Monetary stabilization of sectoral tariffs

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December 28, 2025

Abstract

This paper studies the optimal monetary stabilization of tariffs using a two-country, two-traded sector New Keynesian model, where tariffs imply inefficient sectoral reallocation as well as changes in aggregate output and inflation. The main finding is that nominal exchange rate manipulation can be a useful tool to help offset some of the distortions implied by tariffs. We find that the Ramsey optimal monetary policy response to a tariff on imports of differentiated goods implies a domestic currency appreciation to offset the tariff's distortion on import prices. In our benchmark two-country environment, it is efficient for this exchange rate appreciation to be implemented primarily through monetary expansion in the foreign country; the monetary response in the home country is typically small, and can be either mildly expansionary or contractionary depending on the trade elasticity. In cases where the exchange rate is not an operative tool, such as an environment where prices are sticky in the local currency, or where tariffs impact sectors with flexible prices, the Ramsey optimal response instead tries to stabilize home demand for imports through aggregate demand expansion.

Keywords: macroeconomic effects of tariffs, optimal monetary policy, sectoral reallocation JEL classification: F4

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We thank Javier Bianchi, Louphou Coulibaly, Charles Horioka, Timothy Kehoe, Jinill Kim, Jaewoo Lee, Tommaso Monacelli, and Christopher Waller for helpful comments, as well as participants of the Society for Economic Dynamics meetings in Copenhagen, American Economic Association meetings, Recent Advances in Macroeconomics conference at Korea University, Trade Disputes and their Consequences on the Global Economy conference at Korea University Business School, and seminar participates and Kyungpook University Korea.

1. Introduction

As the world faces large and volatile U.S. tariffs, central banks around the world have grappled with the question of how to design and implement effective stabilization policies. The question is complicated by the fact that central bank objectives of stabilizing inflation and stabilizing employment and output typically conflict in the presence of tariff shocks, as tariffs may simultaneously produce inflationary and recessionary pressures. This question is complicated further by the fact that recent tariffs have targeted different types of U.S. imports, including a broad range of final consumption goods as well as less differentiated materials such as aluminum and steel that were the main targets in the first Trump administration.

This paper studies the Ramsey optimal monetary stabilization of tariff shocks using a two-country New Keynesian model featuring stylized global value chains in production and multiple traded sectors that differ in terms of market structure and price rigidity. The model builds on Bergin and Corsetti (2023), to our knowledge the first paper in the academic literature to study the question of optimal monetary policy response to tariff shocks, showing that an expansion is desirable in a symmetric currency war even though it may worsen headline inflation. The optimality of a countercyclical monetary stance has been recently reconsidered by Bianchi and Coulibaly (2025) and Monacelli (2025), also for the case of unilateral tariff shocks with no retaliation, relying on a small open economy models.

This paper makes two contributions to the academic and policy literature. First, it studies the stabilization of unilateral tariffs in our two-country setting. The shift in the focus of our early analysis, from trade wars with retaliatory tariffs to unilateral shock, reflects the evolution of events, as many countries accept Trump tariffs without retaliating. Relative to the small open economy environments used in the related literature, our global model allows us to study the optimal monetary response by both the tariff "imposer" and the "tariff victim," a question of keen interest around the world. Second, we enrich the model with a sectoral dimension, which is traditionally ignored in the macro analysis of tariffs. In the trade literature, by contrast, the study of tariffs has systematically focused on their implications for the reallocation of production across different sectors. In fact, tariff

¹ For another recent multi-sector treatment of tariffs, see Kalemli-Özcan, et al. (2025).

policies often are motivated by the goal of favoring such reallocation -- say, promoting domestic manufacturing. In our two-sector environment, differentiated goods, often associated with manufacturing, are characterized by monopolistic competition, entry costs, supply chains, and price stickiness. The non-differentiated sector, associated with commodities and agriculture, are associated with competitive markets and flexible prices.

Our model indicates that the country that imposes unilateral tariffs on its imports trades off inflation with international relative price distortions, highlighting the role of the terms of trade, hence the exchange rate, as a key channel through which tariffs affect the economy. A two-country model clarifies that the adjustment in the relative price of output (before and after tariff) results from changes in both domestic and foreign export prices, which are endogenous to both the tariff and the policy response to it. This in turn enhances the exchange rate as a tool policymakers can use to redress the distortionary impact of tariffs.

We show that, in response to unilateral tariffs that raise the relative price of home imports above border prices, it is optimal for monetary authorities to pursue a home currency appreciation to counteract this effect at least in part. Appreciation is optimally engineered with a monetary stance in the country targeted by the tariff that is expansionary *relative* to the stance in the country imposing the tariff. Whether the optimal relative stance policy brings inflation in the home country above or below the natural rate depends on particulars of the model calibration, especially concerning the trade elasticity and the persistence of the shock---as these determine the relevant trade-offs among competing objectives.

Our model suggests that, under cooperation, much of the optimal home appreciation is driven by the expansion in the country targeted by the tariff, i.e., the foreign country. In other words, the policy response of the home country (imposing a unilateral tariff), whether contractionary or expansionary, tends to be small in magnitude compared to the policy response abroad, which is expansionary. The optimal appreciation that results from the difference in overall stance benefits both home and foreign countries, as their specific policies optimally trade off the tariff-related distortions of the international relative prices and cross-border trade, with output gap and inflation distortions also induced by the tariff.

We provide sharp analytical insight on the economics of these results using a tractable version of the model, in which we assume unit trade elasticity and log-linear preferences, and in which we rule out round-about production but keep the two-sector structure of the economy. Analytical solution is facilitated by letting the trade elasticity for one sector approach infinity, thereby replicating the "homogenous world good" sector typically modelled by the trade literature. In our simplified environment, similar to trade models, the exchange rate is pinned down by arbitrage (price equalization) in the (near-)homogenous good sector. The exchange rate hence responds to relative productivity in this sector, tariffs on the homogeneous goods, and relative monetary policy. While this stark result does not generalize in our full model, numerical simulations confirm that it approximates the international transmission mechanism for reasonable calibration of it.

Our stylized environment is especially useful in clarifying how the role of the optimal policy in stabilizing the uncertainty induced by expectations of successive round of volatile tariffs adjustment on the (sticky price) differentiated good sector. In line with Corsetti and Pesenti (2005) and our previous contributions, uncertainty about a home tariff dampening home demand for foreign exports induces foreign firms to preset optimally their supply price at inefficiently high levels, de facto exacerbating monopolistic distortions in the global economy. Under producer currency pricing, firms raise prices of their goods both at home and abroad, which drive up the welfare-relevant price indexes and thereby lowers welfare. By stabilizing demand and relying on the relative price offset of the exchange rate, monetary authorities can compensate for this adverse effect of the tariff. In doing so, they trade off the benefit of stabilizing export demand with the side effects of monetary policy on domestic demand. Because of this trade off, the optimal monetary policy only ensures a partial exchange rate offset of the tariff.

Further insight follows from a comparison this the optimal policy in response to a tariff on the sector producing non-differentiated goods under price flexibility. In this case, the exchange rate directly responds to the tariff in this sector, and this is the source of the problem. The currency appreciation induced by the home tariff on the non-differentiated sector drives up home wages and crowds out the global demand for home differentiated goods. Relatedly, uncertainty about tariffs creates volatility in the currency that reflects in inefficiently high prices. To prevent these adverse effects, the optimal monetary policy

fully offsets the home currency movements induced by the tariff on the non-differentiated good sector, de facto preventing the exchange rate from responding to it.

We show that the optimal policy is also different in an environment in which prices are sticky in the currency of the buyer rather than that of the producer (local currency pricing), and in which one currency is dominant in international trade (dominant currency pricing). In contrast with the benchmark case with producer currency pricing, where optimal policy calls for monetary expansion in response to a tariff in the other country, the local-currency-pricing case flips this result, calling for monetary expansion instead in response to a domestic tariff. This result follows from the fact that home tariffs create uncertainty relevant to the pricing decision of foreign exporters, which weigh on the home average CPI. At the margin, home monetary authorities have an incentive to stabilize the home demand facing foreign exporters, to induce foreign firms to preset lower prices. Further, the case of a dominant currency implies an optimal policy that is a hybrid of the producer and local currency pricing cases – in particular, optimal policy calls for the country with the dominant currency to use expansionary monetary policy in response to either domestic or foreign tariffs, while the non-dominant country now responds to neither.

The paper proceeds as follows. The next section describes the model environment and calibration. Section 3 develops intuition from analytical solution for a simplified model environment. Sections 4 and 5 use impulse responses to a calibrated version of the full model to illustrate results and demonstrate robustness to a more general setting. Section 6 considers tariff retaliation. Section 7 summarizes conclusions and policy implications.

Related Literature. Our analytical and quantitative analysis emphasizes the different mechanisms through which tariff shocks and uncertainty transmit to the economy, as a function of the sector targeted by the trade restrictions and nominal rigidities in the export markets---from a global perspective. Because of this, our analysis offers a different perspective to related literature relying on the small open economy (SOE) model. An important instance is Bianchi and Coulibaly (2025), who find a home stance leading to positive PPI inflation and currency depreciation to be unambiguously optimal for an open economy under that (unexpectedly) imposes import tariffs. This result reflects a diminished role given to the exchange rate as a tool to redress relative price distortions in

their particular model specification, a point central to our model---a home currency depreciation would exacerbate the tariff-induced distortion on the relative import prices, and the SOE abstracts for the likely effects of tariffs on the adjustment in prices and production abroad, when the home country is large in the global economy. If prices and activity in the rest of the world are (taken as) given and constant, monetary policy naturally refocuses on the inefficient fall in the quantity of imports following a tariff, and the implied wedge in the labor. Redressing these inefficiencies requires an expansion raising overall consumption. From the perspective of an open and large economy, however, relative price distortions and the exchange rate channel become more prominent. In a cooperative setting, both the domestic and the foreign monetary policy contribute to engineer a home currency appreciation---requiring the home country imposing the tariff to be *relatively* less expansionary than abroad.

The other instance of closely related work is Monacelli (2025), which studies implementable monetary policy rules as well as the constrained optimal policy response to tariffs. As with Bianchi-Coulibaly (2025), its conclusions contrast with ours, concluding that home optimal monetary policy should promote currency depreciation rather than appreciation, so as to offset output contraction. This difference likely arises from the contrasting, but also valid, choice of studying the optimal monetary policy in a small open economy framework rather than a cooperative policy in a two-country environment.

Most of the early contributions to the literature studying the macroeconomic effects of tariffs relies on either real models or monetary models with a stylized monetary side.² To our knowledge, our previous work Bergin and Corsetti (2023) was first in placing the monetary dimensions of a trade war and the design of an efficient stabilization policy centerstage in the analysis. In our early paper, we studied the optimal monetary policy response to tariff shocks comparing Ramsey policies with monetary rules targeting PPI, CPI and money growth, as well as detailing the nature of tariff shocks in relation to markup and productivity shocks. Our current work differs in two key

² Some leading contributions study macro dynamics in the context of standard monetary policy rules, e.g., Barattieri et al. (2021), Erceg et al. (2018), Jeanne and Son (2024), Kalemli-Özcan et al. (2025), and Auclert, et al. (2025). Caldara et al. (2018) investigates the macroeconomic implications of trade policy uncertainty. Linde and Pescatori (2019) reconsiders the Lerner symmetry. For recent studies of the macroeconomic effects of tariffs abstracting from monetary policy see Baqaee and Malmberg (2025), and Costinot and Werning (2025).

respects. First, while the earlier paper focused on the case of a symmetric trade war, finding optimal policy of monetary expansion, the present paper focuses on the case of a unilateral tariff without retaliation, hence weakening the case for home monetary expansion and instead favoring exchange rate appreciation as a policy response. Historical experience with tariff shocks subsequent to the first paper, surprisingly, has shown the unilateral tariff case to be the more relevant to the current situation. The second key difference is that our current work studies the case of a multi-sector environment where tariffs affect sectoral allocation, a feature of tariffs thought essential within the trade literature that has a longer history than macro of studying tariffs.

In other related work, Werning et al. (2025) emphasizes that the optimal response to tariff shocks is akin to the optimal monetary stabilization of cost-push shocks---an important point that indeed applies generally to a number of shocks in open economy (see Corsetti et al., 2010). Auray, et al. (2025) considers the optimal mix of CPI and PPI inflation targeting in the class of standard monetary policy rules. Auray et al. (2024) study how alternative monetary policies affect an endogenous, strategic tariff policy, turning our question, the choice of optimal monetary policy in the face of an exogenous tariff policy, around. Our present work is distinct from all these papers in considering implications for sectoral allocation.

2. Model

The theoretical framework builds upon the framework of Bergin and Corsetti (2023), augmented with two traded sectors as in the model of Bergin and Corsetti (2020). The model features two countries, home and foreign, each of which produce two types of tradable goods. The first type of good comes in differentiated varieties produced under monopolistic competition, where firm entry requires a sunk investment, and prices are subject to nominal rigidities. The second type of good is modeled according to the standard specification in real business cycle models, assuming perfect substitutability among producers within a country, but imperfect substitutability across countries. In the text to follow, we present the households' and firms' problems as well as the monetary and fiscal policy rules from the vantage point of the home economy, with the understanding that

similar expressions and considerations apply to the foreign economy—foreign variables are denoted with a "*".

2.1. Goods consumption demand and price indexes

In the benchmark version of the model, households consume goods produced in both sectors, and of both domestic and foreign origin. The differentiated goods come in many varieties, produced by a time-varying number of monopolistically competitive firms in the home and foreign country, n_t and n_t^* respectively, each producing a single variety. Each variety is an imperfect substitute for any other variety in this sector, either of home or foreign origin, with elasticity ϕ . The non-differentiated goods come in a home and foreign version, which are imperfect substitutes with elasticity η . However, within each country, all goods in this sector are perfectly substitutable with each other, and are produced in a perfectly competitive environment. We will refer to the differentiated sector as "manufacturing," and denote this sector with a D; we will denote the non-differentiated sector with a N.

Tariffs are specified as ad-valorem duties imposed at the dock. They directly enter the relative prices observed by consumers, and which enter the demand equations. Tariff revenue is collected by the government of the importing country and rebated to domestic consumers, thus canceling out in the consolidated national budget constraint.

The overall consumption index is specified as follows:

$$C_{t} \equiv \left(\theta^{\frac{1}{\xi}} C_{D,t}^{\frac{\xi-1}{\xi}} + \left(1 - \theta\right)^{\frac{1}{\xi}} C_{N,t}^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi-\xi}{\xi-1}},$$

where

$$C_{D,t} \equiv \left(\int_{0}^{n_{t}} c_{t} \left(h\right)^{\frac{\phi-1}{\phi}} dh + \int_{0}^{n_{t}^{*}} c_{t} \left(f\right)^{\frac{\phi-1}{\phi}} df\right)^{\frac{\phi}{\phi-1}}$$

is the index over the endogenous number of home and foreign varieties of the differentiated manufacturing good, $c_t(h)$ and $c_t(f)$, and

$$C_{N,t} = \left(v^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta - 1}{\eta}} + (1 - v)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta - 1}{\eta}}\right)^{\frac{\eta}{\eta - 1}}$$

is the index over goods differentiated only by country of origin, $C_{H,t}$ and $C_{F,t}$ with $v \in [0,1]$

accounting for the weight on domestic goods. The corresponding welfare-based consumption price index is

$$P_{t} = \left(\theta P_{D,t}^{1-\xi} + (1-\theta)(P_{N,t})^{1-\xi}\right)^{\frac{1}{1-\xi}},\tag{1}$$

where

$$P_{D,t} = \left(n_t p_t(h)^{1-\phi} + n_t^* \left(p_t(f) T_{D,t}\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}$$
(2)

is the index over the prices of all varieties of home and foreign manufacturing goods, $p_t(h)$ and $p_t(f)$, and

$$P_{N,t} = \left(\nu P_{H,t}^{1-\eta} + (1-\nu)\left(P_{F,t}T_{N,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$
(3)

is the index over the prices of home and foreign non-differentiated goods. In these indexes, $T_{D,t}$ represents the quantity of 1 plus the ad valorem tariff rate imposed by the home country on imports of foreign differentiated goods, and $T_{N,t}$ represents the quantity of 1 plus the advalorem tariff rate imposed by the home country on imports of foreign non-differentiated goods. In reporting results, we will distinguish between the "ex-tariff" price determined by an exporter, $p_t(f)$, and the "tariff-inclusive" price, $p_t(f)T_{D,t}$, paid by an importer.

