Exchange rates and monetary stabilization of tariff shocks

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January 28, 2026

Abstract
Home currency appreciation can neutralize the relative price distortions created by a tariff, moderating the rise in home import prices. A combination of domestic and foreign monetary policy appropriately managing the exchange rate can thus improve the trade-off between the objectives of supporting domestic demand and containing inflation. Using a New Keynesian two-country model, we analyze the role of the exchange rate in the optimal stabilization of unilateral home tariff shocks hitting, respectively, differentiated sticky-price goods and non-differentiated flexible price goods. In response to a tariff on the former, the cooperative Ramsey optimal monetary policy prescribes home appreciation, implemented mainly through a robust foreign monetary expansion. The monetary response in the home (tariff-imposing) country may be expansionary or contractionary, depending on trade elasticities, but it tends to be modest. If tariffs instead are imposed on flexible-price goods (commodities), the Ramsey optimal monetary response is the opposite, calling for a robust home monetary expansion containing home currency appreciation.

Keywords: macroeconomic effects of tariffs, optimal monetary policy, exchange rate, sectoral reallocation
JEL classification: F42, F44, E52

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We thank Javier Bianchi, Louphou Coulibaly, Charles Horioka, Timothy Kehoe, Jinill Kim, Jaewoo Lee, Tommaso Monacelli, and Christopher Waller for helpful comments, as well as participants of the 2026 American Economic Association meetings, 2025 Society for Economic Dynamics meetings in Copenhagen, the Recent Advances in Macroeconomics conference at Korea University, and the Trade Disputes and their Consequences on the Global Economy conference at Korea University Business School.

1. Introduction

As the world faces large and volatile U.S. tariffs, central banks have grappled with the question of how to design and implement effective stabilization policies. The question is complicated by the fact that the objectives of stabilizing inflation and stabilizing employment and output typically conflict in the presence of tariff shocks, as tariffs may simultaneously produce inflationary and recessionary pressures. We contribute to the growing literature on this question by bringing forward the role of the exchange rate as a channel through with policy can improve macroeconomic stabilization of tariffs. Given that tariffs fundamentally work through distorting international relative prices, monetary policy can in principle neutralize the rise in home import prices by engineering a home currency appreciation. However, such a measure would be in tension with the objective of supporting domestic demand (as emphasized by, e.g., Bianchi and Coulibaly, 2025, and Monacelli, 2025). We characterize the optimal trade-off between competing objectives in a global model, showing how cooperative policies efficiently redress the international and domestic relative price distortions, modulating the intensity of monetary expansion and/or contraction across the country that imposes tariffs, and the country that is targeted by it.

Specifically, this paper studies the Ramsey cooperative optimal monetary stabilization of tariff shocks, focusing on their implications for international relative prices and the currency. It does so relying on a two-country New Keynesian model with two traded sectors that differ in terms of market structure and price rigidity. We associate one sector with manufacturing and the other with commodities, both of which have been targets of recent tariffs. This difference across sectors matters for policy design. In our baseline specification with a high degree of exchange pass through on prices, home currency appreciation is always globally optimal in response to unilateral tariffs that raise the relative price of home imports of differentiated goods above border prices. Appreciation is driven by a robust monetary expansion in the foreign country (the one hit by the tariff) which, while optimally offsetting the fall in the external demand for foreign output, also redresses the relative price misalignment. In the home country, the optimal monetary response may or may not bring output above the natural rate, depending on trade elasticities and the persistence of the tariff shock, but this home response remains modest in either case. Most crucially, however, it remains contractionary relative to the foreign response.

Compared to the effects of standard Taylor rules targeting CPI or PPI inflation, this optimal combination of (cooperative) policies raises welfare in both countries.

In contrast, when tariffs fall on the home non-differentiated sector producing goods with flexible prices, home currency appreciation can no longer offset the misalignment in the relative price of the goods targeted by the tariff. Rather, an appreciation induced by the tariff would have a negative spillover on the other sector, creating misalignment in the relative price of the tariff-free, differentiated goods—as these would become inefficiently expensive in the world market. The Ramsey policy in this case adjusts the exchange rate in the opposite direction, to moderate the currency appreciation, as this serves to offset the relative price distortion in the sticky-price sector.

This distinct and key role of exchange rate misalignment in international markets cannot be fully appreciated using small open-economy, one-sector models, which by construction provide a limited account of the role played by relative prices in shaping stabilization policies, and which miss the effects of monetary adjustment abroad. Indeed, leading contributions in the literature, such as Bianchi and Coulibaly (2025) and Monacelli (2025), find that, from a small open-economy perspective, the dominant effect of unilateral tariffs is inefficient output distortions, which motivates a home expansion; optimal policy in both papers implies home currency depreciation in contrast with the home appreciation of our optimal policy. A distinct result in our contribution is that, in a global model, the optimal foreign expansion that drives home appreciation contributes to redress the relative price misalignment and, by doing so, also contributes to support global demand. The home optimal policy can then focus on keeping own output closer to the natural rate. A second distinct contribution from our analysis consists of clarifying that, when home imposes a unilateral tariff, the social desirability of home currency appreciation depends on the type of goods that are targeted by the trade measures. If these are goods traded in perfectly competitive markets, hence their prices are relatively flexible, letting the home currency appreciate is useless for correcting the effects of the tariff on their demand. On the contrary, it would have negative spillovers reducing the demand for the goods produced in the sector not targeted by the tariff.

To build intuition, after introducing the model we study a stylized tractable version, in which we assume unit trade elasticity in the differentiated sector and log-linear preferences, but retain the two-sector structure of the economy. Specifically, we include a differentiated and a homogenous world good sector. Analytical results provide fundamental insight on the mechanism by which the optimal exchange rate adjustment and stabilization policy in response to tariffs varies with the targeted sector. In addition, our stylized environment is especially useful in clarifying how and why the optimal policy aims to stabilize uncertainty arising from repeated rounds of volatile tariffs. As shown by Corsetti and Pesenti 2009, with nominal rigidities, uncertainty about a home tariff, i.e. uncertainty about home demand for foreign exports, leads foreign firms to preset their supply price at inefficiently high levels, de facto exacerbating monopolistic distortions in the global economy, and thus lowering welfare. By stabilizing demand through the relative price offset of the exchange rate, monetary authorities can moderate this adverse effect of the tariff. In doing so, they trade off the benefit of stabilizing export demand with the side effects of monetary policy on domestic demand. Because of this trade-off, the optimal monetary policy only ensures a partial exchange rate offset of the tariff.

The conclusions from the stylized model are corroborated by quantitative analysis in the full model, in which we assess the macroeconomic transmission of tariffs and welfare contrasting suboptimal policy rules to the optimal policy. We also compare our baseline with alternative specifications of price stickiness in which exchange rates do not pass through to consumer prices, such as local currency price stickiness or dominant currency price stickiness. The disconnect of the exchange rate from allocative prices of imports (at least in the short run) reduces the ability of monetary policy to redress relative prices. The optimal policy calls for buffering the effect of tariffs on import demands by supporting overall domestic demand with a home monetary expansion. In addition, to study the effects of tariffs imposed on different sectors, we also consider scenarios of tariff retaliation, at sectoral level or in the form of trade wars.

Related Literature. In recent years, the literature studying the optimal monetary policy response to tariffs has been flourishing. Many contributions take a small open economy perspective. A leading instances is Bianchi and Coulibaly (2025), who find an expansionary monetary policy and currency depreciation to be unambiguously optimal for an open economy that (unexpectedly) imposes import tariffs. This result reflects a diminished role given to the exchange rate as a tool to redress relative price distortions in their benchmark small open economy model specification, where the relative price of imports to home goods, and also the monetary stance abroad, are taken as exogenous. Given prices, activity and policy in the rest of the world, domestic monetary policy naturally refocuses on the inefficient fall in the quantity of imports following a tariff, and the implied wedge in labor. Redressing these inefficiencies requires an expansion raising overall consumption. From the perspective of an open and large economy, however, relative price distortions and the exchange rate channel become more prominent. In a cooperative setting, both the domestic and the foreign monetary policy contribute to engineer a home currency appreciation—requiring the home country imposing the tariff to be *relatively* less expansionary than abroad.

Another instance of closely related work is Monacelli (2025), which studies implementable monetary policy rules as well as the constrained optimal policy response to tariffs in a small open economy. Its conclusion regarding optimal policy broadly agrees with Bianchi-Coulibaly (2025), inasmuch as it finds monetary policy of the small open economy should be expansionary and promote currency depreciation, in service to the goal of offsetting output contraction. As noted above, a small open economy brings to light key insights on the effects of tariff in the form of an inefficient contraction of domestic demand, which are also present in a global model, where they are nonetheless traded off with relative price considerations.

Most of the early contributions to the literature studying the macroeconomic effects of tariffs rely on either real models or monetary models with a stylized monetary side.[1] To our knowledge, our previous work Bergin and Corsetti (2023) was first in placing the monetary dimensions of a trade war and the design of an efficient stabilization policy centerstage in the analysis. In our early paper, we studied the optimal monetary policy response to tariff shocks comparing Ramsey policies with monetary rules targeting PPI, CPI and money growth, as well as detailing the nature of tariff shocks in relation to markup and productivity shocks. Our current work differs in two key

[1] Some leading contributions study macro dynamics in the context of standard monetary policy rules, e.g., Barattieri et al. (2021), Erceg et al. (2018), Jeanne and Son (2024), Kalemli-Özcan et al. (2025), and Auclert, et al. (2025). Caldara et al. (2020) investigates the macroeconomic implications of trade policy uncertainty. Linde and Pescatori (2019) reconsiders the Lerner symmetry. For recent studies of the macroeconomic effects of tariffs abstracting from monetary policy see Baqaee and Malmberg (2025), and Costinot and Werning (2025).

respects. First, while the earlier paper focused on the case of a symmetric trade war, finding optimal policy of monetary expansion, the present paper focuses on the case of a unilateral tariff without retaliation, hence weakening the case for home monetary expansion and instead favoring exchange rate appreciation as a policy response. Historical experience with tariff shocks subsequent to this paper's publication has shown the unilateral tariff case to be the more relevant scenario. The second key difference is that our current work studies the case of a multi-sector environment where tariffs affect sectoral allocation, a feature of tariffs thought essential within the trade literature that has a longer history than macro of studying tariffs.

In a global model, trade policy is endogenized by Auray et al. (2024), who study how alternative monetary policies affect strategic tariff policy, turning around the question underlying our research, the choice of optimal monetary policy in the face of an exogenous tariff policy. More recently Auray, et al. (2025) also considers the optimal mix of CPI and PPI inflation targeting in the class of standard monetary policy rules.

A paper that, in line with our contribution, stresses the importance of using global multi-sector models is Kalemli-Özcan, Soylu, and Yildirim (2025), who embed a multi-country global network model in a New Keynesian framework. The point of contact is their simplified two-country, five-equation version of the model, with sectors distinguished between flex-price and sticky price—essentially corresponding to our model with roundabout production calibrated to the US data. Their rich dynamics clearly show the importance of moving away from the small-open economy model to capture the macroeconomic effects of tariffs, emphasizing the geographical dimensions of their impact in addition to their impact on consumers, producers and sectors. Different from their work, our smaller model allows us to engage in a close-up analysis of the optimal monetary policy trade-offs between currency appreciation, inflation and activity—specifically focusing on the role of the exchange rate in redressing relative price distortions.

A recent chapter of the literature maps the tariff shocks in the closed-economy monetary stabilization theory. Werning, et al. (2025) for instance, emphasizes that the optimal response to tariff shocks is akin to the optimal monetary stabilization of cost-push shocks—an important point that indeed applies generally to a number of shocks in an open economy (see Corsetti, et al., 2010).

