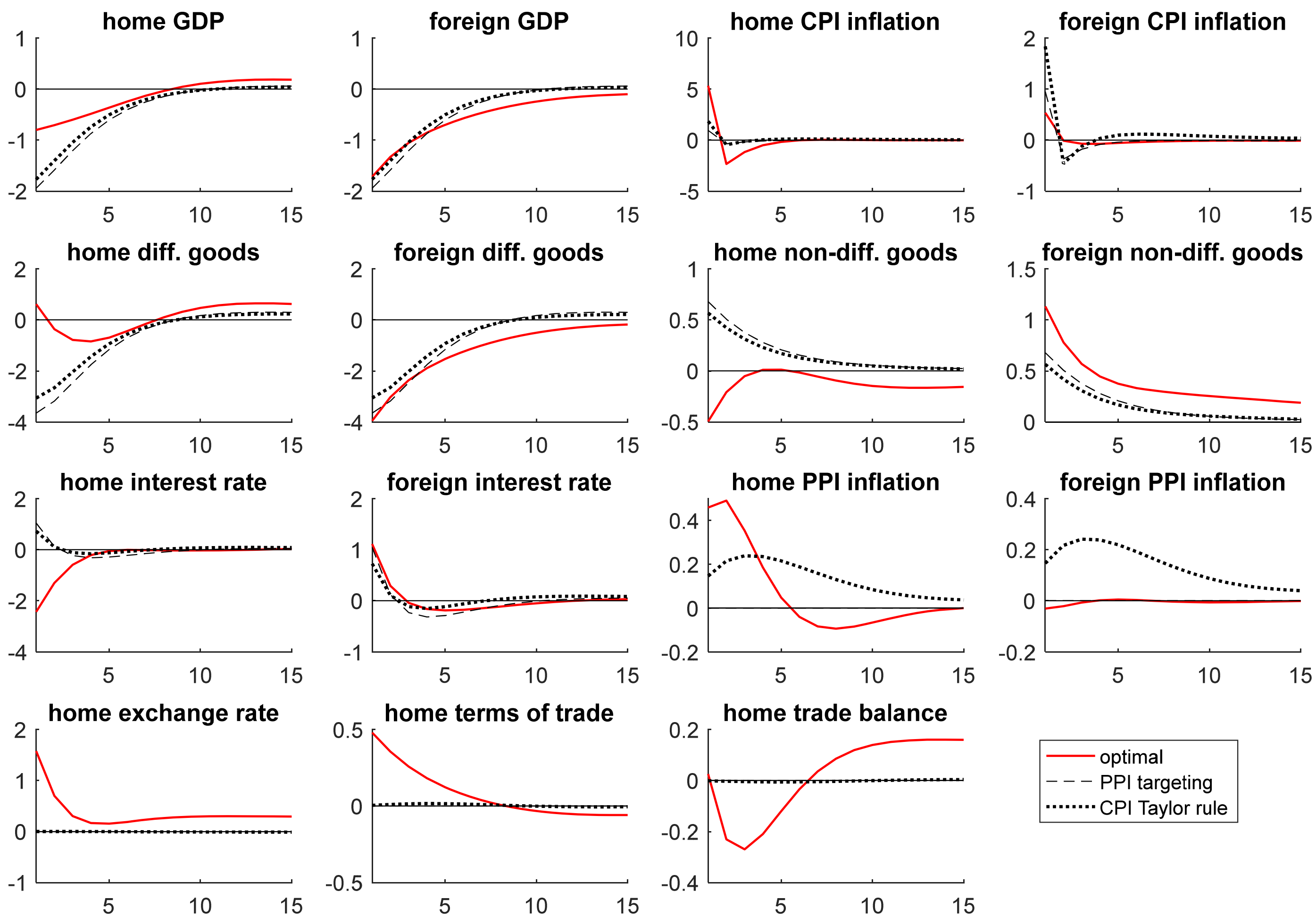
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 7. Impulse responses to a symmetric tariff to differentiated imports in both countries