The relative demand functions for domestic residents implied from our specification of preferences are listed below:

$$C_{D,t} = \theta \left(P_{D,t} / P_t \right)^{-\xi} C_t \tag{4}$$

$$C_{N,t} = (1 - \theta) (P_{N,t} / P_t)^{-\xi} C_t$$
 (5)

$$c_t(h) = (p_t(h)/P_{D,t})^{-\phi} C_{D,t}$$
 (6)

$$c_{t}(f) = (p_{t}(f)T_{D,t}/P_{D,t})^{-\phi}C_{D,t}$$
(7)

$$C_{H,t} = \nu \left(P_{H,t} / P_{N,t} \right)^{-\eta} C_{N,t} \tag{8}$$

$$C_{F,t} = (1 - \nu) (P_{F,t} T_{N,t} / P_{N,t})^{-\eta} C_{N,t}$$
(9)

Note that demand functions for imports (Eqs. (7) and (9)) depend upon the tariff-inclusive price.

2.2 Home households' problem

The representative home household derives utility from consumption (C_t), and from holding real money balances (M_t/P_t); it suffers disutility from labor (l_t). The household budget consists of labor income from working at the nominal wage rate W_t ; profits rebated from home firms denoted with (Π_t) in real terms and defined below, as well as interest income on bonds in home currency ($i_{t-1}B_{H,t-1}$) and foreign currency ($i_{t-1}^*B_{F,t-1}$), where e_t is the nominal exchange rate in units of home currency per foreign. Income is net of lump-sum taxes (T_t), used for monetary transfers and to rebate tariff payments on imports. It is assumed that consumers do not internalize the effects of their consumption decisions on government tariff rebates.

Household optimization for the home country may be written:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U \left(C_t, l_t, \frac{M_t}{P_t} \right)$$

where utility is defined by

$$U_{t} = \frac{1}{1 - \sigma} C_{t}^{1 - \sigma} + \ln \frac{M_{t}}{P_{t}} - \frac{1}{1 + \psi} l_{t}^{1 + \psi},$$

subject to the budget constraint:

$$P_{t}C_{t} + \left(M_{t} - M_{t-1}\right) + \left(B_{Ht} - B_{Ht-1}\right) + e_{t}\left(B_{Ft} - B_{Ft-1}\right) = W_{t}I_{t} + \Pi_{t} + i_{t-1}B_{Ht-1} + i_{t-1}^{*}B_{Ft-1} - P_{t}AC_{Bt} - T_{t}.$$

In the utility function, the parameter σ denotes risk aversion and ψ is the inverse of the Frisch elasticity. The constraint includes a small cost to holding foreign bonds

$$AC_{Bt} = \frac{\psi_B \left(e_t B_{Ft}\right)^2}{2P_t p_{Ht} y_{Ht}},$$

scaled by ψ_B , which is a common device to assure long run stationarity in the net foreign asset position, and resolve indeterminacy in the composition of the home bond portfolio. The bond adjustment cost is a composite of goods that mirrors the consumption index, with analogous demand conditions to Eqs. (4)-(9).

Defining $\mu_t = P_t C_t^{\sigma}$, household optimization implies an intertemporal Euler equation:

$$\frac{1}{\mu_{t}} = \beta \left(1 + i_{t}\right) E_{t} \left[\frac{1}{\mu_{t+1}}\right] \tag{1}$$

a labor supply condition:

$$W_{t} = l_{t}^{\psi} \mu_{t} \tag{11}$$

a money demand condition:

$$M_t = \mu_t \left(\frac{1 + i_t}{i_t} \right), \tag{12}$$

and a home interest rate parity condition:

$$E_{t} \left[\frac{\mu_{t}}{\mu_{t+1}} \frac{e_{t+1}}{e_{t}} \left(1 + i_{t}^{*} \right) \left(1 + \psi_{B} \left(\frac{e_{t} B_{ft}}{p_{Ht} y_{Ht}} \right) \right) \right] = E_{t} \left[\frac{\mu_{t}}{\mu_{t+1}} \left(1 + i_{t} \right) \right]. \tag{13}$$

The problem and first order conditions for the foreign household are analogous.

2.3 Home firm problem and entry condition in the differentiated goods sector

In the manufacturing sector, the production of each differentiated variety follows

$$y_{t}(h) = \alpha_{D} [G_{t}(h)]^{\zeta} [l_{t}(h)]^{1-\zeta}, \qquad (14)$$

where α_D is productivity specific to the production of differentiated goods but common to all firms within that sector, $l_t(h)$ is the labor employed by firm h, and $G_t(h)$ is a composite of differentiated goods used by firm h as an intermediate input. $G_t(h)$ is specified as an index of home and foreign differentiated varieties that mirrors the consumption index specific to differentiated goods ($C_{D,t}$). If we sum across firms, $G_t = n_t G_t(h)$ represents economy-wide demand for differentiated goods as intermediate inputs. Given that the index is the same as for consumption, this implies demands for differentiated goods varieties, $d_{G,t}(h)$ and $d_{G,t}(f)$, analogous to Eqs. (6)–(7).

Differentiated goods firms set prices $p_t(h)$ subject to an adjustment cost:

$$AC_{P,t}(h) = \frac{\psi_P}{2} \left(\frac{p_t(h)}{p_{t-1}(h)} - 1 \right)^2 \frac{p_t(h)y_t(h)}{P_t},$$
(15)

where ψ_P is a calibrated parameter governing the degree of price stickiness. For the sake of tractability, we follow Bilbiie et al. (2008) in assuming that new entrants inherit from

³ See section 1 of the online appendix for the demand equations not listed here.

the price history of incumbents the same price adjustment cost, and so make the same price setting decision.⁴

There is free entry in the sector, but, once active, firms are subject to an exogenous death shock. Since all differentiated goods producers operating at any given time face the same exogenous probability of exit δ , a fraction δ of them exogenously stop operating each period. The number of firms active in the differentiated sector, n_t , at the beginning of each period evolves according to:

$$n_{t+1} = (1 - \delta)(n_t + ne_t),$$
 (16)

where ne_t denotes new entrants.

To set up a firm, managers incur a one-time sunk cost, K_t , and production starts with a one-period lag. This cost is not constant but varies reflecting an entry congestion externality, represented as an adjustment cost that is a function of the number of new firms:

$$K_{t} = \left(\frac{ne_{t}}{ne_{t-1}}\right)^{\lambda} \overline{K}, \qquad (17)$$

where \overline{K} indicates the steady state level of entry cost, and the parameter λ indicates how much the entry cost rises with an increase in entry activity. The congestion externality plays a similar role as the adjustment cost for capital standard in business cycle models, which moderates the response of investment to match dynamics in data. In a similar vein, we calibrate the adjustment cost parameter, λ , to match data on the dynamics of new firm entry. The demands for varieties for use as entry investment, $d_{K,l}(h)$ and $d_{K,l}(f)$, are determined analogously to demands for consumption of differentiated goods.

We now can specify total demand facing a domestic differentiated goods firm:

$$d_{t}(h) = c_{t}(h) + d_{G,t}(h) + d_{K,t}(h) + d_{AC,P,t}(h) + d_{AC,B,t}(h)$$
(18)

which includes the demand for consumption $(c_t(h))$ by households, and the demand by firms for intermediate inputs $(d_{G,t}(h))$, investment (the sunk entry costs) $(d_{K,t}(h))$, and goods absorbed as adjustment costs for prices $(d_{AC,P,t}(h))$ and bonds holding costs

⁴ The price index for adjustment cost is identical to the overall consumption price index, implying demands analogous to those for consumption in Eqs. (4)-(9). See section 1 of the online appendix for the demand equations not listed here.

⁵ The value of steady state entry cost \overline{K} has no effect on the dynamics of the model, and so will be normalized to unity.

 $(d_{AC,B,t}(h))$. There is an analogous demand from abroad $d_t^*(h)$. We assume iceberg trade costs τ_D for exports, so that market clearing for a firm's variety is:

$$y_{t}(h) = d_{t}(h) + (1 + \tau_{D})d_{t}^{*}(h),$$
 (19)

Firm profits are computed as:

$$\pi_{t}(h) = p_{t}(h)d_{t}(h) + e_{t}p_{t}^{*}(h)d_{t}^{*}(h) - mc_{t}y_{t}(h) - P_{t}AC_{p,t}(h).$$
(20)

where $mc_t = \zeta^{-\zeta} (1-\zeta)^{\zeta-1} P_{D,t} \zeta W_t^{1-\zeta} / \alpha_D$ is marginal cost.

Thus the value function of firms that enter the market in period t may be represented as the discounted sum of profits of domestic sales and export sales:

$$v_t(h) = E_t \left\{ \sum_{s=0}^{\infty} (\beta(1-\delta))^s \frac{\mu_{t+s}}{\mu_t} \pi_{t+s}(h) \right\},\,$$

where we assume firms use the discount factor of the representative household, who owns the firm, to value future profits. With free entry, new producers will invest until the point that a firm's value equals the entry sunk cost:

$$v_t(h) = P_{D,t}K_t. \tag{21}$$

By solving for cost minimization we can express the relative demand for labor and intermediates as a function of their relative costs:

$$\frac{P_{D,t}G_t(h)}{W_t l_t(h)} = \frac{\zeta}{1-\zeta}.$$
 (22)

Managers optimally set prices by maximizing the firm value subject to all the constraints specified above. The price setting equation:

$$p_{t}(h) = \frac{\phi}{\phi - 1} m c_{t} + \frac{\psi_{p}}{2} \left(\frac{p_{t}(h)}{p_{t-1}(h)} - 1 \right)^{2} p_{t}(h) - \psi_{p} \frac{1}{\phi - 1} \left(\frac{p_{t}(h)}{p_{t-1}(h)} - 1 \right) \frac{p_{t}(h)^{2}}{p_{t-1}(h)} + \frac{\psi_{p}}{\phi - 1} E_{t} \left[\beta \frac{\Omega_{t+1}}{\Omega_{t}} \left(\frac{p_{t+1}(h)}{p_{t}(h)} - 1 \right) \frac{p_{t+1}(h)^{2}}{p_{t}(h)} \right]$$
(23)

expresses the optimal pricing as a function of the stochastically discounted demand faced by producers of domestic differentiated goods,

$$\begin{split} &\Omega_{t} = \left[\left(\frac{p_{t}(h)}{P_{D,t}} \right)^{-\phi} \left(C_{D,t} + G_{t} + n e_{t} \left(1 - \theta_{K} \right) K_{t} + A C_{P,D,t} + A C_{B,D,t} \right) \right. \\ & \left. + \left(\frac{\left(1 + \tau_{D} \right) T_{D,t}^{*} p_{t}(h)}{e_{t} P_{D,t}^{*}} \right)^{-\phi} \left(1 + \tau_{D} \right) \left(C_{D,t}^{*} + G_{t}^{*} + n e_{t}^{*} \left(1 - \theta_{K} \right) K_{t}^{*} + A C_{P,D,t}^{*} + A C_{B,D,t}^{*} \right) \right] \middle/ \mu_{t} \end{split} .$$

This sums the demand arising from consumption, use as intermediate inputs, sunk entry cost, price adjustment costs, and bond holding costs.

Under the assumption that firms preset prices in own currency, i.e., assuming producer currency pricing, the good price in foreign currency moves one-to-one with the exchange rate, net of trade costs:

$$p_t^*(h) = (1 + \tau_D) p_t(h) / e_t,$$
 (24)

where recall the nominal exchange rate, e, measures home currency units per foreign.

Note that, since households own firms, they receive firm profits but also finance the creation of new firms. In the household budget, the net income from firms may be written:

$$\Pi_{t} = n_{t}\pi_{t}(h) - ne_{t}v_{t}(h).$$

In reporting our quantitative results, we will refer to the overall home gross production of differentiated goods defined as: $y_{D,t} = n_t y_t(h)$.

2.4 Home firm problem in the undifferentiated goods sector

In the second sector firms are assumed to be perfectly competitive in producing a good differentiated only by country of origin. The production function for the home non-differentiated good is linear in labor:

$$y_{H,t} = \alpha_N l_{H,t}, \tag{25}$$

where α_N is productivity specific to this country and sector. It follows that the price of the homogeneous goods in the home market is equal to marginal costs:

$$p_{H,t} = W_t / \alpha_N. \tag{26}$$

An iceberg trade cost specific to the non-differentiated sector implies prices of the home good abroad are

$$p_{H,t}^* = p_{H,t} (1 + \tau_N) / e_t. (27)$$

Analogous conditions apply to the foreign non-differentiated sector.

2.5 Monetary policy

To compute the cooperative Ramsey allocation, we posit that the monetary authority maximizes aggregate welfare of both countries:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\psi} l_t^{1+\psi} \right) + \frac{1}{2} \left(\frac{1}{1-\sigma} C_t^{*_{1-\sigma}} - \frac{1}{1+\psi} l_t^{*_{1+\psi}} \right) \right)$$

under the constraints of the economy defined above. As common in the literature, we write the Ramsey problem by introducing additional co-state variables, which track the value of the planner committing to a policy plan.

For comparison, we also study two alternative nominal specifications. In the first one, we assume a monetary policy of perfect producer price inflation targeting:

$$\frac{p_{t}(h)}{p_{t-1}(h)} = 1. {(28)}$$

In the second one, we assume a standard Taylor rule of the form

$$1 + i_t = \left(1 + i_{t-1}\right)^{\gamma_i} \left[\left(1 + \overline{i}\right) \left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} \left(\frac{\underline{Y}_t}{\overline{Y}}\right)^{\gamma_{\gamma}} \right]^{1 - \gamma_i}, \tag{29}$$

where terms with overbars are steady-state values. In this rule, inflation is defined in terms of the CPI, while Y_t is a measure of GDP defined net of intermediates as:⁶

$$Y_{t} = (n_{t-1}p_{t}(h)y_{t}(h) - P_{D,t}G_{t} + p_{H,t}y_{H,t}) / P_{t}.$$
(30)

Across these different specifications of monetary policy, we will abstract from public consumption expenditure, so that the government uses seigniorage revenues and taxes to finance transfers, assumed to be lump sum. Government transfers are also used to rebate to consumers the tariff duties paid to the government by consumers and firms on imported goods. The government budget constraint thus is specified as follows:

$$T_{t} = (M_{t-1} - M_{t}) + (T_{D,t} - 1)n_{t-1}^{*}d_{t}(f) + (T_{N,t} - 1)(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}).$$
(31)

2.6 Shocks process and equilibrium definition

Shocks are assumed to follow joint log normal distributions:

⁶ For computational simplicity, the Taylor rule is specified in terms of deviations of GDP from its steady state value, which is distinct from the output gap.

$$\begin{bmatrix} \log T_{D,t} - \log \overline{T_D} \\ \log T_{D,t}^* - \log \overline{T_D^*} \\ \log T_{N,t} - \log \overline{T_N^*} \\ \log T_{N,t}^* - \log \overline{T_N^*} \end{bmatrix} = \rho_T \begin{bmatrix} \log T_{D,t-1} - \log \overline{T_D} \\ \log T_{D,t-1}^* - \log \overline{T_D^*} \\ \log T_{N,t-1} - \log \overline{T_N^*} \\ \log T_{N,t-1}^* - \log \overline{T_N^*} \end{bmatrix} + \varepsilon_{Tt}$$

with autoregressive coefficient matrix ρ_T , and the covariance matrix $E\left[\varepsilon_T \dot{\varepsilon_T}\right]$.

To conserve space, the market clearing conditions to close the model are reported in section 2 of the appendix. A competitive equilibrium in our world economy is defined along the usual lines, as a set of processes for quantities and prices in the home and foreign country satisfying: (i) the household and firms optimality conditions; (ii) the market clearing conditions for each good and asset, including money; (iii) the resource constraints—whose specification can be easily derived from the above and is omitted to save space.

2.7 Welfare computation

We report the effects on welfare of a given policy regime configuration relative to the Ramsey allocation. The change in welfare customarily is computed in terms of consumption units that households would be willing to forgo to continue under the Ramsey policy regime. We posit identical initial conditions across different monetary policy regimes using the Ramsey allocation, and we include transition dynamics in the computation to avoid spurious welfare reversals.⁷

2.8 Calibration

Where possible, parameter values are taken from standard values in the literature. Risk aversion is set at $\sigma=2$; labor supply elasticity is set at $1/\psi=1.9$ following Hall (2009). Consistent with a quarterly frequency, $\beta=0.99$.

The price stickiness parameter is set at ψ_p =49, a value which implies in simulations of a productivity shock that approximately half the firms resetting price during the first

⁷ We adopt the methodology created by Giovanni Lombardo and used in Coenen et al. (2010), available from https://www.dropbox.com/s/q0e9i0fw6uziz8b/OPDSGE.zip?dl=0.

year.⁸ The firm death rate is set at δ =0.025. The mean sunk cost of entry is normalized to the value \overline{K} =1, and the adjustment cost parameter for new firm entry, λ , is taken from Bergin and Corsetti (2020). The share of intermediates in differentiated goods production follows Bergin and Corsetti (2020) in setting ζ =1/3.