The paper proceeds as follows. The next section describes the model environment and calibration. Section 3 develops intuition from analytical solution of a simplified model environment. Sections 4 use impulse responses to a calibrated version of the full model to illustrate results for tariffs to the differentiated goods sector. Sections 5 and 6 extend the simulations to cover the case of tariffs to the other, non-differentiated sector, and the case of asymmetric tariff wars. Section 7 draws implications for welfare, and section 8 concludes.

2. Model

The model features two countries, home and foreign, each of which produces two types of tradable goods.[2] The first type of good comes in differentiated varieties produced under monopolistic competition, where firm entry requires a sunk investment, and prices are subject to nominal rigidities. The second type of good is modeled according to the standard specification in real business cycle models, assuming perfect substitutability among producers within a country, but imperfect substitutability across countries. In the text to follow, we present the households' and firms' problems as well as the monetary and fiscal policy rules from the vantage point of the home economy, with the understanding that similar expressions and considerations apply to the foreign economy—foreign variables are denoted with a "*".

2.1. Goods consumption demand and price indexes

In the benchmark version of the model, households consume goods produced in both sectors, of both domestic and foreign origin. The differentiated goods come in many varieties, produced by a time-varying number of monopolistically competitive firms in the home and foreign country, n_t and n_t^* respectively, each producing a single variety. Each variety is an imperfect substitute for any other variety in this sector, either of home or

[2] The macroeconomic model of tariffs builds upon the framework of Bergin and Corsetti (2023), augmented with two traded sectors. The modeling of multiple traded sectors utilizes the framework in Bergin and Corsetti (2020).

foreign origin, with elasticity ϕ. The non-differentiated goods come in a home and foreign version, which are imperfect substitutes with elasticity η. However, within each country, all goods in this sector are perfectly substitutable with each other, and are produced in a perfectly competitive environment. The differentiated sector will be denoted with a *D* and the non-differentiated sector with *N*. We broadly associate the former sector with manufacturing and the latter sector with commodities such as steel, aluminum, or agriculture.

Tariffs are specified as ad-valorem duties imposed at the dock, and directly raise the prices faced by consumers. Tariff revenue is collected by the government and rebated to domestic consumers, hence it cancels out in the consolidated national budget constraint.

The overall consumption index is specified as follows:

$$C_t \equiv \left(\theta^{\frac{1}{\xi}} C_{D,t}^{\frac{\xi-1}{\xi}} + (1-\theta)^{\frac{1}{\xi}} C_{N,t}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}},$$

where

$$C_{D,t} \equiv \left(\int_0^{n_t} c_t(h)^{\frac{\phi-1}{\phi}} dh + \int_0^{n_t^*} c_t(f)^{\frac{\phi-1}{\phi}} df \right)^{\frac{\phi}{\phi-1}}$$

is the index over the endogenous number of home and foreign varieties of the differentiated differentiated good, $c_t(h)$ and $c_t(f)$, and

$$C_{N,t} \equiv \left(\nu^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

is the index over goods differentiated only by country of origin, $C_{H,t}$ and $C_{F,t}$ with $\nu \in [0,1]$ accounting for the weight on domestic goods. The corresponding welfare-based consumption price index is

$$P_t = \left(\theta P_{D,t}{}^{1-\xi} + (1-\theta)\left(P_{N,t}\right)^{1-\xi} \right)^{\frac{1}{1-\xi}}, \tag{1}$$

where

$$P_{D,t} = \left(n_t p_t(h)^{1-\phi} + n_t^* \left(p_t(f) T_{D,t} \right)^{1-\phi} \right)^{\frac{1}{1-\phi}} \tag{2}$$

is the index over the prices of all varieties of home and foreign differentiated goods, $p_t(h)$ and $p_t(f)$, and

$$P_{N,t} = \left(\nu P_{H,t}{}^{1-\eta} + (1-\nu)\left(P_{F,t}T_{N,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3}$$

is the index over the prices of home and foreign non-differentiated goods. In these indexes, $T_{D,t}$ represents the quantity of 1 plus the ad valorem tariff rate imposed by the home country on imports of foreign differentiated goods, and $T_{N,t}$ represents the quantity of 1 plus the ad-valorem tariff rate imposed by the home country on imports of foreign non-differentiated goods. In reporting results, we will distinguish between the "ex-tariff" price determined by an exporter, $p_t(f)$, and the "tariff-inclusive" price, $p_t(f)T_{D,t}$, paid by an importer.

The relative demand functions for domestic residents implied from our specification of preferences are listed below:

$$C_{D,t} = \theta\left(P_{D,t} / P_t\right)^{-\xi} C_t \tag{4}$$

$$C_{N,t} = (1-\theta)\left(P_{N,t} / P_t\right)^{-\xi} C_t \tag{5}$$

$$c_t(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} C_{D,t} \tag{6}$$

$$c_t(f) = \left(\frac{p_t(h)}{e_t p_t^*(f) T_{D,t}}\right)^{\phi} c_t(h) \tag{7}$$

$$C_{H,t} = \nu\left(P_{H,t} / P_{N,t}\right)^{-\eta} C_{N,t} \tag{8}$$

$$C_{F,t} = (1-\nu)\left(P_{F,t}T_{N,t} / P_{N,t}\right)^{-\eta} C_{N,t}\,. \tag{9}$$

Note that demand functions for imports (Eqs. (7) and (9)) depend upon the tariff-inclusive price.

2.2 Home households' problem

The representative home household derives utility from consumption (C_t), and from holding real money balances (M_t/P_t); it suffers disutility from labor (l_t). The household budget consists of labor income from working at the nominal wage rate W_t; profits rebated from home firms denoted with (Π_t) in real terms and defined below, as well as interest income on bonds in home currency ($i_{t-1}B_{H,t-1}$) and foreign currency ($i_{t-1}^*B_{F,t-1}$), where e_t is the nominal exchange rate in units of home currency per foreign. Income is net of lump-sum transfers (negative taxes T_t), through which the government rebates seigniorage and tariff revenue to households (who do not internalize the effects of their consumption and money holding decisions on government rebates).

Household optimization for the home country may be written:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, l_t, \frac{M_t}{P_t}\right),$$

where utility is defined by

$$U_t = \frac{1}{1-\sigma} C_t^{1-\sigma} + \ln \frac{M_t}{P_t} - \frac{1}{1+\psi} l_t^{1+\psi} ,$$

subject to the budget constraint:

$$P_t C_t + \left(M_t - M_{t-1}\right) + \left(B_{Ht} - B_{Ht-1}\right) + e_t\left(B_{Ft} - B_{Ft-1}\right) = W_t l_t + \Pi_t + i_{t-1} B_{Ht-1} + i_{t-1}^* B_{Ft-1} - P_t AC_{Bt} - T_t .$$

In the utility function, the parameter σ denotes risk aversion and ψ is the inverse of the Frisch elasticity. The constraint includes a small cost to holding foreign bonds $AC_{Bt} = \frac{\psi_B \left(e_t B_{Ft}\right)^2}{2 P_t p_{Ht} y_{Ht}}$, scaled by ψ_B, which is a common device to assure long run stationarity in the net foreign asset position and resolve indeterminacy in the composition of the home bond portfolio. The bond adjustment cost is a composite of goods that mirrors the consumption index, with analogous demand conditions to Eqs. (4)-(9).

Defining $\mu_t = P_t C_t^{\sigma}$, household optimization implies an intertemporal Euler equation:

$$\frac{1}{\mu_t} = \beta\left(1+i_t\right) E_t\left[\frac{1}{\mu_{t+1}}\right] \tag{10}$$

a labor supply condition:

$$W_t = l_t^{\psi} \mu_t \tag{11}$$

a money demand condition:

$$M_t = \mu_t\left(\frac{1+i_t}{i_t}\right), \tag{12}$$

and a home interest rate parity condition:

$$\mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}} \frac{e_{t+1}}{e_t}\left(1+i_t^*\right)\left(1+\psi_B\left(\frac{e_t B_{ft}}{p_{Ht} y_{Ht}}\right)\right)\right] = \mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}}\left(1+i_t\right)\right]. \tag{13}$$

The problem and first order conditions for the foreign household are analogous.

2.3 Home firm problem and entry condition in the differentiated goods sector

In the manufacturing sector, the production of each differentiated variety follows

$$y_t(h) = \alpha_D \left[G_t(h)\right]^{\zeta} \left[l_t(h)\right]^{1-\zeta}, \tag{14}$$

where α_D is productivity specific to the production of differentiated goods but common to all firms within that sector, $l_t(h)$ is the labor employed by firm h, and $G_t(h)$ is a composite of differentiated goods used by firm h as an intermediate input. $G_t(h)$ is specified as an index of home and foreign differentiated varieties that mirrors the consumption index specific to differentiated goods ($C_{D,t}$). If we sum across firms, $G_t = n_t G_t(h)$ represents economy-wide demand for differentiated goods as intermediate inputs. Given that the index is the same as for consumption, this implies demands for differentiated goods varieties, $d_{G,t}(h)$ and $d_{G,t}(f)$, analogous to Eqs. (6)–(7).[3]

Differentiated goods firms set prices $p_t(h)$ subject to an adjustment cost:

$$AC_{P,t}(h) = \frac{\psi_P}{2}\left(\frac{p_t(h)}{p_{t-1}(h)} - 1\right)^2 \frac{p_t(h) y_t(h)}{P_t}, \tag{15}$$

where ψ_P is a calibrated parameter governing the degree of price stickiness. For the sake of tractability, we follow Bilbiie et al. (2008) in assuming that new entrants inherit from the price history of incumbents the same price adjustment cost, and so make the same price setting decision.[4]

There is free entry in the sector, but, once active, firms are subject to an exogenous death shock. All differentiated goods producers operating at any given time face the same exogenous probability of exit δ, so that a fraction δ of them exogenously stop operating each period. The number of firms active in the differentiated sector, n_t, at the beginning of each period evolves according to:

$$n_{t+1} = (1-\delta)(n_t + ne_t), \tag{16}$$

where ne_t denotes new entrants.

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[3] See Appendix A for the demand equations not listed here.

[4] The price index for adjustment cost is identical to the overall consumption price index, implying demands analogous to those for consumption in Eqs. (4)-(9). See Appendix A for the demand equations not listed here.

To set up a firm, managers incur a one-time sunk cost, K_t, and production starts with a one-period lag. This cost is not constant but varies reflecting an entry congestion externality, represented as an adjustment cost that is a function of the number of new firms:

$$K_t = \left(\frac{ne_t}{ne_{t-1}}\right)^{\lambda} \overline{K}, \tag{17}$$

where $\overline{K}$ indicates the steady state level of entry cost, and the parameter λ indicates how much the entry cost rises with an increase in entry activity. The congestion externality plays a similar role as the adjustment cost for capital standard in business cycle models, which moderates the response of investment to match dynamics in data. In a similar vein, we calibrate the adjustment cost parameter, λ, to match data on the dynamics of new firm entry.[5] The demands for varieties for use as entry investment, $d_{K,t}(h)$ and $d_{K,t}(f)$, are determined analogously to demands for consumption of differentiated goods.

Bringing all these elements together, the total demand facing a domestic differentiated goods firm:

$$d_t(h) = c_t(h) + d_{G,t}(h) + d_{K,t}(h) + d_{AC,P,t}(h) + d_{AC,B,t}(h) \tag{18}$$

includes the demand for consumption ($c_t(h)$) by households, and the demand by firms for intermediate inputs ($d_{G,t}(h)$), investment (the sunk entry costs) ($d_{K,t}(h)$), and goods absorbed as adjustment costs for prices ($d_{AC,P,t}(h)$) and bonds holding costs ($d_{AC,B,t}(h)$). There is an analogous demand from abroad $d_t^*(h)$. We assume iceberg trade costs τ_D for exports, so that market clearing for a firm's variety is:

$$y_t(h) = d_t(h) + (1+\tau_D) d_t^*(h), \tag{19}$$

Firm profits are computed as:

$$\pi_t(h) = p_t(h) d_t(h) + e_t p_t^*(h) d_t^*(h) - mc_t y_t(h) - P_t AC_{p,t}(h). \tag{20}$$

where $mc_t = \zeta^{-\zeta}(1-\zeta)^{\zeta-1} P_{D,t}{}^{\zeta} W_t^{1-\zeta} / \alpha_D$ is marginal cost.