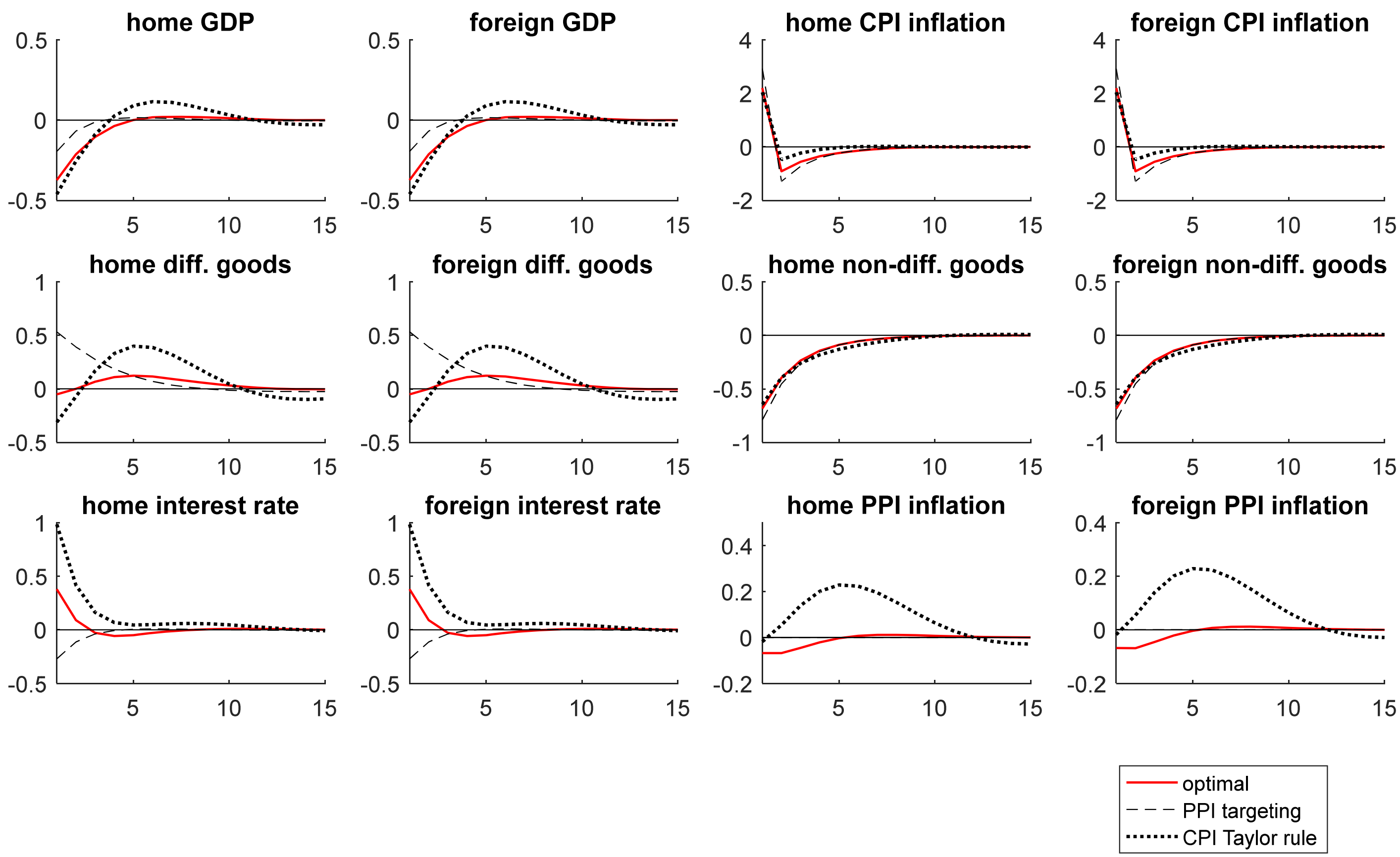
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 8. Impulse responses to a symmetric tariff to differentiated imports in both countries; home currency dominant



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 9. Impulse responses to a symmetric tariff to non-differentiated imports in both countries



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.