To choose parameters for the differentiated and non-differentiated sectors we draw on Rauch (1999). We choose θ so that differentiated goods represent 55 percent of U.S. trade in value: (θ = 0.45). We assume the two countries are of equal size with no exogenous home bias, ν = 0.5, but allow trade costs to determine home bias ratios. The Broda and Weinstein (2006) estimate of the elasticity of substitution between differentiated goods varieties is ϕ =5.2 (the sample period is 1972-1988), though alternative values will be considered in sensitivity analysis. We adopt the same elasticity between home and foreign goods in the non-differentiated sector. We initially adopt a Cobb-Douglas specification for the aggregator function combining the two sectors (ξ \rightarrow 1), but sensitivity analysis will report results for alternative calibrations of this parameter. To set trade costs, we calibrate τ_D so that exports represent 26% of GDP, as is the average in World Bank national accounts data for OECD countries from 2000-2017. This requires a value of τ_D = τ_N =0.44. The contribution of the sectors of the contribution of the sectors are trade costs, we call brack the contribution of the sectors of the contribution of the contribution of the sectors of the contribution of the

Calibration of policy parameters for the historical monetary policy Taylor rule are taken from Coenen, et al. (2010): $\gamma_i = 0.7$, $\gamma_p = 1.7$, $\gamma_y = 0.1$.

The process for tariff shocks is calibrated with a mean value of 1.02 (2 percentage point mean tariff rate) to match U.S. tariff data in Barattieri et al. (2021). The autoregressive parameter is set to 0.56, estimated from Barattieri et al. (2021). The standard deviation of the shock is 0.08 (raising tariff rate from a steady stat of 2% to 10%).

⁸ As is well understood, a log-linearized Calvo price-setting model implies a stochastic difference equation for inflation of the form $\pi_l = \beta E_l \pi_{l+1} + \lambda m c_l$, where mc is the firm's real marginal cost of production, and where $\lambda = (1-q)(1-\beta q)/q$, with q is the constant probability that a firm must keep its price unchanged in any given period. The Rotemberg adjustment cost model used here gives a similar log-linearized difference equation for inflation, but with $\lambda = (\phi - 1)/\kappa$. Under our parameterization, a Calvo probability of q = 0.5 implies an adjustment cost parameter of $\psi_P = 49$.

⁹ See https://data.worldbank.org/indicator/NE.EXP.GNFS.ZS?locations=OE.

¹⁰ To coincide with standard accounting definitions, differentiated goods used as intermediates are included in the measure of exports, and excluded in the measure of GDP, as is appropriate.

3. Insight from an analytically tractable version of the model

This section derives analytical results for a simplified version of the model. While these results do not hold exactly in our full model, numerical simulations suggest that they hold approximately for reasonable calibrations. In other words, our analytical characterization of the equilibrium provides fundamental insight into the economics of monetary stabilization in the face of sectoral tariff shocks and trade policy uncertainty.

3.1 Simplified model environment

To derive analytical solution, we assume that differentiated goods come in distinct country-specific types, and that they are bundled in preferences with a Cobb-Douglas aggregator over the home and foreign types. For the non-differentiated sector, instead, we draw on the trade literature and assume a world homogeneous good, that we approximate by letting the elasticity of substitution between home and foreign varieties approach infinity. In order to study a tariff in such an environment, we assume that the home country is both a producer and an importer of the non-differentiated good (we derive below permissible ranges for productivity levels and tariffs that ensure this is the case in equilibrium). This assumption arguably suits an interpretation of the non-differentiated goods as steel; U.S. tariff policy may promote domestic production of this sector, but an underlying foreign comparative advantage means domestic production will not likely fully replace foreign sources.

Given our simplifying assumptions, the consumption aggregator is:

$$C_{t} \equiv \left(C_{DHt}^{1/2}C_{DFt}^{1/2}\right)^{\theta} \left(C_{Nt}^{1/2}\right)^{1-\theta},$$

with the associated price index: $P_{t} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] P_{DHt}^{-\theta/2} \left(T_{Dt} e_{t} P_{DFt}^{*} \right)^{\theta/2} P_{Nt}^{-1-\theta} .$ The

demand condition for home imports of foreign differentiated goods (7) becomes:

$$c_t(f) = (e_t p_t^*(f) T_{D,t} / P_{D,t})^{-1} C_{D,t}$$

where $p_i^*(f)$ is the foreign currency price of foreign exports, which is fixed in the initial period under producer currency pricing. This equation highlights how, under price stickiness, the effect on relative prices of a rise in tariff, T_D , can in principle be offset by the nominal exchange rate, e. If a tariff distorts prices by making imports more expensive

for home consumers, a home currency appreciation (foreign deprecation) can redress the relative price distortion by reducing the border price in proportion (given nominal rigidities in the producer currency).

As in Corsetti and Pesenti (2005), we posit that household utility is log in consumption and linear in labor $U_t = \ln C_t - \kappa l_t$, and define a variable, $\mu_t = P_t C_t$, which summarizes the effect of monetary policy stance on aggregate nominal spending. Utility maximization implies the standard labor supply condition $W_t = \kappa \mu_t$.

The following is a key condition in our simplified model environment. When both home and foreign-produced homogeneous goods are consumed in home (as we posit by assumption), it must be the case that $P_{NH,t} = e_t T_{N,t} P_{NF,t}^*$. Since firms producing these goods are perfectly competitive, the zero profit condition then implies that prices are equal to marginal costs in both countries, i.e., for the home market $P_{Nt} = W_t / \alpha_N$. In combination with the equilibrium condition in the labor markets, these pricing equations pin down the exchange rate as a function of relative productivity in the homogeneous good sectors, the tariff on this sector and relative monetary stances (same as relative wages).

$$e_{t} = \frac{P_{NH,t}}{P_{NF,t}^{*}T_{N,t}} = \frac{\left(W_{t} / \alpha_{N}\right)}{\left(W_{t}^{*} / \alpha_{N}^{*}\right)T_{N,t}} = \frac{\mu_{t}\alpha_{N}^{*}}{\mu_{t}^{*}\alpha_{N}T_{N,t}}.$$
(32)

The home exchange rate depreciates (proportionally) with a home monetary expansion (rise in μ_i); it appreciates with a home tariff on the non-differentiated sector, as arbitrage induces a rise in home wages and production costs. Strikingly, for given monetary stances, tariffs on the differentiated sector have a fully neutral effect on the exchange rate.

Turning to the differentiated goods sector, we simplify production by abstracting from production chains $y_i(h) = \alpha_i l_i(h)$ ---firms only use labor---but we keep the assumption that firms set prices one period ahead, in producer currency units. With tariffs assumed to be i.i.d. shocks, as shown in the appendix, the optimal price setting of the home differentiated goods firm is

$$P_{DHt} = \frac{\phi}{\phi - 1} E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \frac{\kappa \mu_t}{\alpha_D} \right] / E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \right]. \tag{33}$$

Firm price setting is affected by anticipated shocks and uncertainty in the demand for the firm's exports, which can arise from several sources. Home firms expect a fall in the

demand for their product when, first, they anticipate a rise in foreign tariff on differentiated imports, T_{Dt}^* , as this raises the price foreign households pay for home exports; second, they anticipate a rise in home non-differentiated tariff, $T_{N,t}$, or a home monetary contraction, since both cause currency appreciation (as shown above), which also raises the price foreign consumers pay under prices sticky in the producer currency. Moreover, through the expectation term, a negative correlation of shocks with μ_t induces firms to set higher average prices of the home differentiated goods, facing large fluctuations in foreign demand (see Corsetti and Pesenti, 2005). This observation is consequential for policy design, since μ_t is controlled by policy. With trade policy uncertainty, a policy regime systematically raising μ_t in response to a rise in tariffs can generate a smaller average fluctuation in demand, which lowers prices on averages. Lastly we stress that home tariffs on differentiated goods do not enter the pricing equation (33), since these affect the price foreign consumers pay for home exports neither directly, nor indirectly through the exchange rate (see equation 32).

3.2 Optimal policy under producer currency pricing (PCP)

We compute the optimal Ramsey policy by choosing the policy stance, μ_i^{OP} , to maximize expected utility subject to the equilibrium conditions listed in the preceding section. Appendix B shows that this optimal policy can be characterized in terms of the following rules:¹¹

$$\mu_t^{OP} = a \left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right)^{-1}$$
 (34)

and

$$\mu_{t}^{*OP} = a \left(1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{T_{Nt}}{T_{Dt}} \right)^{-1}, \tag{35}$$

where a is a constant of proportionality. Observe that this policy fully eliminates all fluctuations in the numerator of the optimal firm price setting in equation (33). As noted

¹¹ The Nash and cooperative solutions coincide for this simple environment. As usual, the Nash solution chooses μ_t , to maximize $E_{t-1}[U_t]$ and foreign chooses μ_t^* , to maximize $E_{t-1}[U_t^*]$; the cooperative solution chooses μ_t and μ_t^* jointly to maximize $E_{t-1}[U_t + U_t^*]/2$. See appendix B for details.

above, reducing uncertainty in demand induces firms to set lower prices of the differentiated goods on average. This in turn lowers the home overall price index, which raises welfare.

Drawing on equations (32)-(35), we can provide an articulate account of how policy can rely on the exchange rate "channel" to improve welfare in the face of tariffs. To this goal, we substitute the optimal policies (34-35) into the exchange rate solution (32) to write:

$$e_{t} = \left(\frac{1}{T_{N,t}} + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{Dt}}\right) / \left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right). \tag{36}$$

Consider first a home tariff on import of differentiated goods (which, in the absence of a monetary policy response, would trigger no exchange rate adjustment, as shown by equation (32)). In principle, monetary policy could fully offset the impact on foreign demand via a proportional home currency depreciation. This is not what the optimal policy rules prescribe---according to (36), the optimal home exchange rate depreciation is less than proportional to the tariff. To show this most clearly, we rewrite (36) assuming that the only shock is to foreign tariffs on imported home differentiated goods:

$$e_t = \left(1 + \frac{\alpha_N}{\alpha_N^*}\right) / \left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{Dt}^*}\right)$$
. If countries are nearly symmetric in productivity of the non-

differenated sector ($\alpha_N \to \alpha_N^*$), the optimal policy prescribes that the home exchange rate should depreciate enough to wipe out approximately half of the price rise induced by the tariff---corresponding to the share of differentiated imports in the differentiated consumption aggregate. 12 Intuitively, monetary policy also drives the domestic demand for differentiated goods: the stance required for a full exchange rate offset of the tariff would be suboptimal.

Remarkably, our simplified model environment suggests that the implementation of the optimal policy response to home tariffs on foreign differentiated goods falls entirely on the foreign monetary authority. There is no change in home monetary stance, as these tariffs do not enter the home price setting equation (33)---they do not affect the price domestic and foreign consumers pay for home differentiated goods, either directly,

¹² It is easy to see that in response to a 10% tariff, with symmetric productivity, the Home currency would be optimally appreciated by about 5%.

or indirectly through the exchange rate (see the discussion of equation 32).¹³ Targeting global welfare, the foreign monetary authorities expand to support their demand and redress the relative price distortion.

Full exchange rate offset is instead prescribed in response to a *home tariff on non-differentiated goods*. As noted above, a positive tariff shock induces an appreciation of the home currency, which inefficiently lowers the demand for home exports of the other sector, the differentiated good sector. The optimal policy prevents such an outcome. This is clearly seen by simplifying (36) to include home non-differentiated tariffs only and letting productivities across countries become more symmetric ($\alpha_N \to \alpha_N^*$). We obtain:

$$e_t = \left(\frac{1}{T_{N,t}} + \frac{\alpha_N}{\alpha_N^*}\right) / \left(1 + \frac{\alpha_N^*}{\alpha_N} + \frac{1}{T_{N,t}}\right) \rightarrow 1$$
. Note that the optimal adjustment in the exchange rate

is engineered with a combination of home expansion and foreign contraction, which, as we will show below, will weigh on aggregate economic activity and inflation distortions.

3.3 Admissible range for analytical results

Before proceeding, it is appropriate to characterize the range of admissible productivity levels under which equation (32) holds, i.e. the conditions under which the home country can be simultaneously a producer and an importer of the non-differentiated good. This is as follows:

$$1 < \frac{\alpha_N^*}{\alpha_N T_N, T_{D_t}^*} < \frac{2}{\theta} - 1.$$
 (36)

On one hand, to keep exporting to the home country, foreign firms must enjoy productivity levels high enough to keep an absolute advantage over home in producing non-differentiated goods after accounting for tariffs. On the other hand, to make sure that domestic firms are not crowded out, the foreign advantage cannot be too large.¹⁴

3.4 Optimal policy under local currency pricing (LCP)

¹³ In the full model, this stark result will not hold, if anything because production relies on imported intermediate inputs, whose price reflect exchange rate movements.

¹⁴ For example, if $\theta = \frac{1}{2}$, foreign productivity cannot be more than three times that of home productivity.

Additional insight can be drawn from a version of the model in which prices are sticky in the currency of buyer (the local currency pricing, LCP, case). As shown in Appendix C, optimal policy satisfied the following conditions, for home

$$\frac{1}{\mu_{t}} = \frac{1}{2} \frac{1}{E_{t-1}[\mu_{t}]} + \frac{1}{2} \frac{1}{T_{Dt}} / E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right], \tag{37}$$

and for foreign

$$\frac{1}{\mu_{t}^{*}} = \frac{1}{2} \frac{1}{E_{t-1} \left[\mu_{t}^{*}\right]} + \frac{1}{2} \frac{1}{T_{Dt}^{*}} / E_{t-1} \left[\frac{\mu_{t}^{*}}{T_{Dt}^{*}}\right]. \tag{38}$$

These conditions show that, under LCP, it is the home policy stance that optimally responds to the home tariff on differentiated goods, not to the foreign tariff. This is the *opposite* pattern relative to the PCP case, studied above. As shown in Appendix C, under LCP, the price set by home firms in the home market is no longer affected by uncertainty in export demand in the foreign market. However, uncertainty from the Home tariffs now weighs on the price preset in local currency by *foreign* exporters---on average, the uncertainty associated to these tariffs raises the Home CPI. Among its objectives, the Home monetary policy has a welfare incentive to stabilize the home demand facing foreign exporters, with the goal of lowering the average price they charge home customers (a case discussed early on in Corsetti and Pesenti, 2005). Furthermore, under LCP, policy no longer responds to tariffs on the non-differentiated sector. This follows from the fact that, under LCP, the exchange rate cancels out of the expectation term for price setting.

3.5 Optimal policy under dominant currency pricing (DCP)

Another relevant case is where the home currency is dominant in international trade invoicing, that is, where all export prices are set in the currency of the home country regardless of which country is exporting. We model this case as the home country's firms following the PCP pricing rule above, while the foreign country's firms follow the LCP pricing rules. As derived in Appendix D, this case implies the following optimal policy equation for home

$$\mu_{t} = \frac{1}{2} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]}{\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right)} + \frac{1}{2} \frac{E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right]}{\frac{1}{T_{Dt}}},$$
(39)

and for foreign

$$\mu_t^* = E_{t-1} \left\lceil \mu_t^* \right\rceil. \tag{40}$$

While equation (39) cannot provide a closed form solution (just as in the LCP case), it does imply that the optimal policy now involves both home and foreign tariffs on differentiated goods. The logic, as shown in Appendix D, is a hybrid of the PCP and LCP results studied above. First, since home sets its exports in producer currency terms, it responds to a foreign tariff much like the standard PCP case solved above, where the policy of the exporting country responds to tariffs in the destination country. Home policy wishes to offset the effect of tariffs on uncertain fluctuations in demand for its exports, which introduce a risk premium into the prices of home goods, with implications also for the home price index. Second, since home imports are set in local currency terms, the home policy responds to home tariffs on home imports similarly to the benchmark LCP case above. In this case, policy wants to reduce the risk premium set by foreign exporters by stabilizing home demand for foreign exports.

Equation (40) can be satisfied by any constant foreign monetary rule: $\mu_i^* = a$ for some constant a. This makes clear that foreign policy now does not respond to any tariff. This again can be viewed as a hybrid of the LCP and PCP cases studied above. First, the lack of foreign response to home tariffs was part of the LCP result above. Since export prices for foreign goods are set in a separate currency from domestic sales, uncertainty regarding home tariffs facing foreign exporters has no effect on the foreign price index. Second, the lack of foreign response to own foreign tariffs is part of the PCP result above, where it was left to the other country to manipulate the exchange rate to offset tariffs.