Thus the value function of firms that enter the market in period t may be represented as the discounted sum of profits of domestic sales and export sales:

[5] The value of steady state entry cost $\overline{K}$ has no effect on the dynamics of the model, and so will be normalized to unity.

$$v_t(h) = E_t\left\{\sum_{s=0}^{\infty}\left(\beta(1-\delta)\right)^s \frac{\mu_{t+s}}{\mu_t}\pi_{t+s}(h)\right\},$$

where we assume firms use the discount factor of the representative household, who owns the firm, to value future profits. With free entry, new producers will invest until the point that a firm's value equals the entry sunk cost:

$$v_t(h) = P_{D,t}K_t\ . \tag{21}$$

By solving for cost minimization we can express the relative demand for labor and intermediates as a function of their relative costs:

$$\frac{P_{D,t}G_t(h)}{W_t l_t(h)} = \frac{\zeta}{1-\zeta}\ . \tag{22}$$

Managers optimally set prices by maximizing the firm value subject to all the constraints specified above. The price setting equation:

$$\begin{aligned} p_t(h) = &\frac{\phi}{\phi-1}mc_t + \frac{\psi_P}{2}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)^2 p_t(h) - \psi_P\frac{1}{\phi-1}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)\frac{p_t(h)^2}{p_{t-1}(h)} \\ &+\frac{\psi_P}{\phi-1}E_t\left[\beta\frac{\Omega_{t+1}}{\Omega_t}\left(\frac{p_{t+1}(h)}{p_t(h)}-1\right)\frac{p_{t+1}(h)^2}{p_t(h)}\right] \end{aligned} \tag{23}$$

expresses the optimal pricing as a function of the stochastically discounted demand faced by producers of domestic differentiated goods,

$$\begin{aligned} \Omega_t = &\left[\left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi}\left(C_{D,t} + G_t + ne_t(1-\theta_K)K_t + AC_{P,D,t} + AC_{B,D,t}\right)\right. \\ &\left.+\left(\frac{(1+\tau_D)T^*_{D,t}p_t(h)}{e_t P^*_{D,t}}\right)^{-\phi}(1+\tau_D)\left(C^*_{D,t} + G^*_t + ne^*_t(1-\theta_K)K^*_t + AC^*_{P,D,t} + AC^*_{B,D,t}\right)\right] \Big/ \mu_t \end{aligned}$$

This sums the demand arising from consumption, use as intermediate inputs, sunk entry cost, price adjustment costs, and bond holding costs.

Under the assumption that firms preset prices in own currency, i.e., assuming producer currency pricing, the good price in foreign currency moves one-to-one with the exchange rate, net of trade costs:

$$p^*_t(h) = (1+\tau_D)\,p_t(h)/e_t\,, \tag{24}$$

where recall the nominal exchange rate, *e*, measures home currency units per foreign.

Note that, since households own firms, they receive firm profits but also finance the creation of new firms. In the household budget, the net income from firms may be written:

$$\Pi_t = n_t \pi_t(h) - ne_t v_t(h).$$

In reporting our quantitative results, we will refer to the overall home gross production of differentiated goods defined as: $y_{D,t} = n_t y_t(h)$.

2.4 Home firm problem in the undifferentiated goods sector

In the second sector, producing the home non-differentiated good, firms are perfectly competitive. The production function is linear in labor:

$$y_{H,t} = \alpha_N l_{H,t}, \tag{25}$$

where α_N is productivity specific to this country and sector. It follows that the price of the homogeneous goods in the home market is equal to marginal costs:

$$p_{H,t} = W_t / \alpha_N. \tag{26}$$

An iceberg trade cost specific to the non-differentiated sector implies prices of the home good abroad are

$$p^*_{H,t} = p_{H,t}(1+\tau_N)/e_t. \tag{27}$$

Analogous conditions apply to the foreign non-differentiated sector.

2.5 Monetary and fiscal policy

We solve the model comparing three monetary regimes. The first one supports the cooperative Ramsey allocation, computed by positing that the monetary authority maximizes aggregate welfare of both countries:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}\left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\psi} l_t^{1+\psi} \right) + \frac{1}{2}\left(\frac{1}{1-\sigma} C_t^{*1-\sigma} - \frac{1}{1+\psi} l_t^{*1+\psi} \right) \right)$$

under the constraints of the economy defined above. As common in the literature, we write the Ramsey problem by introducing additional co-state variables, which track the value of the planner committing to a policy plan.

The other two posit either a monetary policy rule that perfectly target producer price inflation:

$$\frac{p_t(h)}{p_{t-1}(h)} = 1, \tag{28}$$

or a standard Taylor rule of the form

$$1 + i_t = (1 + i_{t-1})^{\gamma_i} \left[(1 + \bar{i}) \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_P} \left(\frac{Y_t}{\bar{Y}} \right)^{\gamma_Y} \right]^{1-\gamma_i}, \tag{29}$$

where terms with overbars are steady-state values. In this rule, inflation is defined in terms of the CPI, while Y_t is a measure of GDP defined net of intermediates as:[6]

$$Y_t = \left(n_{t-1} p_t(h) y_t(h) - P_{D,t} G_t + p_{H,t} y_{H,t} \right) / P_t . \tag{30}$$

Across these different specifications of monetary policy, we will abstract from public consumption expenditure. The government rebates seigniorage and any tax revenue (including tariff duties on either sector) to consumers:

$$T_t = \left(M_{t-1} - M_t \right) + \left(T_{D,t} - 1 \right) n^*_{t-1} d_t(f) + \left(T_{N,t} - 1 \right) \left(C_{F,t} + AC_{P,F,t} + AC_{B,F,t} \right). \tag{31}$$

2.6 Shocks process and equilibrium definition

Shocks are assumed to follow joint log normal distributions:

$$\begin{bmatrix} \log T_{D,t} - \log \overline{T_D} \\ \log T^*_{D,t} - \log \overline{T^*_D} \\ \log T_{N,t} - \log \overline{T_N} \\ \log T^*_{N,t} - \log \overline{T^*_N} \end{bmatrix} = \rho_T \begin{bmatrix} \log T_{D,t-1} - \log \overline{T_D} \\ \log T^*_{D,t-1} - \log \overline{T^*_D} \\ \log T_{N,t-1} - \log \overline{T_N} \\ \log T^*_{N,t-1} - \log \overline{T^*_N} \end{bmatrix} + \varepsilon_{Tt}$$

with autoregressive coefficient matrix ρ_T, and the covariance matrix $E\left[\varepsilon_{Tt} \varepsilon'_{Tt} \right]$.

To conserve space, the market clearing conditions to close the model are reported in section 2 of the appendix. A competitive equilibrium in our world economy is defined along the usual lines, as a set of processes for quantities and prices in the home and foreign country satisfying: (i) the household and firms optimality conditions; (ii) the market clearing conditions for each good and asset, including money; (iii) the resource constraints—whose specification can be easily derived from the above and is omitted to save space.

[6] For computational simplicity, the Taylor rule is specified in terms of deviations of GDP from its steady state value, which is distinct from the output gap.

2.7 Welfare computation

We report the effects on welfare of a given policy regime configuration relative to the Ramsey allocation. The change in welfare customarily is computed in terms of consumption units that households would be willing to forgo to continue under the Ramsey policy regime. We posit identical initial conditions across different monetary policy regimes using the Ramsey allocation, and we include transition dynamics in the computation to avoid spurious welfare reversals.[7]

2.8 Calibration

Where possible, parameter values are taken from standard values in the literature. Risk aversion is set at $\sigma = 2$; labor supply elasticity is set at $1/\psi = 1.9$ following Hall (2009). Consistent with a quarterly frequency, $\beta = 0.99$.

The price stickiness parameter is set at ψ_p =49, a value which implies in simulations of a productivity shock that approximately half the firms resetting price during the first year.[8] The firm death rate is set at δ =0.025. The mean sunk cost of entry is normalized to the value $\overline{K}$=1, and the adjustment cost parameter for new firm entry, λ, is taken from Bergin and Corsetti (2020). The share of intermediates in differentiated goods production follows Bergin and Corsetti (2020) in setting ζ =1/3.

To choose parameters for the differentiated and non-differentiated sectors we draw on Rauch (1999). We choose θ so that differentiated goods represent 55 percent of U.S. trade in value: ($\theta = 0.45$). We assume the two countries are of equal size with no exogenous home bias, $\nu = 0.5$, but allow trade costs to determine home bias ratios. The Broda and Weinstein (2006) estimate of the elasticity of substitution between differentiated goods varieties is ϕ=5.2 (the sample period is 1972-1988), though alternative values will

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[7] We adopt the methodology created by Giovanni Lombardo and used in Coenen et al. (2010), available from https://www.dropbox.com/s/q0e9i0fw6uziz8b/OPDSGE.zip?dl=0.

[8] As is well understood, a log-linearized Calvo price-setting model implies a stochastic difference equation for inflation of the form $\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$, where *mc* is the firm's real marginal cost of production, and where $\lambda = (1-q)(1-\beta q)/q$, with q is the constant probability that a firm must keep its price unchanged in any given period. The Rotemberg adjustment cost model used here gives a similar log-linearized difference equation for inflation, but with $\lambda = (\phi - 1)/\kappa$. Under our parameterization, a Calvo probability of $q = 0.5$ implies an adjustment cost parameter of ψ_P = 49.

be considered in sensitivity analysis. We adopt the same elasticity between home and foreign goods in the non-differentiated sector. We initially adopt a Cobb-Douglas specification for the aggregator function combining the two sectors ($\xi \to 1$), but sensitivity analysis will report results for alternative calibrations of this parameter. To set trade costs, we calibrate τ_D so that exports represent 26% of GDP, as is the average in World Bank national accounts data for OECD countries from 2000-2017.[9] This requires a value of $\tau_D = \tau_N$ =0.44.[10]

Calibration of policy parameters for the historical monetary policy Taylor rule are taken from Coenen, et al. (2010): γ_i=0.7, γ_p =1.7, γ_Y=0.1.

The process for tariff shocks is calibrated with a mean value of 1.02 (2 percentage point mean tariff rate) to match U.S. tariff data in Barattieri et al. (2021). The autoregressive parameter is set to 0.56, estimated from Barattieri et al. (2021). The standard deviation of the shock is 0.08—raising tariff rate from a steady state of 2% to 10%.

3. Insight from an analytically tractable version of the model

A simplified version of the model can provide insight into the economics underlying the quantitative results in the main body of the paper, regarding both the transmission of tariff shocks and the welfare implications of tariff uncertainty.