4. Baseline Simulation: unilateral home tariff on foreign differentiated exports

This section uses numerical simulations of the calibrated full model to demonstrate implications of key results and explore robustness in a more general setting. We focus this

section on the case of an unexpected tariff imposed by the home country on its imports of differentiated goods exported from the foreign country. Figure 1 reports impulse responses showing the macroeconomic effects on selected variables under different policy regimes, contrasting the Ramsey optimal policy (solid line), PPI-targeting rule (dashed line), and CPI-based Taylor rule (dotted line).

4.1 Transmission under suboptimal policy

We start our analysis characterizing the natural rate as a benchmark reference (flexible price) allocation. Under the assumptions of our model, this allocation is supported by a monetary policy that perfectly targets producer prices. With this policy in place, the tariff shock generates a modest but sustained fall in home GDP, and a small spike of CPI inflation---an stagflationary outcome that presents a classic challenge to policy makers. Stagflation is also experienced on impact by the foreign country, though the drop in foreign output is steeper, given that the tariff hits the demand for this country's exports.

At the aggregate level the effects of the tariff on activity remain nonetheless moderate---but this outcome masks a large sectoral reallocation. In the foreign country, the percentage fall in the production of differentiated goods is three times the percentage fall in GDP. This is matched by a rise in the production of non-differentiated goods of a similarly large magnitude. Sectoral reallocation is also significant in the home country---inversely mirroring that in the foreign country. The modest fall in home GDP is the net effect of a larger percentage fall in non-differentiated goods production and smaller offsetting rise in non-differentiated production.

PPI targeting results in home currency appreciation---driven by a combination of higher nominal policy rates in home, and lower rates in foreign. The rate of appreciation however is not large enough to offset the impact of the tariff on the relative price of home exports to home imports.¹⁵ The home tariff-inclusive relative price---defined here as the relative price of home differentiated exports to differentiated imports---rises, reflecting the

¹⁵ Linde and Pescatori (2019) have pointed out that, in its stronger form, Lerner symmetry fails in many macroeconomic contexts, depending on the structure of financial markets and nominal rigidities.

effect on the import prices of the tariff. The home country experiences a temporary trade surplus. ¹⁶

Compared to strict PPI targeting, a standard Taylor rule (dotted line in Figure 1) dampens the interest rate response, due to the substantial interest rate smoothing term in this rule. The smaller response means that home GDP now rises, and PPI inflation turns positive; correspondingly, in the foreign country GDP falls slightly more and inflation turns negative. The sectoral reallocation is larger.

4.2 Transmission under the optimal policy

In Figure 1, economic dynamics under the optimal (cooperative) policy are depicted with a solid (red) line. Relative to PPI targeting, the policy response consists of a stronger expansion in the foreign country implying a fall in PPI and a lower interest rate; in the home country, the optimal policy remains close to PPI targeting, with a modest fall of PPI below target. Remarkably, simulation results broadly corroborate the analytical result from the simplified model in the preceding section, in which optimal policy called for a large monetary expansion in the foreign country, with no response in the home country. Indeed, as in the analytical model, a primary outcome of the optimal policy is a foreign currency depreciation, which works to offset the effect of the tariff on the relative prices of home imports. In figure 1, the exchange rate offsets approximately half of the effect of the tariff on the relative price of home goods to the tariff-inclusive price of imports (labeled here as terms of trade), which again broadly corresponds to the result in the analytical model. The optimal policy substantially dampens the sector reallocation observed under the PPI targeting rule. It thus substantially undercuts the tariff related distortions in both relative prices and sectoral allocation.

We stress that a Home currency appreciation requires the monetary stance at home to be contractionary *relative* to foreign. The optimal exchange rate adjustment can thus be engineered also with an asymmetric expansion in both countries, provided the stance is relatively less expansionary in home than in foreign. Appendix Figure 2 shows this

¹⁶ Appendix Figure 1 reports responses for additional variables. Home consumption falls and foreign rises, reflecting interest rates in each country. Firm entry rises at home and falls in foreign, reflecting movements in differentiated goods output. Employment moves likewise; wages fall in both countries reflecting the fall in GDP.

possibility, modelling a smaller trade elasticity that serves to dampen the effectiveness of the exchange rate as a mechanism of adjustment (we lower $\phi = 5.2$ to 3.8, a value common in some parts of the firm dynamics literature (see Ghironi and Melitz 2003).¹⁷ This change in parametrization is sufficient to flip the sign of the PPI inflation response from negative to positive. Home GDP also rises relative to the PPI targeting benchmark.

4.3 Extension to the case of dominant currency and local currency pricing

A large body of work has documented the relevance of dollar invoicing in international trade, motivating model specifications assuming that export prices are set and sticky in a dominant currency. The analytical section showed that this case can be analyzed as a hybrid, where the home firm is characterized by producer currency pricing and the foreign firm by local currency pricing. Our results highlight how the specification of price stickiness in exports has fundamental implications for the optimal monetary policy response to tariffs.

In Figure 2, we present impulse responses to a home tariff shock under the assumption that the prices of both home imports and exports of differentiated goods are set in home currency units, reflecting the role of the dollar as a dominant currency.¹⁸

Note first that, under PPI targeting, the dynamics of sectoral output and inflation are similar to the benchmark model with stickiness in the currency of the producer, that is, they are nearly the same as in Figure 1. The main reason is that tariffs are added on to import prices at the dock (after prices have been set by firms), hence the pass through of tariffs to final user price level is unaffected by the specification of price stickiness at the border. Yet, the dynamic of home GDP is different. Under dominant currency pricing, there is no (downward) adjustment at the border reflecting the exchange rate, hence tariffs on differentiated goods raise the final price of these goods relative to non-differentiated by more. In the aggregation of overall GDP, this gives more weight to the rise in home

¹⁷ This experiment also lowered the trade elasticity for non-differentiated goods (η =1.5), to satisfy Blanchard-Kahn condition of dynamic stability. The market structure for the differentiated sector is taken from the trade literature and implies that the parameter governing the elasticity of substitution between home and foreign varieties is the same parameter governing the elasticity between different home varieties and hence firm markup. So we are limited in the range of trade elasticities admissible.

¹⁸ To save space, the case of local currency price stickiness applied symmetrically to both countries is reported in Appendix Figure 3.

production of differentiated goods relative to the fall in home production of nondifferentiated goods.

Regarding optimal policy, the simplified analytical model suggested that, when home import prices are sticky in home currency, the home policy should systematically respond to home tariff shocks by stabilizing domestic demand. At the margin, a stable demand induces foreign exporters to lower their average prices in the home market. The simulation generalizes this analytical result---clarifying that the home optimal policy response to own tariff is more expansionary than under PPI targeting. The policy supports overall demand, tolerating positive PPI inflation (while placing less weight on redressing the relative price of exports). Indeed, the policy does nothing to dampen the sectoral reallocation induced by the tariff. Observe that under the optimal policy CPI inflation is higher in both countries, relative to PPI inflation. ¹⁹ While the analytical result for the simplified model implied no foreign policy response to the home tariff, the more richly parameterized and calibrated simulation model implies a mild foreign expansion, with interest rate below that implied by the PPI targeting rule, and slightly positive PPI inflation.

For comparison, Appendix Figure 3 shows impulse responses for an environment with local currency pricing (LCP) in both countries. In this case, consistent with our analytical section, the home interest is lower than that implied by the PPI targeting rule, implying a relative expansion, and home PPI inflation as sharply positive. While the analytical result does not indicate any response in foreign monetary policy stance, in the richer environment of the simulation model, the optimal foreign response to the home tariff is mildly contractionary—the interest rate is above that implied by the PPI target, and there is a moderate fall in PPI inflation.

We also analyze the case in which the dominant currency is issued by the country targeted by tariff, i.e., the foreign country. The dynamic responses are shown in Appendix Figure 4. The analytical result in the simplified model implied that optimal policy in the country with the dominant currency in this case should resemble that under PCP. In the case of the present simulation, this prescription implies the foreign (dominant) country should expand in response to the home (non-dominant) tariff. Indeed, Figure 4 shows a

¹⁹ The effect on GDP again is complicated by the change in sectoral relative prices used in aggregating output over the two sectors.

sharp foreign expansion, with interest rate falling more than that implied by PPI targeting, and with a substantial PPI inflation. While the analytical result did not call for any policy response from the non-dominant country (home country in this case), the simulation in Figure 4 shows a mild contraction, with interest rate slightly above that of the PPI targeting rule, and a small reduction in PPI inflation.

4.4 Welfare implications and sensitivity

We show the welfare gains from pursuing the optimal policy both relative to a suboptimal Taylor Rule, in Table 2, and relative to the PPI targeting rule, in Table 4---all measured in units of steady state consumption. To compute these gains, we conduct a stochastic simulation of the model in which unilateral home tariff shocks are mean zero (shocks include both hikes and cuts in tariff rates relative to their mean level).²⁰

For our benchmark simulation, in Figure 1, the optimal policy improves world welfare by 0.096 percent relative to the Taylor Rule. The optimal policy raises welfare in both countries, but not symmetrically. Home gains more, 0.162 percent vs. 0.031 percent in foreign (first line of Table 2). A key reason for this asymmetry is that CPI targeting is particularly harmful in the home country, as it results in alternative excessive contractions and expansions in response to random tariff shock. Namely, instead of accommodating the `natural' relative price adjustment created by tariff hikes, it forces an inefficient adjustment by cutting demand for domestic differentiated goods, inducing their producers to pay the cost of lowering their (sticky) supply prices (vice versa in response to random tariff cuts). A policy that corrects this policy randomness is particularly beneficial to the home country, as final user prices are directly affected by tariffs. Indeed, the gains from optimal policy relative to PPI targeting, shown in Table 3, are smaller: 0.020 percent for the world welfare, 0.024 for home, and 0.016 for foreign. PPI targeting moderate the policy response to headline inflation.

Welfare gains are overall smaller under the assumption of home dominant currency (0.046 percent and 0.042 percent relative to Taylor and PPI targeting, respectively), see line 2 of the tables. This reflects the limited ability of monetary policy to remediate the distortion of the tariff on (home) relative prices.²¹ Welfare gains are fairly similar under the specification of

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²⁰ Perturbation solution methods require that shocks be mean zero.

²¹ Relative to the PPI targeting rule, the welfare gains of the cooperative optimal policy are negative for the home country, suggesting that a cooperative policy might be harder to sustain for this case. Relative to the standard Taylor rule, the welfare gains remain positive for both countries.

symmetric LCP price stickiness.

We offer further insights conducting sensitivity analysis for welfare gains to alternative parametrizations of the model, especially with regard to elasticities, with results also reported in Tables 2 and 3. Most crucially, welfare gains are somewhat higher if the two sectors are modeled as complements (ξ =0.5), somewhat lower if the two sectors are substitutes (ξ =1.4). The case of complementarity nicely reflects the possibility that non-differentiated goods like steel and commodities are combined with differentiated goods as complementary inputs in producing the final consumption good. Higher complementarity implies that, in equilibrium, relative prices are more sensitive to relative quantities, and addressing relative price (and reallocation) distortions is therefore more consequential. As discussed above, this is where CPI targeting is the least efficient regime. The market structure for the non-differentiated sector in our model nests model specification with a homogeneous good (common in the trade literature, η large), or with a country-specific good (in the macro real business cycle literature often calibrated setting η =1.5. Welfare gains from optimal policy are slightly larger in the former case and slightly smaller in the latter (see lines 6 and 7 of Tables 2 and 3).

Arguably much more relevant empirically, in light of the U.S. trade policy following the tariff initiatives in both 2018 and 2025, is the case of persistent tariffs ($\rho\tau$ = 0.95). In this case, welfare gains from the optimal policy quadruple, up to 0.44% percent, relative to CPI targeting. As shown in Appendix Figure 5, in response to a persistent tariff, the optimal nominal interest rate response is more symmetric---the foreign cut is mirrored by a slightly smaller rise in home---driving PPI inflation in opposite directions but serving to engineer the optimal home currency appreciation. Relative to CPI targeting, the optimal policy contains distortions in both the aggregate GDP and the sectoral allocation---but not CPI movements. Remarkably, CPI inflation is negative (in spite of the tariff) in home, positive in foreign.

Finally, it is useful to contrast the results in our model with the standard assumption in the macro literature that the second sector is nontraded. While overall welfare gains in table 2 and 3 are similar to the benchmark case (with smaller welfare gains specific to the home country), Appendix Figure 6 shows that the sectoral reallocation is a fraction of that in our benchmark case with two traded sectors. Without a second traded sector to offer alternative specialization of the foreign country, the home country cannot achieve as much specialization in the differentiated sector.

5. Contrasting the effects of tariffs on differentiated and non-differentiated goods

In Figure 3, we bring our model to bear on the case of a home tariff on imports of the foreign non-differentiated good, in contrast to the differentiated good. As shown in this figure, under a PPI-targeting rule, the sectoral reallocation induced by the tariff is precisely the opposite of the benchmark case in Figure 1, shifting home demand and production away from differentiated goods toward non-differentiated goods, with offsetting sectoral reallocation in the foreign country. As predicted by the analytical results, this reallocation is facilitated by a home currency appreciation induced by the home tariff, which makes home exports of differentiated goods less competitive. Also in contrast with Figure 1, home overall GDP rises for this tariff. The rise in home non-differentiated production is larger in percentage terms than the fall in differentiated production. Different from Figure 1, also, the dynamics of differentiated production are positive *and* smoother over time, due to the slow adjustment of the sticky prices in the sector.

The direction of optimal policy also differs for the non-differentiated sectoral tariff compared to the benchmark case of a differentiated sectoral tariff, coinciding with the prediction of the simple analytical model. Home monetary policy is now more expansionary than the PPI-targeting rule, with interest rate rising less and positive PPI inflation. Foreign monetary policy, in contrast, is now more contractionary than the PPI-targeting rule, with interest rate falling less, and negative PPI inflation. As a result, the home exchange rate now appreciates less due to the policy and the terms of trade movement induced by the tariff is larger than under PPI-targeting. This policy serves to significantly moderate the fluctuation in differentiated goods production in the two countries, though there is essentially no impact of the policy to moderate sectoral reallocation in the foreign country. The reason is that the latter sector has flexible prices, and this is insulated from the effect of monetary policy, which can do little to remediate the effects of the tariff on this sector. But sticky prices in the differentiated goods sector mean that monetary policy can remediate the side effects of this policy on the differentiated sector.

In sum, we can add this case, a home tariff on non-differentiated goods, to the list of cases calling for optimal home monetary expansion, along with home dominant currency and low trade elasticity.

6. Tariff with retaliation

While the focus of this paper is on a unilateral home (U.S.) tariff, it remains possibility that central banks will be called upon to deal with a situation in which the foreign country reciprocates U.S. tariffs with tariffs of its own in a tariff war.

6.1 Symmetric Tariff war

Appendix Figure 7 shows the case of symmetric tariff hikes imposed on the differentiated goods exports of both countries. The fall in home aggregate GDP now applies to both countries. In addition, the contraction in activity is largely driven by the fall in differentiated goods production. The production of non-differentiated goods actually rises somewhat, but not enough to compensate for the fall in overall output coming from the differentiated sector. In a symmetric tariff war, there is no shift in sectoral specialization across countries---rather, the tariff distortions result in a shift in the sectoral composition of output at a global level.

The optimal monetary policy stance is expansionary in both countries, despite the inflationary impact of the tariff. Given that a symmetric tariff war cannot be remedied by a currency depreciation, the optimal policy aims at resolving the distortion created by the tariff between differentiated and non-differentiated prices within each country. An expansionary monetary stance mitigates the contraction in the differentiated good sector, driving up overall aggregate demand as well as the prices of non-differentiated goods, which are flexible.²²

Tables 2 and 3 (row 10) suggest that the welfare loss of a symmetric tariff war is lower than for tariffs that are not coincident. This can be attributed to the fact that a symmetric tariff does not imply a large asymmetric sectoral reallocation across countries associated with a shift in comparative advantage, as found in the case of a unilateral tariff in the analysis above. Row 11 shows that under home dominant currency pricing, optimal policy favors the foreign country, but welfare changes remain small. (See Appendix Figure 8 for dynamics).

²² Appendix Figure 2 show the case of a symmetric trade war where firms in both countries set export prices in local currency (LCP stickiness). Monetary policy has minimal impact on trade flows.

A similarly diminished welfare loss is associated with a symmetric tariff war launched on exports of non-differentiated goods of both countries. (See row 13 of Tables 2 and 3 for welfare, and Appendix Figure 9 for impulse responses.)

6.2 Asymmetric Tariff war

We next consider a hybrid case where home tariffs on differentiated goods imports are met by foreign retaliation in the form of tariffs of an equal percentage on imports of less differentiated goods. This scenario could be interpreted as representing U.S. tariffs on manufacturing imports, while foreign countries retaliate with tariffs on US agricultural commodities (such as Chinese limits on imports of U.S. soybeans). Figure 4 shows that under PPI targeting, the sectoral reallocation is larger than in the case of a unilateral foreign tariff in Figure 1, as the foreign tariff further shifts production of non-differentiated goods to foreign, reinforcing the reallocation induced by home tariffs promoting home manufacturing (differentiated goods). In contrast with Figure 1, the fall in overall GDP in Figure 4 now is larger in the home country than the foreign.

Nonetheless, the optimal policy is qualitatively similar to that in the case of the unilateral foreign tariff in Figure 1, calling for a foreign expansion and home contraction. Since the non-differentiated sector is characterized by flexible prices, monetary policy has little power to affect demand in that sector. So optimal policy is driven by the goal of offsetting the home tariffs on differentiated goods.

Tables 2 and 3 (see row 14) show that welfare gains from optimal policy are asymmetric, favoring the home country more than foreign, and world welfare is somewhat higher than the benchmark case of the unilateral differentiated goods tariff.

7. Conclusion

Central banks around the world have grappled with the question of how to respond to the mix of inflation and output implications of tariffs, with the potential of igniting a trade war across and within geopolitical regions. The question is compounded by the fact that the recent rounds of tariffs may fall on a very broad range of goods, from final consumption goods to materials such as aluminum and steel, and tariff rates may be set quite high.

Using a New Keynesian model enriched with elements from the trade literature, including global value chains in production, and comparative advantage between multiple traded sectors that differ in terms of market structure and price rigidity, we find that the optimal monetary response can be either expansionary or contractionary, depending in particular on the sector targeted by tariffs and on the nature of price stickiness.

In the case of tariffs targeting differentiated final consumption goods, characterized with monopolistic competition and price stickiness at the producer level, a specific objective of monetary policy is to redress the distortionary effects of tariffs on relative prices between home and foreign goods, at least in part, via foreign currency depreciation. A foreign expansion with depreciation counteracts the impact of tariffs on both aggregate GDP and sectoral reallocation. In the case of non-differentiated goods, their price flexibility implies that a currency depreciation cannot significantly redress the distortionary effects of the tariff on relative prices, and an expansion will induce a strong price rather than a quantity response by firms in the sector. However, monetary stabilization can compensate the loss of production due to the fall in the foreign demand for non-differentiated goods (due to the tariff) by raising the demand and production of differentiated goods. Indeed, by pursuing an optimal expansion cum depreciation to hasten the reallocation toward the differentiated goods, monetary policy promotes welfare by amplifying the reallocation induced by the tariff rather than offsetting it.

As a note of caution, however, we find that the role for optimal policy is altered when prices are sticky in the currency of the buyer, or when the country imposing the tariffs has the advantage of a currency dominant in global trade. In particular, optimal policy for a country with a dominant currency tends to be expansionary in response to both domestic and foreign tariffs.

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Table 1. Parameter Values

<u>Preferences</u>

Risk aversion	$\sigma = 2$
Time preference	$\beta = 0.99$
Labor supply elasticity	$1/\psi=1.9$
Differentiated goods share	$\theta = 0.45$
Non-differentiated goods home bias	v = 0.5

Differentiated goods elasticity $\phi = 5.2 \text{ (also } 3.8)$ Non-differentiated goods elasticity $\eta = 5.2 \text{ (also } 1.5, 15)$ Substitution between sectors $\xi = 1 \text{ (also } 0.5, 1.5)$

Technology

Firm death rate	$\delta = 0.025$
Price stickiness	$\psi_P = 49$
Intermediate input share	$\varsigma = 1/3$
Differentiated goods trade cost	$\tau_D = 0.44$
Non-differentiated goods trade cost	$\tau_N = 0.44$
Mean sunk entry cost	$\overline{K} = 1$
Firm entry adjustment cost	$\lambda = 0.10$
Bond holding cost	$\psi_{B} = 10^{-6}$

Monetary Policy (for Taylor Rule)

Interest rate smoothing	$\gamma_i = 0.7$
Inflation response	$\gamma_p = 1.7$
GDP response	$\gamma_{v}=0.1$

Tariff shock

Mean	$T_D = T_N = 1.02$
Standard deviation	$\sigma_{\varepsilon_T} = 0.08$
Autoregressive parameter	$\rho_{T} = 0.56$

Table 2. Welfare gains from Ramsey Optimal policy relative to Taylor Rule (percent change, in consumption units):

	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.096	0.162	0.031
2.	home DCP	0.046	0.066	0.026
3.	both countries LCP	0.042	0.037	0.047
4.	sectors complements (ξ =0.5)	0.113	0.185	0.040
5.	sectors substitutes (ξ =1.4)	0.093	0.151	0.034
6.	nondifferentiated goods BKK (η=1.5)	0.092	0.148	0.036
7.	nondiff. goods homogeneous (η =15)	0.108	0.118	0.099
8.	persistent tariff ($\rho T = 0.95$)	0.444	0.194	0.694
9.	nontraded non-differentiated good	0.110	0.116	0.103
10.	symmetric tariff on differentiated goods	0.027	0.027	0.027
11.	home DCP	0.057	-0.100	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	0.035	-0.006
13.	symmetric tariff on non-differentiated goods	0.013	0.013	0.013
14.	home diff. tariff; foreign non-diff. tariff	0.121	0.174	0.067

Table 3. Welfare gains from Ramsey Optimal policy relative to PPI targeting Rule (percent change, in consumption units):

	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.020	0.024	0.016
2.	home DCP	0.017	-0.083	0.117
3.	both countries LCP	0.023	-0.127	0.172
4.	sectors complements (ξ =0.5)	0.023	0.032	0.014
5.	sectors substitutes (ξ =1.4)	0.016	0.016	0.017
6.	nondifferentiated goods BKK (η=1.5)	0.019	0.024	0.013
7.	nondiff. goods homogeneous (η =15)	0.018	0.010	0.025
8.	persistent tariff ($\rho \tau = 0.95$)	0.053	0.034	0.071
9.	nontraded non-differentiated good	0.017	0.010	0.024
10.	symmetric tariff on differentiated goods	0.071	0.028	0.028
11.	home DCP	0.034	-0.096	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	-0.015	0.025
13.	symmetric tariff on non-differentiated goods	0.001	0.001	0.001
14.	home diff. tariff; foreign non-diff. tariff	0.034	0.068	0.000

Figure 1. Impulse responses to a rise in home tariff on differentiated imports

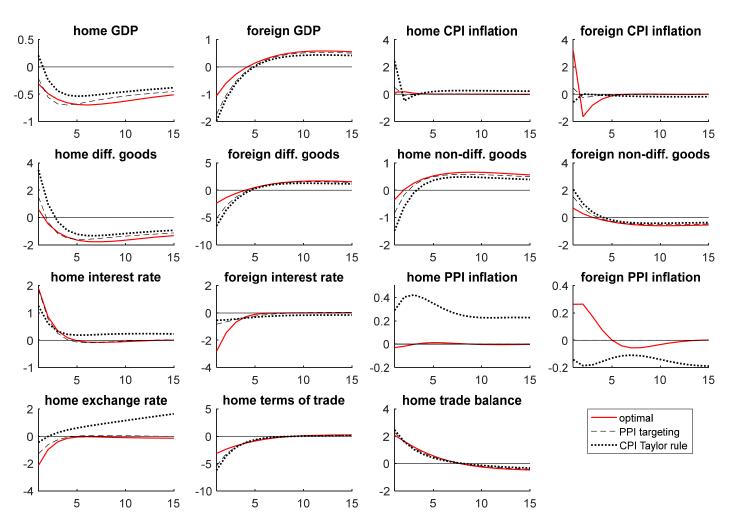
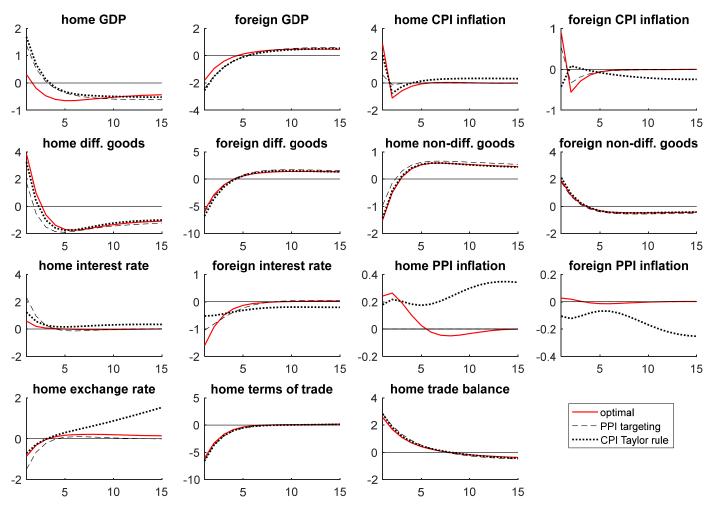
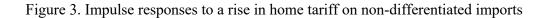


Figure 2. Impulse responses to a rise in home tariff on differentiated imports; Home currency dominant





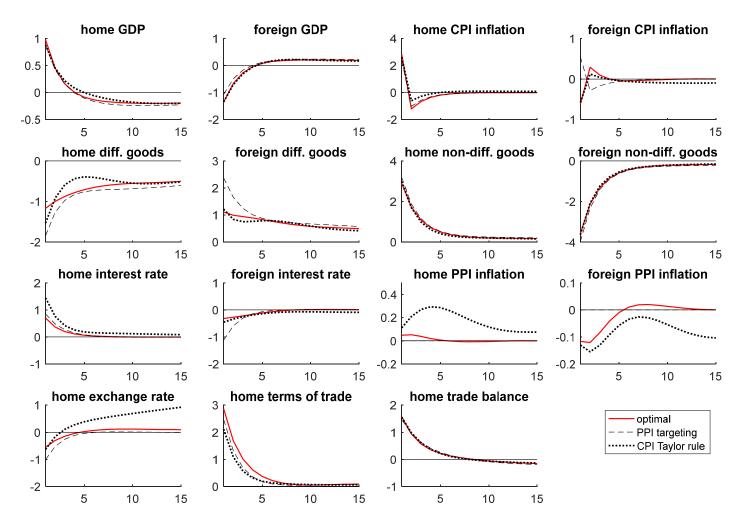
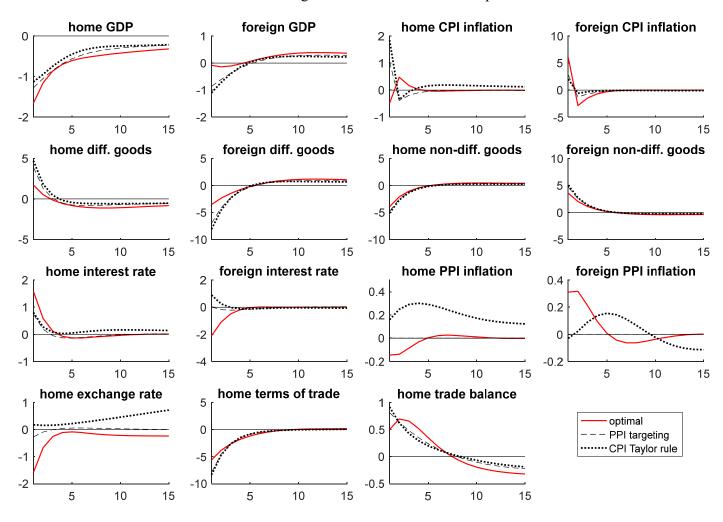


Figure 4. Impulse responses to a hybrid case: home tariff on differentiated imports; foreign tariff non-differentiated imports



Appendix

For "Monetary stabilization of sectoral tariffs"

by
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Appendix A. Demand equations from the benchmark model not listed in text

The composition of expenditure on adjustment costs, both for prices and bond holding, follows the same preferences as for consumption, and the associated demands mirror Eqs. (4)-(9). Adjustment costs for bond holding are as follows:

$$AC_{B,D,t} = \theta P_t AC_{B,t} / P_{D,t}$$

$$AC_{B,N,t} = (1-\theta) P_t AC_{B,t} / P_{N,t}$$

$$d_{AC,B,t}(h) = (p_t(h) / P_{D,t})^{-\phi} AC_{B,D,t}$$

$$d_{AC,B,t}(f) = (p_t(f) T_{D,t} / P_{D,t})^{-\phi} AC_{B,D,t}$$

$$AC_{B,H,t} = \nu (P_{H,t} / P_{N,t})^{-\eta} AC_{B,N,t}$$

$$AC_{B,F,t} = (1-\nu) (P_{F,t} T_{N,t} / P_{N,t})^{-\eta} AC_{B,N,t}.$$

The economy-wide demand for goods arising from price adjustment costs sums across the demand arising among n home firms: $AC_{P,t} = n_t AC_{P,t}(h)$. This is allocated as follows:

$$AC_{P,D,t} = \theta P_t AC_{P,t} / P_{D,t}$$

$$AC_{P,N,t} = (1-\theta)P_t AC_{P,t} / P_{N,t}$$

$$d_{AC,P,t}(h) = (p_t(h)/P_{D,t})^{-\phi} AC_{P,D,t}$$

$$d_{AC,P,t}(f) = (p_t(f)T_{D,t}/P_{D,t})^{-\phi} AC_{P,D,t}$$

$$AC_{P,H,t} = \nu (P_{H,t}/P_{N,t})^{-\eta} AC_{P,N,t}$$

$$AC_{P,F,t} = (1-\nu)(P_{F,t}T_{N,t}/P_{N,t})^{-\eta} AC_{P,N,t}.$$

The demand for differentiated goods for use as intermediates in production mirrors Eqs. (6)-(7), as follows:

$$d_{G,t}(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} G_t$$

$$d_{G,t}(f) = (p_t(f)T_{D,t} / P_{D,t})^{-\phi} G_t.$$

The demand for differentiated goods for use in the sunk entry investment of new firms mirrors Eqs. (6)-(7), as follows:

$$d_{K,t}(h) = \left(p_t(h) / P_{D,t} \right)^{-\phi} ne_t K_t$$

$$d_{K,t}(f) = \left(p_t(f)T_{D,t}/P_{D,t}\right)^{-\phi} ne_t K_t.$$

2. Market clearing conditions not listed in the text

Market clearing for the non-differentiated goods market requires:

$$y_{H,t} = C_{H,t} + AC_{P,H,t} + AC_{B,H,t} + (1+\tau_N)(C_{H,t}^* + AC_{P,H,t}^* + AC_{B,H,t}^*)$$

$$y_{F,t} = (1 + \tau_N^*)(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}) + C_{F,t}^* + AC_{P,F,t}^* + AC_{B,F,t}^*.$$

The market clearing condition for the manufacturing goods market is given in Eq. (19) in the main text.