3.1 Simplified model environment

To derive analytical solution, we adopt the following simplifying assumptions. Differentiated goods, that come in distinct country-specific types, are bundled in final consumption with a Cobb-Douglas aggregator. For the non-differentiated sector, instead, we let the elasticity of substitution between home and foreign varieties approach infinity, de facto assuming a world homogenous good (a common assumption in the trade literature). Importantly, we restrict productivity levels and tariffs such that the home country is both a producer and an importer of the non-differentiated good.[11] This

[9] See https://data.worldbank.org/indicator/NE.EXP.GNFS.ZS?locations=OE.
[10] To coincide with standard accounting definitions, differentiated goods used as intermediates are included in the measure of exports, and excluded in the measure of GDP, as is appropriate.
[11] This condition is $1 < \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta} - 1$. On one hand, to keep exporting to the home country, foreign firms must enjoy productivity levels high enough to keep an absolute advantage over home in producing non-differentiated goods after accounting for tariffs. On the other hand, to make sure that domestic firms are not

assumption arguably suits an interpretation of the non-differentiated sector encompassing a number of goods such as steel; U.S. tariff policy may promote domestic production of this sector, but an underlying foreign comparative advantage means domestic production will not likely fully replace foreign sources. Moreover, as in Corsetti and Pesenti (2005), we posit that household utility is log in consumption and linear in labor $U_t = \ln C_t - \kappa l_t$, and define a variable, $\mu_t = P_t C_t$, which summarizes the effect of monetary policy stance on aggregate nominal spending. Utility maximization implies the standard labor supply condition $W_t = \kappa \mu_t$. Finally, in the differentiated good sector, production employs labor only, $y_t(h) = \alpha_t l_t(h)$, and prices are preset one period in advance, in producer currency units.

It is worth noting that, given our simplifying assumptions of a Cobb Douglas consumption aggregator:

$$C_t \equiv \left(C_{DHt}^{1/2} C_{DFt}^{1/2}\right)^{\theta} C_{Nt}^{1-\theta},$$

the price index is $P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right] P_{DHt}{}^{\theta/2}\left(T_{Dt} e_t P_{DFt}^{*}\right)^{\theta/2} P_{Nt}{}^{1-\theta}$, and the demand condition for home imports of foreign differentiated goods (7) becomes:

$$c_t(f) = \frac{p_t(h)}{e_t p_t^{*}(f) T_{D,t}} c_t(h)$$

where $p_t^{*}(f)$ is the foreign currency price of foreign exports—fixed during the period under producer currency pricing. This equation highlights that, as both $p_t(h)$ and $p_t^{*}(f)$ are preset, the effect on relative prices of a rise in the tariff, T_D can in principle be offset by an equivalent percent fall in the nominal exchange rate, *e*. If a tariff distorts prices by making imports more expensive for home consumers, a home currency appreciation (foreign deprecation) can redress the relative price distortion by reducing the border price in proportion (given nominal rigidities in the producer currency). We will see below that perfect offset is not optimal.

The key condition in our simplified model concerns exchange rate determination. When the homogeneous good is both produced and consumed in each country (as we posit

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crowded out, the foreign advantage cannot be too large. For example, if θ = ½, foreign productivity cannot be more than three times that of home productivity.

by assumption), it must be the case that the law of one price holds, that is, $P_{NH,t} = e_t T_{N,t} P^*_{NF,t}$. Since firms producing these goods are perfectly competitive, by the zero-profit condition prices are equal to marginal costs in both countries, i.e., for the home market $P_{Nt} = W_t / \alpha_N$. In combination with the equilibrium condition in the labor markets, these equations together pin down the exchange rate as a function of relative productivity in the homogeneous good sectors, the tariff on this sector and relative monetary stances (same as relative wages).

$$e_t = \frac{P_{NH,t}}{P^*_{NF,t} T_{N,t}} = \frac{\left(W_t / \alpha_N\right)}{\left(W^*_t / \alpha^*_N\right) T_{N,t}} = \frac{\mu_t \alpha^*_N}{\mu^*_t \alpha_N T_{N,t}}. \quad (32)$$

The home exchange rate depreciates (proportionally) with a home monetary expansion (rise in μ_t); it appreciates with a home tariff on the non-differentiated sector, as arbitrage induces a rise in home wages and production costs. Strikingly, as long as monetary stances remain constant, the exchange rate does not respond to tariffs on the other (differentiated-good) sector.

3.2 Uncertainty, pricing and welfare

In the model, tariff uncertainty (and thus monetary stabilization) impacts welfare via its effect on ex-ante pricing by firms operating under nominal rigidities (in the differentiated goods sector). To see this most clearly, posit that tariff shocks are i.i.d. As shown in Appendix B, the optimal price setting of the home differentiated goods firm is

$$P_{DHt} = \frac{\phi}{\phi - 1} E_{t-1}\left[\left(1 + \frac{\alpha^*_N}{\alpha_N} \frac{1}{T_{N,t} T^*_{Dt}}\right) \frac{\kappa \mu_t}{\alpha_D}\right] \Big/ E_{t-1}\left[\left(1 + \frac{\alpha^*_N}{\alpha_N} \frac{1}{T_{N,t} T^*_{Dt}}\right)\right]. \quad (33)$$

Firm price setting is affected both by anticipated shocks to—and uncertainty in the demand for—the firm's exports. Regarding anticipation effects: home firms expect a fall in the demand for their product when, first, they anticipate a rise in foreign tariff on differentiated imports, T^*_{Dt}, as this raises the price foreign households pay for home exports; second, they anticipate a rise in home non-differentiated tariff, $T_{N,t}$, since, as shown above, this causes currency appreciation which raises the price foreign consumers pay under prices sticky in the producer currency. The same applies to anticipation of a home monetary contraction. Note that home tariffs on differentiated goods do not enter the pricing equation (33), since these affect the price foreign consumers pay for home exports neither directly, nor indirectly through the exchange rate (see equation 32).

Most consequential for our welfare results is the pricing response to uncertainty. Through the expectation term, a negative correlation of shocks with μ_t, corresponding to large fluctuations in foreign demand, induces firms to set higher average prices of the home differentiated goods. As first discussed by Corsetti and Pesenti (2005) this observation is crucial for policy design, since μ_t is controlled by policy. With trade policy uncertainty, a policy regime systematically raising μ_t in response to a rise in tariffs can generate a smaller average fluctuation in demand, which lowers prices on average. Lower average prices correspond to higher average output, in turn raising welfare. This mechanism is a key driver of our quantitative results.

3.3 Optimal policy and exchange rate determination

To study the role of policy in redressing the distortions a tariff creates on international prices and the optimal trade off with internal output distortion, we write the optimal cooperative policy, which, thanks to our simplification, can be characterized in terms of two seemingly inward-looking national rules:[12]

$$\mu_t^{OP} = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1} \tag{34}$$

and

$$\mu_t^{*OP} = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}, \tag{35}$$

where a is a constant of proportionality. To start with, a key property of these rules is that they fully eliminate all fluctuations in the numerator of the optimal firm price setting in equation (33). As noted above, reducing uncertainty in demand leads firms to set lower prices of the differentiated goods on average, with positive effects on the price levels and welfare.

Second, together the equilibrium exchange rate (32), these expressions for optimal

[12] The Nash and cooperative solutions coincide for this simple environment. As usual, the Nash solution chooses μ_t, to maximize $E_{t-1}\left[U_t\right]$ and foreign chooses μ_t^*, to maximize $E_{t-1}\left[U_t^*\right]$; the cooperative solution chooses μ_t and μ_t^* jointly to maximize $E_{t-1}\left[U_t+U_t^*\right]/2$. See appendix B for details.

policy highlight the distinct role of the exchange rate in the stabilization of tariff shocks, depending on their sectoral composition. Upon substituting the optimal policies (34-35) into (32) we obtain:

$$e_t = \left(\frac{1}{T_{N,t}} + \frac{\alpha_N}{\alpha_N^*} \frac{1}{T_{Dt}} \right) \Big/ \left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*} \right). \tag{36}$$

Consider first a *home tariff on imports of differentiated goods* (risen in T_{Dt}). As shown by (34) and (35), the implementation of the response to home tariffs on foreign differentiated goods falls entirely on the *foreign* monetary authority. There is no change in home monetary stance, as these tariffs do not enter the home price setting equation (33)—they do not affect the price domestic and foreign consumers pay for home differentiated goods, either directly, or indirectly through the exchange rate (see the discussion of equation 32).[13] Targeting global welfare, the foreign monetary authorities expand to support their demand and redress the relative price distortion. We will see that this asymmetry in the optimal policy stance will be preserved in our quantitative exercises.

In principle, monetary policy could fully offset the impact on foreign demand—the foreign expansion could be large enough to cause a proportional home currency appreciation (rise in e_t), neutralizing the effect of a rise in $T_{D,t}$ on the relative price $\frac{p_t(h)}{e_t p_t^*(f) T_{D,t}}$. But this is not what the optimal policy rules prescribe: according to (36), the optimal home exchange rate appreciation is less than proportional to the tariff. To see this most clearly, rewrite (36) assuming that the only shock is to home tariffs on imported foreign differentiated goods: $e_t = \left(1 + \frac{\alpha_N}{\alpha_N^*} \frac{1}{T_{Dt}} \right) \Big/ \left(1 + \frac{\alpha_N^*}{\alpha_N} \right)$. If countries are nearly symmetric in productivity of the non-differenated sector ($\alpha_N \to \alpha_N^*$), the optimal policy prescribes that the home exchange rate should appreciate enough to wipe out approximately half of the rise in home prices due to the tariff—a percentage that corresponds to the share of

[13] In the full model, this stark result will not hold, if anything because production relies on imported intermediate inputs, whose price reflect exchange rate movements.

differentiated imports in the differentiated consumption aggregate.[14] However, monetary policy also drives the overall *domestic* demand for differentiated goods, so the stance required for a full exchange rate offset of the tariff would be suboptimal.

Full exchange rate offset is instead optimal in response to a *home tariff on non-differentiated goods*. As noted above, a positive tariff shock leads to appreciation of the home currency, which inefficiently lowers the demand for home exports of the other (sticky price) sector, the differentiated good sector. The optimal policy prevents such an outcome. This is clearly seen by simplifying (36) to include home non-differentiated tariffs only and letting productivities across countries become symmetric, such that $e_t = \left(\frac{1}{T_{N,t}} + \frac{\alpha_N}{\alpha_N^*}\right) \Big/ \left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}}\right) \to 1$. Note that the optimal policy completely offsets the impact appreciation of the home currency, that is, it *depreciates the home currency relative to this impact.* This is implemented by a combination of home and foreign policies that move in opposite direction relative to the previous case: home is expansionary, foreign contractionary.

3.4 Optimal policy under alternative specifications of nominal rigidities in export prices

The optimal monetary rules seen above are, however, sensitive to pricing behavior impacting on the degree of exchange rate pass through. Consider a version of the model in which prices are sticky in the currency of buyer (the *local currency pricing, LCP, case*). As shown in Appendix C, optimal policy satisfies the following conditions, for home

$$\frac{1}{\mu_t} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} + \frac{1}{2}\frac{1}{T_{Dt}} \Big/ E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right], \tag{37}$$

and for foreign

$$\frac{1}{\mu_t^*} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t^*\right]} + \frac{1}{2}\frac{1}{T_{Dt}^*} \Big/ E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]. \tag{38}$$

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[14] It is easy to verify that in response to a 10% tariff, (T_{Dt} =1.10), with symmetric productivity, the home currency would be optimally appreciated by about 5% (e_t = 0.954).

Under LCP, the optimal policy no longer responds to tariffs on the non-differentiated sector. This follows from the fact that, under LCP, the exchange rate does not move domestic prices of the differentiated goods, and so it cancels out of the expectation term for price setting. Most crucially, the home policy stance no longer responds to foreign tariffs on home differentiated goods: it optimally responds only to the home tariff on these goods. This is the *opposite* pattern relative to the PCP case studied above. As shown in Appendix C, under LCP, the price set by home firms in the home market is no longer affected by uncertainty in export demand in the foreign market. However, uncertainty from the home tariffs now weighs on the price preset in local currency by *foreign* exporters—on average, the uncertainty associated to these tariffs raises home CPI. So, the home monetary policy has a welfare incentive to stabilize the home demand facing foreign exporters, with the goal of lowering the average price they charge home customers (a case discussed early on in Corsetti and Pesenti, 2005).