Labor market clearing requires:

$$\int_{0}^{n_{t}} l_{t}(h)dh + l_{H,t} = l_{t}.$$

Bond market clearing requires:

$$B_{Ht} + B_{Ht}^* = 0$$

$$B_{Ft} + B_{Ft}^* = 0.$$

Balance of payments requires:

$$\int_{0}^{n_{t}} p_{t}^{*}(h)(d_{t}^{*}(h))dh - \int_{0}^{n_{t}^{*}} p_{t}(f)(d_{t}(f))df + P_{Ht}^{*}(C_{H,t}^{*} + AC_{P,H,t}^{*} + AC_{B,H,t}^{*})$$

$$-P_{F,t}(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}) - i_{t-1}B_{H,t-1}^{*} + e_{t}i_{t-1}^{*}B_{F,t-1} = (B_{H,t}^{*} - B_{H,t-1}^{*}) + e_{t}(B_{F,t} - B_{F,t-1}).$$

Appendix B: Derivation of Analytical Results, Producer Currency Pricing

a) Demands

The modified consumption index implies the following demands:

$$C_{DHt} = \frac{1}{2} \frac{P_{Dt}C_{Dt}}{P_{DHt}} = \frac{\theta}{2} \frac{P_{t}C_{t}}{P_{DHt}}$$

$$C_{DFt} = \frac{1}{2} \frac{P_{Dt}C_{Dt}}{T_{Dt}e_{t}P_{DFt}^{*}} = \frac{\theta}{2} \frac{P_{t}C_{t}}{T_{Dt}e_{t}P_{DFt}^{*}}$$

$$C_{DHt}^{*} = \frac{\theta}{2} \frac{e_{t}P_{t}^{*}C_{t}^{*}}{T_{Dt}^{*}P_{DHt}} = \frac{\theta}{2} \frac{P_{t}C_{t}}{T_{Dt}^{*}P_{DHt}}$$

$$C_{Dt} = \theta \frac{P_{t}C_{t}}{P_{Dt}}$$

$$C_{Nt} = (1 - \theta) \frac{P_{t}C_{t}}{P_{Nt}}$$

$$c_{t}(h) = (p_{t}(h)/P_{DH,t})^{-\phi}C_{DH,t}$$

b) Optimal price setting differentiated good

The home firm maximizes

$$E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left[\left(p_{t}(h) - \frac{W_{t}}{\alpha_{D}} \right) \left(p_{t}(h) / P_{DH,t} \right)^{-\phi} C_{DH,t} + \left(p_{t}(h) - \frac{W_{t}}{\alpha_{D}} \right) \left(\frac{T_{t}^{*} p_{t}(h)}{e_{t}} / \left(\frac{T_{t}^{*} P_{DH,t}}{e_{t}} \right) \right)^{-\phi} C_{DH,t}^{*} \right] \right]$$
or
$$E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left(p_{t}(h) - \frac{W_{t}}{\alpha_{D}} \right) \left(p_{t}(h) / P_{DH,t} \right)^{-\phi} \left(C_{DH,t} + C_{DH,t}^{*} \right) \right],$$

implying the price setting rule

$$P_{Ht} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left(C_{DH,t} + C_{DH,t}^{*} \right) \frac{W_{t}}{\alpha_{D}} \right]}{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left(C_{DH,t} + C_{DH,t}^{*} \right) \right]}.$$

Substitute in demands from above

$$P_{Ht} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_t} \left(\frac{\theta}{2} \frac{P_t C_t}{P_{DHt}} + \frac{\theta}{2} \frac{e_t P_t^* C_t^*}{T_{Dt}^* P_{DHt}} \right) \frac{W_t}{\alpha_D} \right]}{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_t} \left(\frac{\theta}{2} \frac{P_t C_t}{P_{DHt}} + \frac{\theta}{2} \frac{e_t P_t^* C_t^*}{T_{Dt}^* P_{DHt}} \right) \right]},$$

and substitute in for μ_i and exchange rate

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left(\frac{\theta}{2} \frac{\mu_{t}}{P_{DHt}} + \frac{\theta}{2} \frac{e_{t}\mu_{t}^{*}}{T_{Dt}^{*}P_{DHt}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left(\frac{\theta}{2} \frac{\mu_{t}}{P_{DHt}} + \frac{\theta}{2} \frac{e_{t}\mu_{t}^{*}}{T_{Dt}^{*}P_{DHt}} \right) \right]}.$$

Use $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$ from the main text:

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \frac{\kappa \mu_t}{\alpha_D} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \right]}.$$

The foreign firm counterpart is:

$$P_{DFt}^{*} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta \left[\frac{\mu_{t-1}^{*}}{\mu_{t}^{*}} \left(C_{DFt}^{*} + C_{DF,t} \right) \frac{W_{t}^{*}}{\alpha_{D}^{*}} \right]}{E_{t-1}\beta \left[\frac{\mu_{t-1}^{*}}{\mu_{t}^{*}} \left(C_{DFt}^{*} + C_{DF,t} \right) \right]}.$$

Substitute in for in μ_{ι}^{*} mu and exchange rate:

$$\begin{split} P_{DFt}^* &= \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta \left[\frac{\mu_{t-1}^*}{\mu_t^*} \left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*} \right) \frac{\mu_t^*}{\alpha_D^*} \right]}{E_{t-1}\beta \left[\frac{\mu_{t-1}^*}{\mu_t^*} \left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*} \right) \right]} \\ P_{DFt}^* &= \frac{\phi}{\phi - 1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}} \right) \frac{\mu_t^*}{\alpha_D^*} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}} \right) \right]} \end{split}$$

So the home price index can be written:

$$P_t = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] P_{DHt}^{\theta/2} \left(T_{Dt} e_t P_{DFt}^* \right)^{\theta/2} P_{NHt}^{-1-\theta}.$$

Use $P_{NHt} = \frac{W_t}{\alpha_N} = \frac{\kappa \mu_t}{\alpha_N}$ to write the price index in terms of exogenous variables:

$$P_{t} = 2^{\theta} \left[\left(\frac{1 - \theta}{\theta} \right)^{\theta - 1} + \left(\frac{1 - \theta}{\theta} \right)^{\theta} \right] P_{DHt}^{\theta/2} \left(T_{Dt} e_{t} p_{t}^{*} \left(f \right) \right)^{\theta/2} \left(\frac{\kappa \mu_{t}}{\alpha_{N}} \right)^{(1 - \theta)}$$

$$P_{t} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right]} \right)^{\theta/2} \left(T_{Dt} e_{t} \frac{\phi}{\phi-1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{Dt}} \right) \frac{\mu_{t}^{*}}{\alpha_{Dt}^{*}} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{Dt}} \right) \right]} \right)^{\theta/2} \left(\frac{\kappa \mu_{t}}{\alpha_{N}} \right)^{(1-\theta)}$$

c) labor

Given the homogenous second sector, the easiest way to derive equilibrium labor is from the household budget constraint, which under balanced trade, implies labor income equals total nominal expenditure minus profits from the home differentiated sector.

Write the household budget constraint:

$$W_{i}l_{i}+\pi_{i}=P_{i}C_{i}$$

where π is profits of home differentiated goods firms, used in the firm maximization problem above to determine price setting. Use labor supply condition to substitute out wage: $\kappa \mu_i l_i + \pi_i = \mu_i$, and use this to compute the term in welfare including labor, $\kappa E_{t-1}[l_i]$:

$$\kappa E_{t-1} \left[l_t \right] = 1 - E_{t-1} \left[\frac{\pi_t}{\mu_t} \right].$$

Compute profit for the home differentiated good producer:

$$E_{t-1} \left[\frac{\pi_t}{\mu_t} \right] = E_{t-1} \left[\frac{1}{\mu_t} \frac{1}{\phi} \left(P_{DHt} C_{DHt} + P_{DHt} C_{DHt}^* \right) \right].$$

Use $C_{DHt} + C_{DHt}^* = \frac{\theta}{2} \frac{\mu_t}{P_{DHt}} + \frac{\theta}{2} \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}} \frac{\mu_t^*}{T_{Dt}^* P_{DHt}}$ from price setting derivation

$$E_{t-1} \left[\frac{\pi_{t}}{\mu_{t}} \right] = E_{t-1} \left[\frac{1}{\mu_{t}} \frac{1}{\phi} \left(P_{DHt} \frac{\theta}{2} \frac{\mu_{t}}{P_{DHt}} + P_{DHt} \frac{\theta}{2} \frac{\mu_{t} \alpha_{N}^{*}}{\mu_{t}^{*} \alpha_{N} T_{N,t}} \frac{\mu_{t}^{*}}{T_{Dt}^{*} P_{DHt}} \right) \right]$$

$$E_{t-1} \left[\frac{\pi_{t}}{\mu_{t}} \right] = \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right]$$

So
$$\kappa E_{t-1}[l_t] = 1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} \right].$$

Note that there is no μ_t left in this term, so the labor term in the welfare condition will have no bearing on the optimal monetary policy under our specification.

d) Home optimal policy, Nash

Write home welfare, and express as a function of exogenous variables.

$$W_{t} = E_{t-1} \ln C_{t} - E_{t-1} \kappa l_{t}$$

$$W_{t} = E_{t-1} [\ln \mu_{t}] - E_{t-1} [\ln P_{t}] - E_{t-1} \kappa l_{t}$$

$$\begin{split} W_{l} &= E_{l-1} \Big[\ln \mu_{l}\Big] - E_{l-1} \Bigg[\ln \Big(2 \Big[\Big(\frac{1-\theta}{\theta}\Big)^{\theta-1} + \Big(\frac{1-\theta}{\theta}\Big)^{\theta}\Big] \Bigg] \frac{\phi}{\phi - 1} \frac{E_{l-1} \Big[\Big(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big) \frac{\kappa \mu_{l}}{\alpha_{D}} \Big]}{E_{l-1} \Big[\Big(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big) \Big] \frac{\theta}{\alpha_{D}} \Bigg] \frac{\theta}{\phi} - 1 \frac{E_{l-1} \Big[\Big(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big) \Big]}{E_{l-1} \Big[\Big(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big) \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] \Bigg) - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,I}T_{Dl}^{*}} \Big] - \Big(1 - \frac{1}{\theta} \frac{\theta}{2} E_{l-$$

Take the derivative of home welfare with respect to the home monetary policy variable μ :

$$\frac{1}{\mu} - \frac{\theta}{2} \frac{\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right) \frac{\kappa}{\alpha_D}}{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right) \frac{\kappa \mu_t}{\alpha_D}\right] - \left(\frac{\theta}{2} + 1 - \theta\right) \frac{1}{\mu} = 0$$

$$\mu = \frac{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \mu \right]}{\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right)}$$

Conjecture the solution: $\mu = a \left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right)^{-1}$,

which is easily verified by substituting this in the equation immediately above.

e) Foreign optimal policy, Nash

The foreign price index is:

$$\begin{split} P_{t}^{*} &= 2^{\theta} \Bigg[\bigg(\frac{1-\theta}{\theta} \bigg)^{\theta-1} + \bigg(\frac{1-\theta}{\theta} \bigg)^{\theta} \Bigg] \Bigg(\frac{\phi}{\phi-1} \frac{E_{t-1} \Bigg[\bigg(1 + \frac{\alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{Dt}} \bigg) \frac{\mu_{t}^{*}}{\alpha_{D}^{*}} \bigg) \Bigg)^{\theta/2} \Bigg(T_{Dt}^{*} \frac{\phi}{\phi-1} \frac{E_{t-1} \Bigg[\bigg(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \bigg) \frac{\kappa \mu_{t}}{\alpha_{D}} \bigg) - \ell_{t} \Bigg)^{\theta/2} \Bigg(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \bigg)^{(1-\theta)} \\ P_{t}^{*} &= 2^{\theta} \Bigg[\bigg(\frac{1-\theta}{\theta} \bigg)^{\theta-1} + \bigg(\frac{1-\theta}{\theta} \bigg)^{\theta} \bigg) \Bigg(\frac{\phi}{\phi-1} \frac{E_{t-1} \Bigg[\bigg(1 + \frac{\alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{Dt}} \bigg) \frac{\mu_{t}^{*}}{\alpha_{D}^{*}} \bigg) \bigg)^{\theta/2} \Bigg(T_{Dt}^{*} \frac{\phi}{\phi-1} \frac{E_{t-1} \Bigg[\bigg(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \bigg) \frac{\kappa \mu_{t}}{\alpha_{D}} \bigg) \frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \Bigg) \Bigg] \Bigg(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \bigg)^{(1-\theta)} \Bigg) \Bigg(\frac{\phi}{\alpha_{N}^{*}} \frac{E_{t-1} \Bigg[\bigg(1 + \frac{\alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{Dt}} \bigg) \frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \bigg)^{(1-\theta)} \Bigg) \Bigg(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \bigg)^{(1-\theta)} \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \bigg)^{(1-\theta)} \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}} \bigg) \Bigg) \Bigg(\frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t}^{*} \alpha_{N}^{*}}$$

Foreign labor:

$$\kappa E_{t-1} \left[l_{t}^{*} \right] = 1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*} T_{D,t} T_{Nt}^{*}} \right]$$

Welfare:

$$\begin{split} & W_{t}^{*} = E_{t-1} \ln C_{tt}^{*} - E_{t-1} \kappa I_{t}^{*} \\ & W_{t}^{*} = E_{t-1} \Big[\ln \mu_{t} \Big] - E_{t-1} \Big[\ln P_{t}^{*} \Big] - E_{t-1} \kappa I_{t}^{*} \\ & W_{t}^{*} = E_{t-1} \Big[\ln \mu_{t}^{*} \Big] - E_{t-1} \Big[\ln P_{t}^{*} \Big] - E_{t-1} \kappa I_{t}^{*} \\ & W_{t}^{*} = E_{t-1} \Big[\ln \mu_{t}^{*} \Big] - E_{t-1} \Big[\ln \rho_{t}^{*} \Big] - E_{t-1} \Big[\ln P_{t}^{*} \Big] - E_{t-1} \Big[\left(\frac{1 - \theta}{\theta} \right)^{\theta - 1} + \left(\frac{1 - \theta}{\theta} \right)^{\theta} \Big] \Big[\frac{\phi}{\phi - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N} T_{N,t}}{\alpha_{N}^{*} T_{D,t}} \right) \frac{\mu_{t}^{*}}{\phi - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t}^{*} T_{D,t}^{*}} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}^{*} T_{N,t}} \frac{1}{T_{N,t}^{*}} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}^{*} T_{N,t}^{*}} \frac{1}{\rho - 1} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{\mu_{t}^{*}}{\rho - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}^{*} T_{N,t}} \frac{1}{\rho - 1} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{\mu_{t}^{*}}{\rho - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}^{*} T_{N,t}} \frac{1}{\rho - 1} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{\mu_{t}^{*}}{\rho - 1} \frac{\mu_{t}^{*}}{\rho - 1} \frac{E_{t-1} \Big[\left(\frac{1 + \alpha_{N}^{*}}{\alpha_{N}^{*} T_{N,t}} \frac{1}{\rho - 1} \right) \frac{\mu_{t}^{*}}{\rho - 1} \frac{\mu_{t}^{$$

Which is directly analogous to home, with solution $\mu_t^* = a \left(1 + \frac{\alpha_N}{\alpha_N^*} \frac{T_{Nt}}{T_{Dt}} \right)^{-1}$

f) Cooperative policy

Take derivative of sum of home and foreing welfare with respect to μ_i

$$\frac{1}{\mu} - \frac{\theta}{2} \frac{\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa}{\alpha_{D}}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa \mu_{t}}{\alpha_{D}}\right]} - \left(\frac{\theta}{2} + 1 - \theta\right) \frac{1}{\mu} + \frac{\theta}{2} \frac{1}{\mu} - \frac{\theta}{2} \frac{\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa}{\alpha_{D}}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa \mu_{t}}{\alpha_{D}}\right]} = 0$$

$$\mu = \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa \mu_{t}}{\alpha_{D}}\right]}{\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}}\right) \frac{\kappa}{\alpha_{D}}}.$$

Conjecture solution: $\mu = a \left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right)^{-1}$, which is easily verified

We note that the cooperative solution is same as Nash in this case.

Now take the derivative with respect to μ_t^* :

$$\begin{split} &\frac{1}{\mu_{t}^{*}} - \frac{\theta}{2} \frac{\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{1}{\alpha_{D}^{*}}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{\mu_{t}^{*}}{\alpha_{D}^{*}}\right]} - \left(\frac{\theta}{2} + 1 - \theta\right) \frac{1}{\mu} + \frac{\theta}{2} \frac{1}{\mu} - \frac{\theta}{2} \frac{\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{1}{\alpha_{D}^{*}}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{\mu_{t}^{*}}{\alpha_{D}^{*}}\right]} = 0 \\ &\theta \frac{1}{\mu_{t}^{*}} - \theta \frac{\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{1}{\alpha_{D}^{*}}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \frac{\mu_{t}^{*}}{\alpha_{D}^{*}}\right]} = 0 \\ &\mu_{t}^{*} = \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right) \mu_{t}^{*}\right]}{\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right)} \\ &\frac{1}{\left(1 + \frac{\alpha_{N}T_{N,t}}{\alpha_{N}^{*}T_{Dt}}\right)} \mu_{t}^{*} \\ &\frac{1}{\alpha_{N}^{*}T_{Dt}} \\ &\frac{1}{\alpha_{N}^{*}T_{Dt}} \frac{1}{\alpha_{N}^{*}T_{Dt}} \\ &\frac{1}{\alpha_{N}^{*}T_{Dt}} \frac{1}{\alpha_{N}^{*}T_{Dt}} \frac{1}{\alpha_{N}^{*}T_{Dt}} \\ &\frac{1}{\alpha_{N}^{*}T_{Dt}} \frac{1}{\alpha_{N}^{*}T_{Dt}} \frac{1}{\alpha_{N}^{*}T_{$$

Conjecture same solution as before: $\mu_t^* = a \left(1 + \frac{\alpha_N}{\alpha_N^*} \frac{T_{Nt}}{T_{Dt}} \right)^{-1}$, verified.

This is also same as Nash solution above.

g) Defining conditions under which home is both producer and net importer of homogeneous good

We can easily compute home consumption of the non-differentiated good:

$$C_{Nt} = (1 - \theta) \frac{P_t C_t}{P_{Nt}},$$

where $P_{Nt} = e_t T_{N,t} P_{Nt}^*$ and $P_{Nt} = \frac{W_t}{\alpha_N} = \frac{\kappa \mu_t}{\alpha_N}$, $P_{Nt}^* = \frac{W_t^*}{\alpha_N^*} = \frac{\kappa \mu_t^*}{\alpha_N^*}$.