The case in which the home currency is dominant in international trade invoicing, that is, where all export prices are set in the currency of the home country regardless of which country is exporting, is a combination of the PCP and LCP case. Under *Dominant Currency Pricing*, of *DCP*, the home country's firms follow the PCP pricing rule above, while the foreign country's firms follow the LCP pricing rules. As derived in Appendix D, we can write:

$$, \mu_t = \frac{1}{2}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)}+\frac{1}{2}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{\frac{1}{T_{Dt}}} \tag{39}$$

and for foreign

$$\mu_t^* = E_{t-1}\left[\mu_t^*\right]. \tag{40}$$

While there is no closed form solution to equation (39) (just as in the LCP case), the optimal policy rule now responds to both home and foreign tariffs on differentiated goods. The logic combines the economics of the PCP and LCP results studied above. First, since home sets its exports in producer currency terms, the home monetary policy responds to a foreign tariff cutting global demand for home exports much like the standard PCP case solved above. Again, home policy aims to offset the effect of tariffs uncertainty, translating into fluctuations in demand for its firms' exports, as this would tend to increase inefficiently also the home price index. By the same token, since home imports are set in local currency, similar to the LCP case above, the home monetary policy aims to contain the average price of imports that foreign exporters set higher when facing a variable demand in the home market.

In the DCP world, all the adjustment falls optimally on the home policy. The optimal foreign policy does not respond at all to any tariff, home or foreign. Equation (40) can be satisfied by any constant foreign monetary rule: $\mu_t^* = a$ for some constant a.

4. Baseline Simulation: unilateral home tariff on foreign differentiated exports

This section generalizes our analysis of the transmission of tariffs and their optimal stabilization using numerical simulations of the calibrated full model, starting from our baseline case of an unexpected tariff imposed by the home country on its imports of differentiated goods exported from the foreign country. Figure 1 reports impulse responses of selected variables under different policy regimes. The figure contrasts the allocation under the Ramsey optimal policy (solid line) against the allocations under a strict PPI-targeting rule (dashed line), and a CPI-based Taylor rule (dotted line).

4.1 Transmission under suboptimal policy

In our baseline with producer currency pricing, a monetary policy that perfectly targets producer prices also supports the natural rate (flexible price) allocation. With strict PPI targeting in place, thus, the allocation is identical to the one under flexible prices. As shown in the figure, the tariff shock generates a modest but sustained fall in home GDP, and a small spike of CPI inflation—a stagflationary outcome that presents a classic challenge to policy makers. Stagflation is also experienced on impact by the foreign country, where the drop in output is nonetheless steeper, given that the tariff hits the demand for this country's exports.

While at the aggregate level the effects of the tariff on activity remain moderate, both countries experience significant sectoral reallocation. In the foreign country, the percentage fall in the production of differentiated goods is three times the percentage fall in GDP. But this is matched by a rise in the production of non-differentiated goods of a similarly large magnitude. In the home country, sectoral reallocation inversely mirrors that in the foreign country. The modest fall in home GDP is the net effect of a larger percentage fall in non-differentiated goods production and smaller offsetting rise in non-differentiated production.

PPI targeting results in home nominal and real currency appreciation, driven by a combination of higher nominal policy rates in home, and lower rates in foreign. The rate of appreciation however is not large enough to offset the impact of the tariff on the relative price of home exports to home imports.[15] Figure 1 reports the "differentiated terms of trade", defined as the relative price of home differentiated goods to the tariff-inclusive price of home imports of foreign differentiated goods, $p_t(h)/\left(e_t p_t^*(f) T_{D,t}\right)$. This is the same relative price found in the relative demand condition (7). As shown in the figure, this price falls, reflecting the effect on the import prices of the tariff. The home country experiences a temporary trade surplus.[16]

It is also instructive to compare the above with the allocation under a standard Taylor rule (dotted line in Figure 1). This rule actually dampens the interest rate response, due to the substantial interest rate smoothing term in our calibrated version of this rule. A smaller monetary contraction means that home GDP now rises, and PPI inflation turns positive. For the same reason however, in the foreign country GDP falls slightly more and inflation turns negative. The sectoral reallocation is larger.

4.2 Transmission under the optimal policy

Since tariffs create real misalignment, supporting the natural rate is not optimal. As shown in Figure 1, the optimal (Ramsey) policy, depicted by a solid (red) line, deviates from the natural rate. Relative to strict PPI targeting, the monetary stance in the foreign country is strongly expansionary—PPI inflation rises well above zero. In the home country, instead, the optimal policy remains close to PPI targeting—the country only experiences a

[15] Linde and Pescatori (2019) have pointed out that, in its stronger form, Lerner symmetry fails in many macroeconomic contexts, depending on the structure of financial markets and nominal rigidities.
[16] Appendix Figure 1 reports responses for additional variables. Home consumption falls and foreign rises, reflecting interest rates in each country. Firm entry rises at home and falls in foreign, reflecting movements in differentiated goods output. Employment moves likewise; wages fall in both countries reflecting the fall in GDP.

modest fall of PPI below target. Remarkably, these simulation results broadly corroborate the analytical results from the simplified model in the preceding section, in which optimal policy calls for a large monetary expansion in the foreign country, with no response in the home country.

The home currency optimally appreciates, more than under the alternative regimes. Figure 1 shows that, under the optimal policy, the home exchange rate appreciation offsets approximately half of the effect of the tariff on this relative price (labeled as diff. terms of trade), which again broadly corresponds to the result in the analytical model. Moreover, the optimal policy substantially dampens the sector reallocation observed under the PPI targeting rule. So, the Ramsey policy undercuts the tariff-related distortions in both relative prices and sectoral allocation.

It is worth stressing that the optimal home currency appreciation requires the monetary stance at home to be contractionary *relative* to foreign. Depending on parameter values, the optimal exchange rate adjustment may result from an asymmetric expansion in both countries, provided the stance is relatively less expansionary in home than in foreign. Appendix Figure 2 explores this possibility, modelling a smaller trade elasticity that serves to dampen the effectiveness of the exchange rate as a mechanism of adjustment (we lower ϕ =5.2 to 3.8, a value common in some firm dynamics literature, see Ghironi and Melitz 2003).[17] This change in parametrization is sufficient to flip the sign of the Ramsey optimal PPI inflation response at home from negative to positive, and bring this country's GDP above the case of PPI targeting.

4.3 The case of dominant currency pricing

As shown in the analytical section, the way monetary policy responds to the tariff-induced misalignment changes when import prices in domestic currency become less sensitive to the exchange rate. In Figure 2 we analyze the economy under dominant currency pricing, i.e., under the assumption that the prices of both home imports and

[17] This experiment also lowered the trade elasticity for non-differentiated goods (η=1.5), to satisfy Blanchard-Kahn condition of dynamic stability. The market structure for the differentiated sector is taken from the trade literature and implies that the parameter governing the elasticity of substitution between home and foreign varieties is the same parameter governing the elasticity between different home varieties and hence firm markup. So we are limited in the range of trade elasticities admissible.

exports of differentiated goods are set in home currency units (leaving analysis of the case of Local Currency Pricing to Appendix E).[18]

Under DCP, PPI targeting no longer supports the natural rate allocation. So, while the dynamics of sectoral output and inflation are similar to the benchmark model with stickiness in the currency of the producer shown in Figure 1, the dynamics of home GDP are different. Under dominant currency pricing, the home currency appreciation does not reduce the border price of home differentiated imports in domestic currency. Tariffs on differentiated goods raise the final price of these goods relative to non-differentiated goods by more. This impinges on the aggregation of sectoral output into the overall GDP, as it gives more weight to the rise in home production of differentiated goods relative to the fall in home production of non-differentiated goods.

From the simplified analytical model, we have seen that, when home import prices are sticky in home currency, the home policy should systematically respond to home tariff shocks by stabilizing domestic demand. The simulation generalizes this analytical result. In the full model the home optimal policy response to own tariff is more expansionary than under PPI targeting, to support overall demand, tolerating positive PPI inflation. A low exchange-rate pass through reduces the weight the optimal policy places on redressing the relative price of exports. Indeed, the policy does nothing to dampen the sectoral reallocation induced by the tariff. While the analytical result for the simplified model implied no foreign policy response to the home tariff, the more richly parameterized and calibrated simulation model implies a mild foreign expansion, producing a slightly positive PPI inflation. Observe that under the optimal policy CPI inflation is higher in both countries, relative to PPI inflation.[19]

5. Tariffs on non-differentiated goods

The nature of misalignment, optimal monetary policy and macroeconomic dynamics are all sharply different in the case of a home tariff on imports of the foreign non-differentiated good, which we broadly associate with commodities subjected to recent

[18] The case of local currency price stickiness applied symmetrically to both countries is reported in Appendix Figure 3. The case of foreign currency dominance is reported in Appendix Figure 4.
[19] The effect on GDP again is complicated by the change in sectoral relative prices used in aggregating output over the two sectors.

tariffs such as steel, aluminum and agriculture. As shown in Figure 3, under a PPI-targeting rule (that is, in the natural rate allocation in our baseline), the sectoral reallocation induced by the tariff is precisely the opposite of Figure 1: tariffs shift home demand and production away from differentiated goods toward non-differentiated goods, with offsetting sectoral reallocation in the foreign country. As predicted by our analytical results, this reallocation is driven by a home currency appreciation induced by the home tariff, which makes home exports of differentiated goods less competitive. The home terms of trade for differentiated goods (inclusive of the tariffs) however now worsen, instead of appreciating. Also in contrast with Figure 1, home overall GDP rises for this tariff. The rise in home non-differentiated production is larger in percentage terms than the fall in differentiated production. Finally, different from Figure 1, the dynamics of differentiated production are positive *and* smoother over time, due to the slow adjustment of the sticky prices in the sector.

The direction of optimal policy is also fully in line with the predictions of the simple analytical model. Home monetary policy is now markedly more expansionary than the PPI-targeting rule, with interest rate rising less and positive PPI inflation. Foreign monetary policy, in contrast, is now more contractionary than the PPI-targeting rule, with interest rate falling less, and negative PPI inflation. As a result, the optimal policy appreciates the home exchange rate *less*, up to worsening the terms of trade more than under PPI-targeting.

The optimal policy nonetheless contains somewhat the fluctuations in differentiated goods production in the two countries (relative to PPI targeting), without significantly altering the response in the other sector (relative to both PPI and CPI targeting). The reason is that the flexible-price sector is insulated from the effects of monetary policy, which can do little to remediate the effects of the tariff on this sector. But price stickiness in the differentiated goods sector enables monetary policy to remediate the side effects of tariffs (through home appreciation), supporting the demand for these goods.

6. Tariff with sector-specific retaliation

We revisit the topic of trade wars with retaliation studied in our previous work, though here with two sectors; we even study the case of asymmetric retaliation across different sectors, reflecting recent experiences with tariffs.

6.1 Symmetric Tariff war

Symmetric tariff hikes imposed on the differentiated goods exports of both countries have clear recessionary effects on aggregate GDP at global level. As shown in Appendix Figure 7, in both countries the contraction in activity is largely driven by the fall in differentiated goods production. The production of non-differentiated goods actually rises somewhat, but not enough to compensate for the fall in overall output coming from the differentiated sector. In a symmetric tariff war, there is no shift in sectoral specialization across countries—rather, the tariff distortions result in a shift in the sectoral composition of output at a global level.

In line with our previous work, the optimal monetary policy stance is expansionary in both countries, despite the inflationary impact of the tariff. Given that a symmetric tariff war cannot be remedied by a currency depreciation, the optimal policy aims at resolving the distortion created by the tariff between differentiated and non-differentiated prices within each country. An expansionary monetary stance mitigates the contraction in the differentiated good sector, driving up overall aggregate demand as well as the prices of non-differentiated goods, which are flexible.[20]

6.2 Asymmetric Tariff war

Our model allows us to study the effect of a home tariff on differentiated goods imports met by foreign retaliation in the form of tariffs (of an equal percentage) on imports of less differentiated goods. This scenario, novel in the literature, is empirically relevant—in some occasion, U.S. tariffs on manufacturing imports, were met by foreign tariffs on U.S. agricultural commodities (such as Chinese limits on imports of U.S. soybeans). Figure 4 shows that under PPI targeting, the sectoral reallocation is larger than in the case of a unilateral foreign tariff in Figure 1, as the foreign tariff further shifts

[20] Appendix Figure 3 show the case of a symmetric trade war where firms in both countries set export prices in local currency (LCP stickiness). Monetary policy has minimal impact on trade flows.

production of non-differentiated goods to foreign, reinforcing the reallocation induced by home tariffs promoting home manufacturing (differentiated goods). In contrast with Figure 1, the fall in overall GDP in Figure 4 is larger in the home country.

Nonetheless, the optimal policy is qualitatively similar to that in the case of the unilateral foreign tariff in Figure 1, calling for a foreign expansion and home contraction. Since the non-differentiated sector is characterized by flexible prices, monetary policy has little power to affect demand in that sector. So optimal policy is driven by the goal of offsetting the home tariff on differentiated goods.

7. Welfare

Results in our analytical section suggest that, when the home policy systematically responds to home tariff shocks by stabilizing demand, reduced uncertainty leads firms to set lower prices on average, with positive effects on social welfare. We conclude this section with an assessment of the impact of the optimal monetary policy on welfare in the presence of tariff uncertainty. Tables 2 and 3 report the welfare gains from pursuing the optimal policy relative to, respectively, a suboptimal Taylor Rule regime, and the PPI targeting rule—all measured in units of steady state consumption. To compute these gains, we conduct a stochastic simulation of the model in which unilateral home tariff shocks are mean zero (shocks include both hikes and cuts in tariff rates relative to their mean level).[21]

While quantitatively the benefits from the optimal policy relative to suboptimal rules are small in our baseline, they are much larger when tariff shocks are persistent—arguably a relevant case in light of the U.S. trade policy in both 2018 and 2025.

Throughout the tables, the largest gains are against the Taylor rule regime. For our benchmark simulation, in Figure 1, the optimal policy improves world welfare by 0.096 percent relative to this. The optimal policy improves welfare in both countries, though not symmetrically. Home gains more, 0.162 percent vs. 0.031 percent in foreign (first line of Table 2). Even though the cooperative optimal policy exacerbates a fall in home production, the home country benefits more from a policy that offsets a home tariff's distortions on the relative price of imports and contains inflation. Conversely, the gains from optimal policy relative to PPI targeting, shown in Table 3, are smaller: 0.020 percent for the world welfare, 0.024 for home,

[21] Perturbation solution methods require that shocks be mean zero.

and 0.016 for foreign.

World welfare gains from the optimal policy are nonetheless smaller overall under the assumption of home currency dominance (0.046 percent and 0.042 percent relative to Taylor and PPI targeting, respectively, in line 2 of the tables). This reflects the limited ability of monetary policy to remediate the distortion of the tariff on (home) relative prices.[22] Welfare gains are fairly similar under the specification of symmetric LCP price stickiness.

Welfare results also vary with elasticities. Welfare gains are somewhat higher if the two sectors are modeled as complements (ξ=0.5), somewhat lower if the two sectors are substitutes (ξ=1.4). The case of complementarity nicely reflects the fact that many non-differentiated goods like steel and other commodities are combined with differentiated goods as complementary inputs in producing the final consumption good. Our model specification nests the case of a homogeneous good (common in the trade literature, η large), or the case of country-specific goods traded in competitive markets (in the macro real business cycle literature often calibrated setting η=1.5). Relative to our baseline, welfare gains from optimal policy are slightly larger in the former case and slightly smaller in the latter (see lines 6 and 7 of Tables 2 and 3).

Welfare gains are substantially higher when tariff shocks are persistent. Setting ρ_T = 0.95, welfare gains from the optimal policy quadruple, up to 0.44% percent, relative to CPI targeting. As shown in Appendix Figure 5, in response to a persistent tariff, the optimal policy engineers a larger home appreciation, acting more strongly to correct relative prices. Relative to CPI targeting, the optimal policy contains distortions in both the aggregate GDP and the sectoral allocation—but not CPI movements. Remarkably, CPI inflation is negative (in spite of the tariff) in home, positive in foreign.

Our two-sector model differs from models, standard in the macro literature, assuming one tradable and one nontradable good sector. For comparison, we include a version of this model in our assessment of welfare in tables 2 and 3. While welfare results are similar to the benchmark case (with smaller gains accruing to the home country), Appendix Figure 6 shows that the sectoral reallocation is a fraction of what predicted by our two-traded sector model. Without a second traded sector to offer alternative specialization of the foreign country, the home country cannot achieve as much specialization in the differentiated sector.

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[22] Relative to the PPI targeting rule, the welfare gains of the cooperative optimal policy are negative for the home country, suggesting that a cooperative policy might be harder to sustain for this case. Relative to the standard Taylor rule, the welfare gains remain positive for both countries.

The welfare loss of a symmetric tariff war is actually lower relative to the case of a unilateral tariff (see row 10 of Tables 2 and 3). This can be attributed to the fact that a symmetric tariff does not imply a large asymmetric sectoral reallocation across countries associated with a shift in comparative advantage, as found in the case of a unilateral tariff in the analysis above. Row 11 shows that under home dominant currency pricing, optimal policy favors the foreign country, but welfare changes remain small. (See Appendix Figure 8 for dynamics).

Results are similar for the case of a symmetric tariff war launched on exports of non-differentiated goods of both countries. (See row 13 of Tables 2 and 3 for welfare, and Appendix Figure 9 for impulse responses.)

Regarding the asymmetric tariff war depicted in Figure 4 discussed above, the welfare gains from optimal policy are asymmetric (see row 14 of Tables 2 and 3), favoring the home country more than foreign, and world welfare is somewhat higher than the benchmark case of the unilateral differentiated goods tariff.

8. Conclusion

Using a New Keynesian model enriched with elements from the trade literature, including global value chains in production and multiple traded sectors, we study the monetary trade-offs created by the distortionary effects of tariffs on the international prices of imports—especially relevant when the country imposing the tariff is large in the global economy. We also allow for different market structure and degree of nominal rigidities across sectors, bringing a novel sectoral dimension to the study of the optimal stabilization of tariff shocks.

We show that, in the case of tariffs targeting differentiated final consumption goods, characterized by monopolistic competition and price stickiness at the producer level, a specific objective of monetary policy is to redress the distortionary effects of unilateral tariffs on relative prices between home and foreign goods. Policy in both countries moves to ensure that, at least in part, the exchange rate offsets the misalignment in these prices. The optimal response is mainly driven by a foreign expansion, causing positive PPI inflation, with the home country remaining close to PPI stability. This combination serves to counteract the impact of tariffs on both aggregate GDP and sectoral reallocation. In the case of non-differentiated goods, price flexibility in this sector prevents currency movements to redress the distortionary effects of the tariff on relative prices. However, monetary stabilization can compensate for the loss of aggregate production due to the fall in the demand for non-differentiated goods in response to the tariff, by raising the demand and production of differentiated goods. The optimal policy is supported by a combination of home and foreign stances that moderates the home appreciation, causing negative PPI inflation in the foreign country.

The main takeaway from our study is that management of the exchange rate is a key channel for redressing the relative price distortions created by tariffs. Importantly, rather than offering an interpretation aligned with the logic of "competitive devaluation" (which could pose the risk of igniting a tariff-cum-currency war), we portray a currency realignment restoring competitiveness as a product of cooperative optimal policy–one which serves to improve welfare for the residents in both countries.

References

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub 2025. “The Macroeconomics of Tariff Shocks,” NBER working paper 33726.

Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem, 2024. “Trade Wars and Currency Wars,” *Review of Economic Studies*, doi:10.1093/restud/rdae075.

Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem, 2025. “Trade Wars and the Optimal Design of Monetary Rules,” *Journal of Monetary Economics*, 103726.

Baqaee, David and Hannes Malmberg, 2025. “Long-Run Effects of Trade Wars,” NBER Working Paper 33702.

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi, 2021. “Protectionism and the Business Cycle,” *Journal of International Economics* 129, 103417.

Bergin, Paul R. and Giancarlo Corsetti, 2020. “Beyond Competitive Devaluations: The Monetary Dimensions of Comparative Advantage,” *American Economic Journal: Macroeconomics* 12(4), 246-86.

Bergin, Paul R. and Giancarlo Corsetti, 2023. “The Macroeconomic Stabilization of Tariff Shocks: What is the Optimal Monetary Response?” *Journal of International Economics* 143, 103758.

Bianchi, Javier and Louphou Coulibaly, 2025. ”The Optimal Monetary Policy Response to Tariffs,” NBER Working Paper 33560.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, 2008. “Monetary Policy and Business Cycles with Endogenous Entry and Product Variety,” in Acemoglu, D., K. S. Rogoff, and M. Woodford, eds*., NBER Macroeconomics Annual 2007*, Univ. of Chicago Press, Chicago, 299-353.

Broda, Christian and David E. Weinstein, 2006. “Globalization and the Gains from Variety,” *The Quarterly Journal of Economics* 121, 541-585.

Caldara, Dario, Matteo Iacoviello, Patrick Molligo, Andrea Prestipino and Andrea Raffo, 2020. "The Economic Effects of Trade Policy Uncertainty," *Journal of Monetary Economics* 109, 38-59.

Coenen, Gunter, Giovanni Lombardo, Frank Smets and Ronald Straub, 2010. “International Transmission and Monetary Policy Cooperation,” in Jordi Galí and Mark J. Gertler, eds. *International Dimensions of Monetary Policy,* Chicago: University of Chicago Press, 157-192.

Corsetti, Giancarlo, and Paolo Pesenti, 2005. “International Dimensions of Optimal Monetary Policy,” Journal of Monetary Economics 52, 281-305.

Corsetti, Giancarlo and Paolo Pesenti, 2009. “The Simple Geometry of Transmission and Stabilization in Closed and Open Economies,” *NBER International Seminar on Macroeconomics 2007*, ed by Richard Clarida and Francesco Giavazzi, Francesco: University of Chicago press, Chicago, 65-116.

Costinot, Arnaud and Iván Werning, 2025. “How Tariffs Affect Trade Deficits,” NBER Working Paper 33709.

Erceg, Christopher, Andrea Prestipino, and Andrea Raffo, 2018. "The Macroeconomic Effects of Trade Policy," Board of Governors of the Federal Reserve System International Finance Discussion Papers, Number 1242.

Ghironi, Fabio and Marc J. Melitz, 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms" *The Quarterly Journal of Economics* 120(3), 865-915.

Hall, Robert E., 2009. "By How Much Does GDP Rise If the Government Buys More Output?" *Brookings Papers on Economic Activity* 2, 183–231.

Jeanne, Olivier and Jeongwon Son, 2024. "To What Extent are Tariffs Offset by Exchange Rates?" *Journal of International Money and Finance* 103015.

Kalemli-Özcan, Sebnem, Can Soylu, and Muhammed A. Yildirim, 2025. "Global Networks, Monetary Policy and Trade," NBER Working paper 33686.

Lindé, Jesper and Andrea Pescatori, 2019. "The Macroeconomic Effects of Trade Tariffs: Revisiting the Lerner Symmetry Result," *Journal of International Money and Finance* 95(C), 52-69.