So
$$C_{Nt} = (1 - \theta) \frac{P_t C_t}{\kappa \mu_t} = (1 - \theta) \frac{\mu_t}{\kappa \mu_t} = (1 - \theta) \frac{\alpha_N}{\kappa}$$
.

Note this is consant, unaffected by tariffs.

Now compute the level of home production in this sector based on labor allocation. Recall total labor allocation above:

$$l_{t} = \frac{1}{\kappa} - \frac{1}{\mu_{t}} \frac{1}{\phi \kappa} \left(P_{DHt} C_{DHt} + P_{DHt} C_{DHt}^{*} \right).$$

We next subtract labor for the differentiated goods sector: $\frac{C_{DHt} + C^*_{DHt}}{\alpha_{D}}$.

So the labor allocation for the non-differentiated sector becomes:

$$l_{Nt} = \frac{1}{\kappa} - \frac{1}{\mu_{t}} \frac{1}{\phi \kappa} \left(P_{DHt} C_{DHt} + P_{DHt} C^{*}_{DHt} \right) - \frac{C_{DHt} + C^{*}_{DHt}}{\alpha_{D}}$$

$$\begin{split} & l_{t} = \frac{1}{\kappa} - \frac{1}{\phi \kappa} \frac{\theta}{2} \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] - \frac{1}{\alpha_{D}} \left(\frac{\theta}{2} \frac{\mu_{t}}{P_{DHt}} + \frac{\theta}{2} \frac{\mu_{t} \alpha_{N}^{*}}{\mu_{t}^{*} \alpha_{N} T_{N,t}} \frac{\mu_{t}^{*}}{T_{Dt}^{*} P_{DHt}} \right) \\ & l_{t} = \frac{1}{\kappa} - \left(\frac{1}{\phi \kappa} \frac{\theta}{2} + \frac{1}{\alpha_{D}} \frac{\theta}{2} \frac{\mu_{t}}{P_{DHt}} \right) \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] \\ & l_{Nt} = \frac{1}{\kappa} - \left(\frac{1}{\phi \kappa} \frac{\theta}{2} + \frac{1}{\alpha_{D}} \frac{\theta}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \mu_{t}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right] \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right]. \end{split}$$

So output in the sector may be written:

$$y_{Nt} = \frac{\alpha_{N}}{\kappa} - \left[\frac{\alpha_{N}}{\phi \kappa} \frac{\theta}{2} + \frac{\alpha_{N}}{\alpha_{D}} \frac{\theta}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \mu_{t}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right]} \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right].$$

We conclude that the condition for where $C_{Nt} > y_{Nt}$ may be written:

$$(1-\theta) \frac{\alpha_{N}}{\kappa} - \frac{\alpha_{N}}{\kappa} + \left[\frac{\alpha_{N}}{\phi \kappa} \frac{\theta}{2} + \frac{\alpha_{N}}{\alpha_{D}} \frac{\theta}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \mu_{t}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right] } \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] > 0$$

$$(-1) + \left[\frac{1}{\phi} \frac{1}{2} + \frac{1}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]} \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] > 0$$

$$\left[\frac{1}{\phi} \frac{1}{2} + \frac{1}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]} \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] > 1.$$

Evaluate this condition under perfect foresight, abstracting from risk premium in pricing of the sticky price good:

$$\left(\frac{1}{\phi} \frac{1}{2} + \frac{1}{2} \frac{\phi - 1}{\phi}\right) \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}}\right] > 1$$

$$1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} > 2$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} > 1.$$

Derive the condition under which we also can guarantee positive home production of the non-differentiated good:

$$y_{Nt} = \frac{\alpha_{N}}{\kappa} - \left[\frac{\alpha_{N}}{\phi \kappa} \frac{\theta}{2} + \frac{\alpha_{N}}{\alpha_{D}} \frac{\theta}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \mu_{t}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right] \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] > 0$$

$$y_{Nt} = 1 - \left[\frac{1}{\phi} \frac{\theta}{2} + \frac{\theta}{2} \frac{\phi - 1}{\phi} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \mu_{t}}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right] \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] > 0.$$

Again, evaluate in perfect foresight, abstracting from the risk premium in pricing of sticky-price goods:

$$y_{Nt} = 1 - \left(\frac{1}{\phi} \frac{\theta}{2} + \frac{\theta}{2} \frac{\phi - 1}{\phi}\right) \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$y_{Nt} = 1 - \frac{\theta}{2} \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} < \frac{2}{\theta}$$

$$\frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} < \frac{2}{\theta} - 1.$$

Appendix C: Derivation of Analytical Results, Local Currency Pricing

a) Price setting:

Home good now has distinct prices in home and foreign market, P_{DHt} and P_{DHt}^* , with foreign price in foreign currency.

The home firm maximizes:

$$E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left[\left(p_{t}(h) - \frac{W_{t}}{\alpha_{t}} \right) \left(p_{t}(h) / P_{DH,t} \right)^{-\phi} C_{DH,t} + \left(e_{t} p_{t}^{*}(h) - \frac{W_{t}}{\alpha_{t}} \right) \left(\frac{T_{t}^{*} p_{t}^{*}(h)}{T_{t}^{*} P_{DH,t}^{*}} \right)^{-\phi} C_{DH,t}^{*} \right] \right]$$

$$E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} \left[\left(p_{t}(h) - \frac{W_{t}}{\alpha_{t}} \right) \left(p_{t}(h) / P_{DH,t} \right)^{-\phi} C_{DH,t} + \left(e_{t} p_{t}^{*}(h) - \frac{W_{t}}{\alpha_{t}} \right) \left(p_{t}^{*}(h) / P_{DH,t}^{*} \right)^{-\phi} C_{DH,t}^{*} \right] \right]$$

With respect to $p_{i}(h)$:

$$E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \right] = E_{t-1}\beta \left[\phi \frac{1}{P_{DHt}} \frac{\mu_{t-1}}{\mu_{t}} \left(P_{DHt} - \frac{W_{t}}{\alpha_{D}} \right) (C_{DH,t}) \right]$$

$$P_{dHt}E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \right] = P_{DHt}E_{t-1}\beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \right] - E_{t-1}\beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \right]$$

$$(\phi - 1)P_{DHt}E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \right] = E_{t-1}\beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} (C_{DH,t}) \frac{W_{t}}{\alpha_{D}} \right]$$

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_{t}} C_{DH,t} \frac{W_{t}}{\alpha_{D}} \right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_{t}} C_{DH,t} \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}$$

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_{t}} \frac{\theta}{2} \frac{\mu_{t}}{P_{DH,t}} \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_{t}} \frac{\theta}{2} \frac{\mu_{t}}{P_{DH,t}} \right]}$$

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_{-}} E_{t-1} \left[\mu_{t} \right]$$

Maximizing with respect to $p_t^*(h)$:

$$\begin{split} E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} e_{t} C^{*}_{DH,t} \right] &= E_{t-1}\beta \left[\phi \frac{1}{P_{DH,t}^{*}} \frac{\mu_{t-1}}{\mu_{t}} \left(e_{t} P_{DH,t}^{*} - \frac{W_{t}}{\alpha_{D}} \right) C^{*}_{DH,t} \right] \\ E_{t-1}\beta \left[\frac{\mu_{t-1}}{\mu_{t}} e_{t} C^{*}_{DH,t} \right] &= E_{t-1}\beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} e_{t} C^{*}_{DH,t} \right] - \frac{1}{P_{DH,t}^{*}} E_{t-1}\beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} C^{*}_{DH,t} \frac{W_{t}}{\alpha_{D}} \right] \end{split}$$

$$\begin{split} \left(\phi-1\right) P_{DH,t}^{*} E_{t-1} \beta \left[\frac{\mu_{t-1}}{\mu_{t}} e_{t} C^{*}_{DH,t}\right] &= E_{t-1} \beta \left[\phi \frac{\mu_{t-1}}{\mu_{t}} C^{*}_{DH,t} \frac{W_{t}}{\alpha_{D}}\right] \\ P_{DH,t}^{*} &= \frac{\phi}{\phi-1} \frac{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} C^{*}_{DH,t} \frac{W_{t}}{\alpha_{D}}\right]}{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} e_{t} C^{*}_{DH,t}\right]} \\ P_{DH,t}^{*} &= \frac{\phi}{\phi-1} \frac{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} \frac{\theta}{2} \frac{P_{t}^{*}}{T_{Dt}^{*} P_{DHt}^{*}} C_{t}^{*} \frac{W_{t}}{\alpha_{D}}\right]}{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} e_{t} \frac{\theta}{2} \frac{P_{t}^{*}}{T_{Dt}^{*} P_{DHt}^{*}} C_{t}^{*}\right]}. \end{split}$$

Use the property of a homogeneous sector above: $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$:

$$P_{DH,t}^{*} = \frac{\phi}{\phi - 1} \frac{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} \frac{\theta}{2} \frac{\mu_{t}^{*}}{T_{Dt}^{*} P_{DHt}^{*}} \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1} \left[\frac{\mu_{t-1}}{\mu_{t}} \frac{\mu_{t} \alpha_{N}^{*}}{\mu_{t}^{*} \alpha_{N} T_{N,t}} \frac{\theta}{2} \frac{\mu_{t}^{*}}{T_{Dt}^{*} P_{DHt}^{*}} \right]}$$

and cancel terms:

$$P_{DH,t}^* = \frac{\phi}{\phi - 1} \frac{\frac{\kappa}{\alpha_D} E_{t-1} \left[\frac{\mu_t^*}{T_{Dt}^*} \right]}{E_{t-1} \left[\frac{\alpha_N^*}{\alpha_N T_{Nt} T_{Dt}^*} \right]}.$$

Analogously for foreign differentiated good:

$$P_{Ft}^* = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} E_{t-1} \left[\mu_t^* \right]$$

$$P_{Ft} = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} \frac{E_{t-1} \left[\frac{\mu_t}{T_{Dt}} \right]}{E_{t-1} \left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}} \right]}.$$

Non-differentiated prices are the same as in PCP case above.

Substitute into the home price index:

$$P_{t} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}} E_{t-1} \left[\mu_{t} \right] \right)^{\theta/2} \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} \frac{E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right]}{E_{t-1} \left[\frac{\alpha_{N} T_{Nt}}{\alpha_{N}^{*} T_{Dt}} \right]} \right)^{\theta/2} \left(\frac{\kappa \mu_{t}}{\alpha_{N}} \right)^{(1-\theta)}.$$

Analogously for foreign price index:

$$P_{t}^{*} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} E_{t-1} \left[\mu_{t}^{*} \right] \right)^{\theta/2} \left(\frac{\phi}{\phi-1} \frac{\frac{\kappa}{\alpha_{D}} E_{t-1} \left[\frac{\mu_{t}^{*}}{T_{Dt}^{*}} \right]}{E_{t-1} \left[\frac{\alpha_{N}^{*}}{\alpha_{N} T_{Nt} T_{Dt}^{*}} \right]} \right)^{\theta/2} \left(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \right)^{(1-\theta)}$$

Home equilibrium labor:

Use labor supply condition to substitute out wage:

$$\kappa \mu_t l_t + \pi_t = \mu_t.$$

Use this to compute term for labor required in the welfare function: $\kappa E_{t-1}[l_t]$:

$$\kappa E_{t-1}[l_t] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home D good producer:

$$E_{t-1}\left[\frac{\pi_{t}}{\mu_{t}}\right] = E_{t-1}\left[\frac{1}{\mu_{t}}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + e_{t}P_{DHt}^{*}C_{DHt}^{*}\right)\right].$$

Use demands from above: $C_{DH,t} = \frac{\theta}{2} \frac{P_t C_t}{P_{DH,t}}$, $C_{DF,t} = \frac{\theta}{2} \frac{P_t C_t}{T_{Dt} P_{DF,t}}$, $C_{DHt}^* = \frac{\theta}{2} \frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}$

$$E_{t-1}\left[\frac{\pi_{t}}{\mu_{t}}\right] = E_{t-1}\left[\frac{1}{\mu_{t}}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{P_{t}C_{t}}{P_{DH,t}} + e_{t}P_{DHt}^{*}\frac{\theta}{2}\frac{P_{t}^{*}C_{t}^{*}}{T_{Dt}^{*}P_{DHt}^{*}}\right)\right]$$

$$E_{t-1} \left[\frac{\pi_t}{\mu_t} \right] = E_{t-1} \left[\frac{1}{\mu_t} \frac{1}{\phi} \left(\frac{\theta}{2} \mu_t + e_t \frac{\theta}{2} \frac{\mu_t^*}{T_{Dt}^*} \right) \right].$$

Sub in for exchange rate:

$$E_{t-1}\left[\frac{\pi_{t}}{\mu_{t}}\right] = E_{t-1}\left[\frac{1}{\mu_{t}}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_{t} + \frac{\mu_{t}\alpha_{N}^{*}}{\mu_{t}^{*}\alpha_{N}T_{N,t}}\frac{\theta}{2}\frac{\mu_{t}^{*}}{T_{Dt}^{*}}\right)\right]$$

So
$$\kappa E_{t-1}[l_t] = 1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N_t} T_{D_t}^*} \right].$$

Note this is the same as under PCP pricing.

Compute welfare:

Home:

$$W_{t} = E_{t-1} \ln C_{t} - E_{t-1} \kappa l_{t}$$

$$W_{t} = E_{t-1} \left[\ln \mu_{t} \right] - E_{t-1} \left[\ln P_{t} \right] - E_{t-1} \kappa l_{t}$$

$$\begin{split} W_{t} &= E_{t-1} \Big[\ln \mu_{t} \Big] - E_{t-1} \Bigg[\ln \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}} E_{t-1} \Big[\mu_{t} \Big] \right)^{\theta/2} \Bigg(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} \frac{E_{t-1} \Big[\frac{\mu_{t}}{T_{Dt}} \Big]}{E_{t-1} \Big[\frac{\alpha_{N} T_{Nt}}{\alpha_{N}^{*} T_{Dt}} \Big]} \Bigg)^{\theta/2} \left(\frac{\kappa \mu_{t}}{\alpha_{N}} \right)^{(1-\theta)} \Bigg) \Bigg] \\ &- \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Bigg[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t}} T_{Dt}^{*} \Bigg] \Bigg) \\ W_{t} &= E_{t-1} \Big[\ln \mu_{t} \Big] - \frac{\theta}{2} E_{t-1} \ln \Big(E_{t-1} \Big[\mu_{t} \Big] \Big) - \frac{\theta}{2} E_{t-1} \ln \Big(E_{t-1} \Big[\frac{\mu_{t}}{T_{Dt}} \Big] \Big) - (1-\theta) E_{t-1} \Big[\ln \mu_{t} \Big] \\ &- E_{t-1} \Bigg[\ln \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}} \right)^{\theta/2} \Bigg(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} \frac{1}{E_{t-1} \Big[\frac{\alpha_{N} T_{Nt}}{\alpha_{N}^{*} T_{Dt}} \Big]} \Bigg)^{\theta/2} \Bigg(\frac{\kappa}{\alpha_{N}} \Big)^{(1-\theta)} \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \Big] \Bigg) \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{N,t}^{*}} \Big] \Bigg) \Bigg] \Bigg] \Bigg] - \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{N,t}^{*}} \Big] \Bigg] \Bigg] \Bigg] \Bigg] \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t}^{*}} \Big] \Bigg] \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \Big[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t}^{*}} \Big] \Bigg) \Bigg] \Bigg(1 - \frac{1}{\phi} \frac{\theta}{2} E_$$

Compute derivative of home welfare with respect to μ :

$$\frac{1}{\mu_{t}} - \frac{\theta}{2} \frac{1}{E_{t-1}[\mu_{t}]} - \frac{\theta}{2} \frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_{t}}{T_{Dt}}\right]} - (1 - \theta) \frac{1}{\mu_{t}} = 0$$

$$\frac{\theta}{\mu_{t}} - \frac{\theta}{2} \frac{1}{E_{t-1}[\mu_{t}]} - \frac{\theta}{2} \frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_{t}}{T_{Dt}}\right]} = 0$$

$$\frac{1}{\mu_{t}} = \frac{1}{2} \frac{1}{E_{t-1}[\mu_{t}]} + \frac{1}{2} \frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_{t}}{T_{Dt}}\right]}$$

$$\begin{split} W_{t}^{*} &= E_{t-1} \ln C_{tt}^{*} - E_{t-1} \kappa I_{t}^{*} \\ W_{t}^{*} &= E_{t-1} \Big[\ln \mu_{t} \Big] - E_{t-1} \Big[\ln P_{t}^{*} \Big] - E_{t-1} \kappa I_{t}^{*} \\ W_{t}^{*} &= E_{t-1} \Big[\ln \mu_{t}^{*} \Big] - E_{t-1} \Bigg[\ln \left(2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} E_{t-1} \left[\mu_{t}^{*} \right] \right)^{\theta/2} \left(\frac{\phi}{\phi-1} \frac{\kappa}{E_{t-1}} \left[\frac{\mu_{t}^{*}}{T_{Dt}^{*}} \right] \right)^{\theta/2} \left(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}} T_{Nt}^{*} \right)^{(1-\theta)} \right) \Bigg] \\ - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*} T_{D,t}} T_{Nt}^{*} \right] \right) \end{split}$$

This is directly analogous to home. Now W* include only μ_t^* and no μ_t . So directly analogous optimality condition:

$$\frac{1}{\mu_{t}^{*}} = \frac{1}{2} \frac{1}{E_{t-1} \left[\mu_{t}^{*}\right]} + \frac{1}{2} \frac{\frac{1}{T_{Dt}^{*}}}{E_{t-1} \left[\frac{\mu_{t}^{*}}{T_{Dt}^{*}}\right]}.$$

Appendix D: Derivation of Analytical Results, Home Dominant Currency Pricing

Home exporters face the PCP problem from above, with the implied price setting rule:

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \frac{\kappa \mu_t}{\alpha_D} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \right]},$$

which implies the export price:

$$P_{DHt}^* = \frac{1}{e_t} \frac{\phi}{\phi - 1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \frac{\kappa \mu_t}{\alpha_D} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right) \right]}$$

where the exchange rate still follows: $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$.