Monacelli, Tommaso, 2025. "Tariffs and Monetary Policy," mimeo, Bocconi.

Rauch, James E., 1999. "Networks Versus Markets in International Trade," *Journal of International Economics* 48, 7–35.

World Bank, 2017. "World Bank National Accounts Data: Exports of Goods and Services (% of GDP)." https://data.worldbank.org/indicator/NE.EXP.GNFS.ZS?locations=OE (accessed September 16, 2019).

Werning, Ivan, Guido Lorenzoni, and Veronica Guerrieri, 2025. "Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy," NBER working paper 33772.

Table 1. Parameter Values

Preferences	
Risk aversion	$\sigma = 2$
Time preference	$\beta = 0.99$
Labor supply elasticity	$1/\psi = 1.9$
Differentiated goods share	$\theta = 0.45$
Non-differentiated goods home bias	$\nu = 0.5$
Differentiated goods elasticity	$\phi = 5.2$ (also 3.8)
Non-differentiated goods elasticity	$\eta = 5.2$ (also 1.5, 15)
Substitution between sectors	$\xi = 1$ (also 0.5, 1.5)
Technology	
Firm death rate	$\delta = 0.025$
Price stickiness	$\psi_P = 49$
Intermediate input share	$\varsigma = 1/3$
Differentiated goods trade cost	$\tau_D = 0.44$
Non-differentiated goods trade cost	$\tau_N = 0.44$
Mean sunk entry cost	$\overline{K} = 1$
Firm entry adjustment cost	$\lambda = 0.10$
Bond holding cost	$\psi_B = 10^{-6}$
Monetary Policy (for Taylor Rule)	
Interest rate smoothing	$\gamma_i = 0.7$
Inflation response	$\gamma_p = 1.7$
GDP response	$\gamma_Y = 0.1$
Tariff shock	
Mean	$\overline{T_D} = \overline{T_N} = 1.02$
Standard deviation	$\sigma_{\varepsilon_T} = 0.08$
Autoregressive parameter	$\rho_T = 0.56$

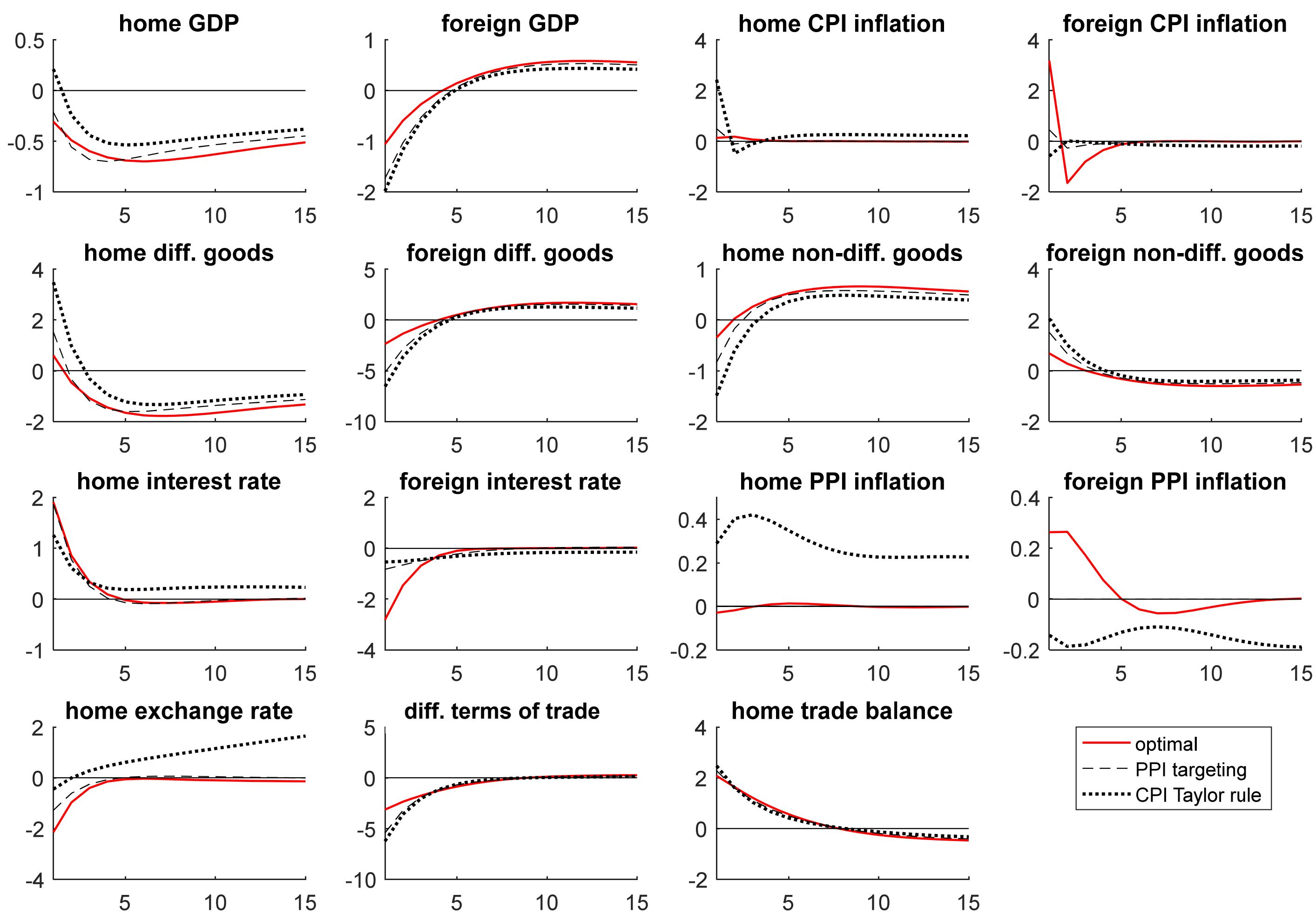
Table 2. Welfare Gains from Ramsey Optimal Policy Relative to Taylor Rule
(percent change, in consumption units):

	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.096	0.162	0.031
2.	home DCP	0.046	0.066	0.026
3.	both countries LCP	0.042	0.037	0.047
4.	sectors complements (ξ=0.5)	0.113	0.185	0.040
5.	sectors substitutes (ξ=1.4)	0.093	0.151	0.034
6.	nondifferentiated goods BKK (η=1.5)	0.092	0.148	0.036
7.	nondiff. goods homogeneous (η=15)	0.108	0.118	0.099
8.	persistent tariff ($\rho_T = 0.95$)	0.444	0.194	0.694
9.	nontraded non-differentiated good	0.110	0.116	0.103
10.	symmetric tariff on differentiated goods	0.027	0.027	0.027
11.	home DCP	0.057	-0.100	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	0.035	-0.006
13.	symmetric tariff on non-differentiated goods	0.013	0.013	0.013
14.	home diff. tariff; foreign non-diff. tariff	0.121	0.174	0.067

Table 3. Welfare Gains from Ramsey Optimal Policy Relative to PPI Targeting Rule
(percent change, in consumption units):

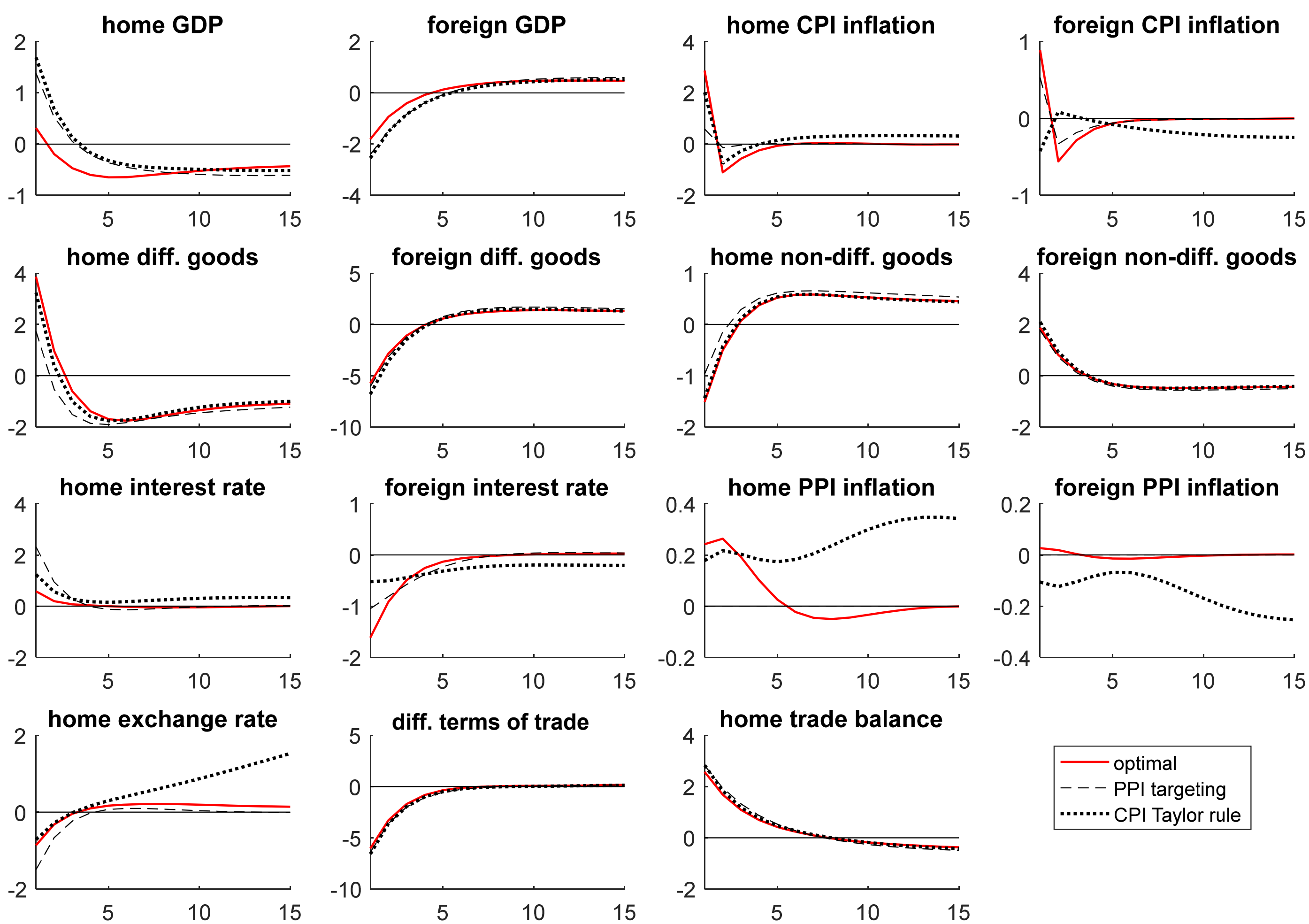
	tariff shock	world	home	foreign
1.	unilateral home tariff on differentiated goods	0.020	0.024	0.016
2.	home DCP	0.017	-0.083	0.117
3.	both countries LCP	0.023	-0.127	0.172
4.	sectors complements (ξ=0.5)	0.023	0.032	0.014
5.	sectors substitutes (ξ=1.4)	0.016	0.016	0.017
6.	nondifferentiated goods BKK (η=1.5)	0.019	0.024	0.013
7.	nondiff. goods homogeneous (η=15)	0.018	0.010	0.025
8.	persistent tariff ($\rho_T = 0.95$)	0.053	0.034	0.071
9.	nontraded non-differentiated good	0.017	0.010	0.024
10.	symmetric tariff on differentiated goods	0.071	0.028	0.028
11.	home DCP	0.034	-0.096	0.215
12.	unilateral home tariff on non-differentiated goods	0.215	-0.015	0.025
13.	symmetric tariff on non-differentiated goods	0.001	0.001	0.001
14.	home diff. tariff; foreign non-diff. tariff	0.034	0.068	0.000