Foreign exporters follow the price setting rules derived for the LCP case above:

$$P_{Ft}^* = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} E_{t-1} \left[\mu_t^* \right]$$

$$P_{Ft} = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} \frac{E_{t-1} \left[\frac{\mu_t}{T_{Dt}} \right]}{E_{t-1} \left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}} \right]}.$$

These prices imply the home price index:

$$P_{t} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right]} \right)^{\theta/2} \left(T_{Dt} \frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} \frac{E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right]}{E_{t-1} \left[\frac{\alpha_{N} T_{Nt}}{\alpha_{N}} \right]} \right)^{\theta/2} \left(\frac{\kappa \mu_{t}}{\alpha_{N}} \right)^{(1-\theta)}$$

Note that this is a hybrid of cases above, since the home good prices are PCP, while home import prices are LCP.

The foreign price index is:

$$P_{t}^{*} = 2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi-1} \frac{\kappa}{\alpha_{D}^{*}} E_{t-1} \left[\mu_{t}^{*} \right] \right)^{\theta/2} \left(T_{Dt}^{*} \frac{\phi}{\phi-1} \frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right]}{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right]} \frac{\mu_{t}^{*} \alpha_{N} T_{N,t}}{\mu_{t} \alpha_{N}^{*}} \right)^{\theta/2} \left(\frac{\kappa \mu_{t}^{*}}{\alpha_{N}^{*}} \right)^{(1-\theta)}$$

This too is a hybrid case, since the foreign good prices are LCP, while foreign imports of home goods are PCP.

Home labor supply is the same as the PCP case:

$$\kappa E_{t-1}[l_t] = 1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} \right],$$

and foreign the same as the LCP case:

$$\kappa E_{t-1} [l_t] = 1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} \right].$$

Home welfare may be computed:

$$\begin{split} W_{t} &= E_{t-1} \Big[\ln \mu_{t} \Big] - \ln \left(2 \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \right) - \theta \ln \left(\frac{\phi}{\phi-1} \right) - \frac{\theta}{2} \ln \left(E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa \mu_{t}}{\alpha_{D}} \right] \right) \\ &+ \frac{\theta}{2} \ln \left(E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \right] \right) - \frac{\theta}{2} E_{t-1} \Big[\ln \left(T_{Dt} \right) - \ln \alpha_{D}^{*} \right] - \frac{\theta}{2} \ln \left(E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right] \right) + \frac{\theta}{2} \ln \left(E_{t-1} \left[\frac{\alpha_{N} T_{Nt}}{\alpha_{N}^{*} T_{Dt}} \right] \right) \\ &- \left(1 - \theta \right) E_{t-1} \left(\ln \kappa + \ln \mu_{t} - \ln \alpha_{N} \right) - 1 + \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}^{*}}{\alpha_{N} T_{N,t} T_{Dt}^{*}} \right] \end{split}$$

Note that the home policy variable, μ , interacts with both home and foreign tariffs, T_{Dt} and T_{Dt}^* , inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will involve both tariffs. Note also that the foreign policy variable, μ_t^* , does not appear in this welfare computation for home.

Foreign welfare is

$$W_{t}^{*} = E_{t-1} \Big[\ln \mu_{t}^{*} \Big] - E_{t-1} \Bigg[\ln \left(2^{\theta} \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_{D}^{*}} E_{t-1} \left[\mu_{t}^{*} \right] \right)^{\theta/2} \\ - \left[\left(\frac{1-\theta}{\theta} \right)^{\theta-1} + \left(\frac{1-\theta}{\theta} \right)^{\theta} \right] \left(\frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_{D}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \frac{\kappa}{\alpha_{D}} \frac{1}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{N,t}^{*}} \Big] - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right] - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right] \right) - \left(1 - \frac{1}{\phi} \frac{\theta}{2} E_{t-1} \left[1 + \frac{\alpha_{N}}{\alpha_{N}^{*}}$$

Note that the foreign policy variable, μ_t^* , does not interact with either home or foreign tariffs, T_{Dt} and T_{Dt}^* , inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will not involve either tariff.

The optimality conditions for both Nash and cooperative problems are

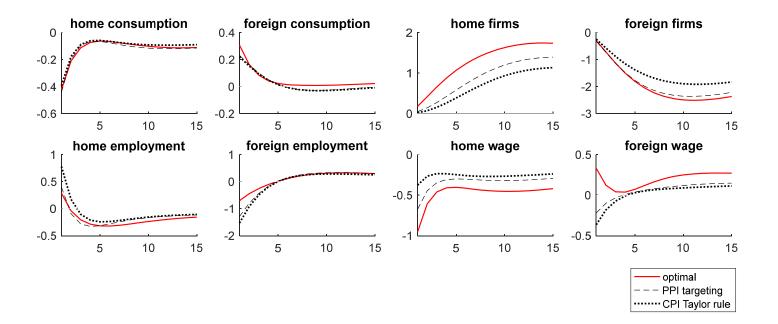
$$\mu_{t} = \frac{1}{2} \left[\frac{E_{t-1} \left[\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right) \mu_{t} \right]}{\left(1 + \frac{\alpha_{N}^{*}}{\alpha_{N}} \frac{1}{T_{N,t} T_{Dt}^{*}} \right)} + \frac{E_{t-1} \left[\frac{\mu_{t}}{T_{Dt}} \right]}{\frac{1}{T_{Dt}}} \right]$$

for home policy, and

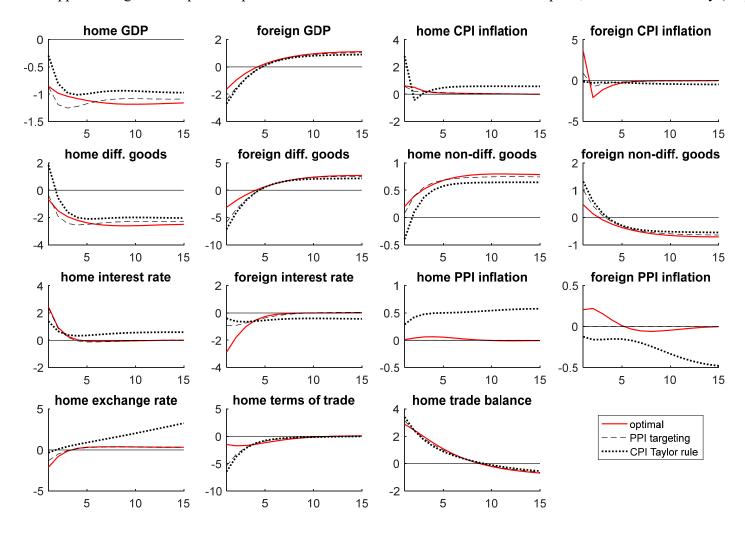
$$\mu_t^* = E_{t-1} \left[\mu_t^* \right]$$

for foreign.

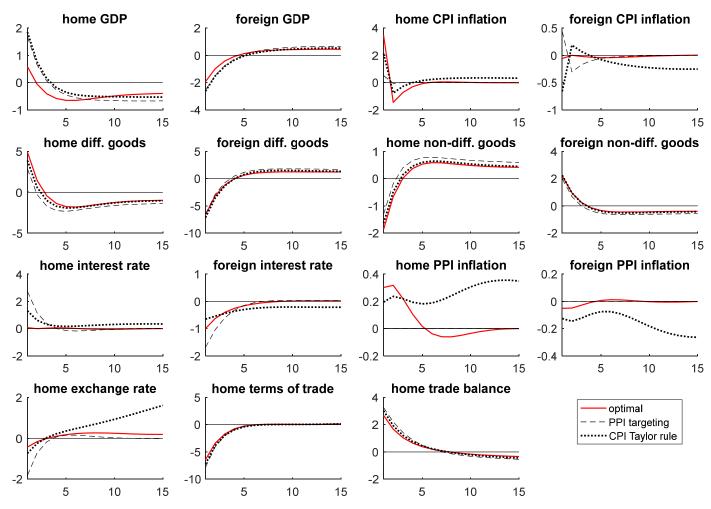
Appendix Figure 1. Additional Impulse responses for benchmark case: home tariff on differentiated imports



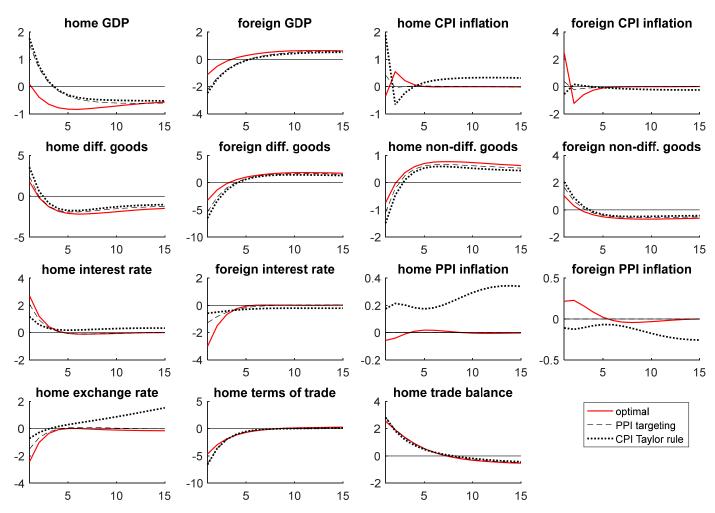
Appendix Figure 2. Impulse responses to a rise in home tariff on differentiated imports, lower trade elasticity (3.8)



Appendix Figure 3. Impulse responses to a rise in home tariff on differentiated imports; LCP price stickiness in both countries

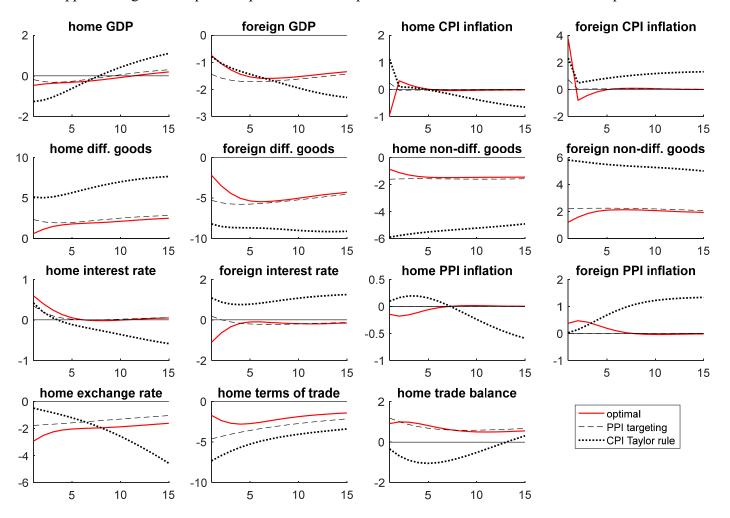


Appendix Figure 4. Impulse responses to a rise in home tariff on differentiated imports; foreign currency dominant

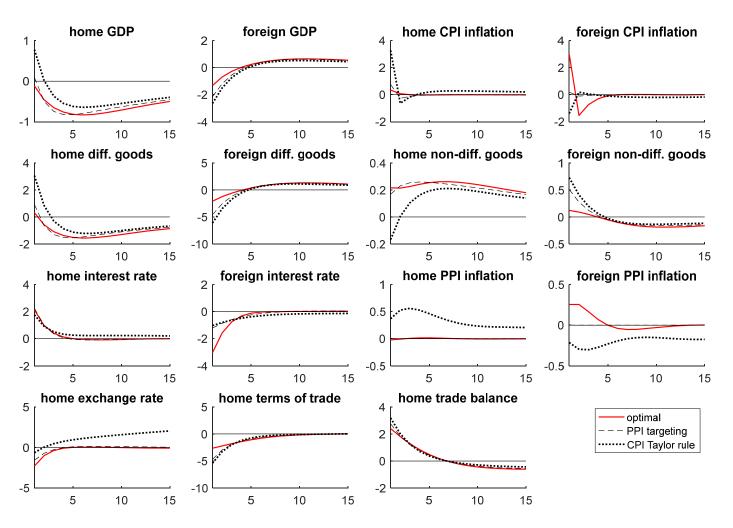


Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 5. Impulse responses to a more persistent tariff on home differentiated imports

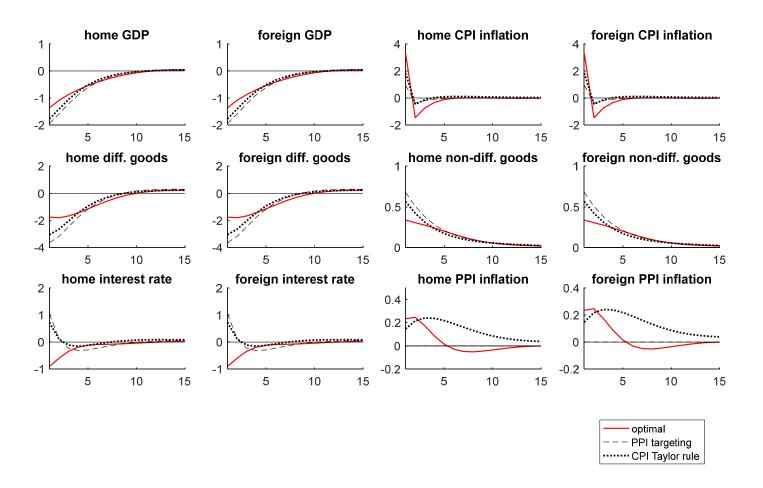


Appendix Figure 6. Impulse responses to home tariff on differentiated imports, nontraded non-diff. goods

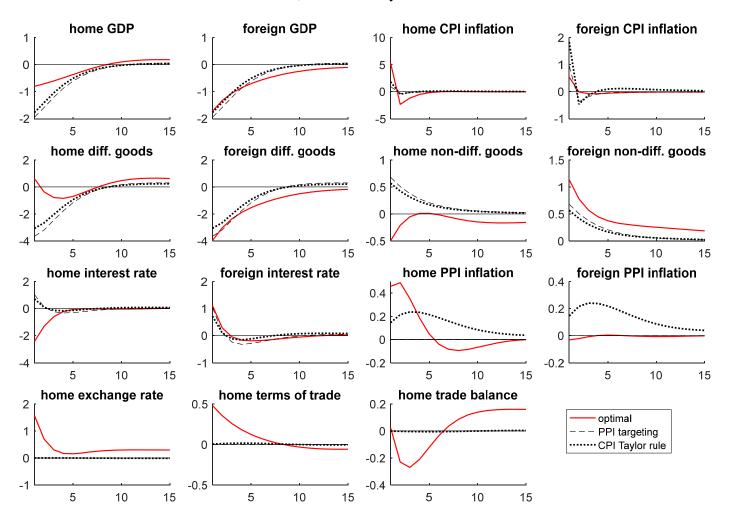


Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 7. Impulse responses to a symmetric tariff to differentiated imports in both countries



Appendix Figure 8. Impulse responses to a symmetric tariff to differentiated imports in both countries; home currency dominant



Appendix Figure 9. Impulse responses to a symmetric tariff to non-differentiated imports in both countries