Figure 1. Impulse responses to a rise in home tariff on differentiated imports



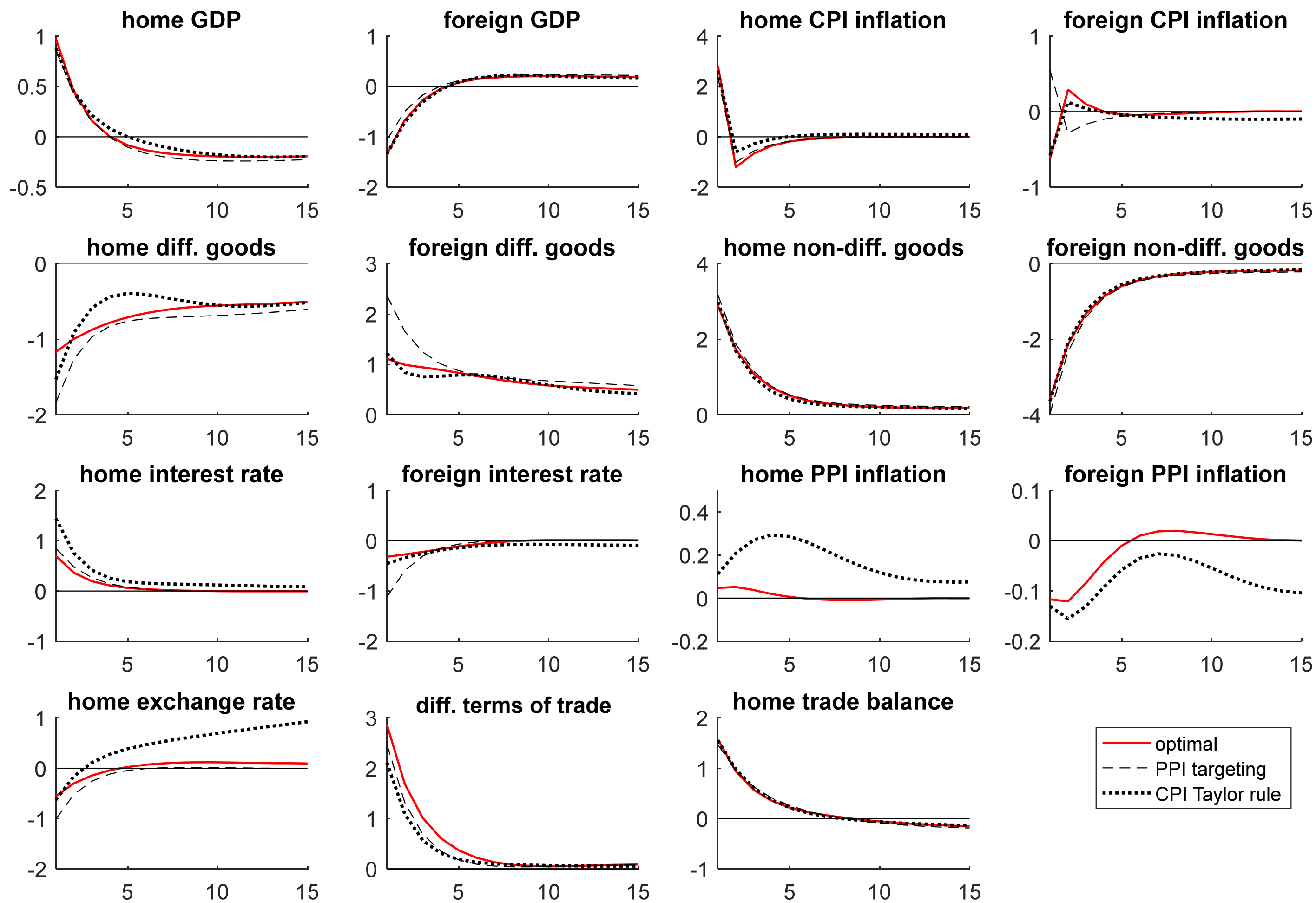
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 2. Impulse responses to a rise in home tariff on differentiated imports; home currency dominant



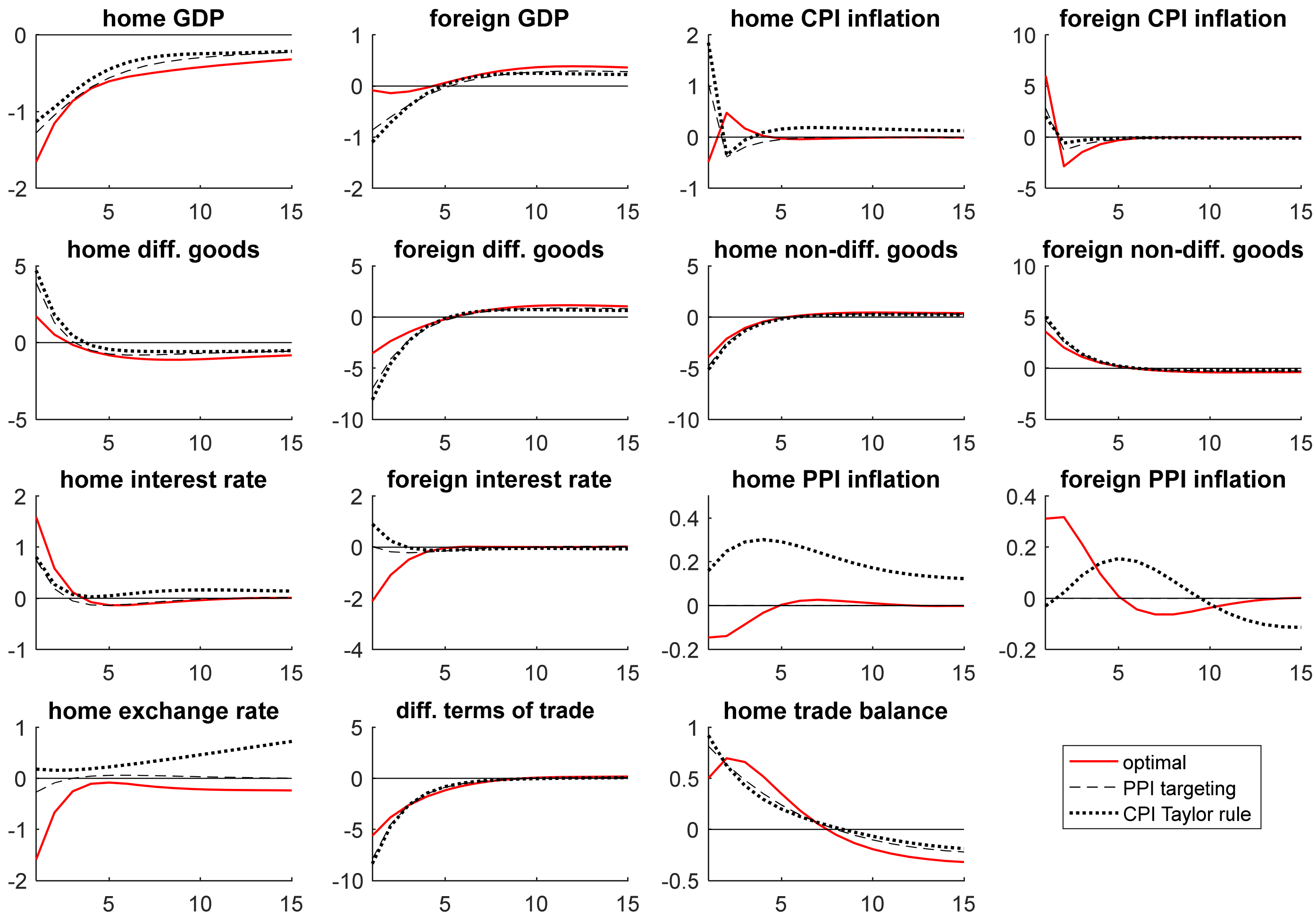
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 3. Impulse responses to a rise in home tariff on non-differentiated imports



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Figure 4. Impulse responses to a hybrid case: home tariff on differentiated imports;
foreign tariff non-differentiated imports



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix

For
"Monetary stabilization of sectoral tariffs"

by
Paul R. Bergin
and
Giancarlo Corsetti

Appendix A. Equations from the benchmark model not listed in the main text

1. Demand equations

The composition of expenditure on adjustment costs, both for prices and bond holding, follows the same preferences as for consumption, and the associated demands mirror Eqs. (4)-(9). Adjustment costs for bond holding are as follows:

$$AC_{B,D,t} = \theta P_t AC_{B,t} / P_{D,t}$$

$$AC_{B,N,t} = (1-\theta) P_t AC_{B,t} / P_{N,t}$$

$$d_{AC,B,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$d_{AC,B,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$AC_{B,H,t} = \nu \left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t}$$

$$AC_{B,F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t} .$$

The economy-wide demand for goods arising from price adjustment costs sums across the demand arising among *n* home firms: $AC_{P,t} = n_t AC_{P,t}(h)$. This is allocated as follows:

$$AC_{P,D,t} = \theta P_t AC_{P,t} / P_{D,t}$$

$$AC_{P,N,t} = (1-\theta) P_t AC_{P,t} / P_{N,t}$$

$$d_{AC,P,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$d_{AC,P,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$AC_{P,H,t} = \nu \left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t}$$

$$AC_{P,F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t} .$$

The demand for differentiated goods for use as intermediates in production mirrors Eqs. (6)-(7), as follows:

$$d_{G,t}(h)=\left(p_t\left(h\right)/P_{D,t}\right)^{-\phi}G_t$$

$$d_{G,t}(f)=\left(p_t\left(f\right)T_{D,t}/P_{D,t}\right)^{-\phi}G_t.$$

The demand for differentiated goods for use in the sunk entry investment of new firms mirrors Eqs. (6)-(7), as follows:

$$d_{K,t}(h)=\left(p_t\left(h\right)/P_{D,t}\right)^{-\phi}ne_tK_t$$

$$d_{K,t}(f)=\left(p_t\left(f\right)T_{D,t}/P_{D,t}\right)^{-\phi}ne_tK_t.$$

2. Market clearing conditions

Market clearing for the non-differentiated goods market requires:

$$y_{H,t}=C_{H,t}+AC_{P,H,t}+AC_{B,H,t}+\left(1+\tau_N\right)\left(C^*_{H,t}+AC^*_{P,H,t}+AC^*_{B,H,t}\right)$$

$$y_{F,t}=\left(1+\tau^*_N\right)\left(C_{F,t}+AC_{P,F,t}+AC_{B,F,t}\right)+C^*_{F,t}+AC^*_{P,F,t}+AC^*_{B,F,t}.$$

Labor market clearing requires:

$$\int_0^{n_t}l_t\left(h\right)dh+l_{H,t}=l_t.$$

Bond market clearing requires:

$$B_{Ht}+B^*_{Ht}=0$$

$$B_{Ft}+B^*_{Ft}=0.$$

Balance of payments requires:

$$\int_0^{n_t}p^*_t\left(h\right)\left(d^*_t\left(h\right)\right)dh-\int_0^{n^*_t}p_t\left(f\right)\left(d_t\left(f\right)\right)df+P^*_{Ht}\left(C^*_{H,t}+AC^*_{P,H,t}+AC^*_{B,H,t}\right)$$
$$-P_{F,t}\left(C_{F,t}+AC_{P,F,t}+AC_{B,F,t}\right)-i_{t-1}B^*_{H,t-1}+e_ti^*_{t-1}B_{F,t-1}=\left(B^*_{H,t}-B^*_{H,t-1}\right)+e_t\left(B_{F,t}-B_{F,t-1}\right).$$

Appendix B: Derivation of Analytical Results, Producer Currency Pricing

1. Demands

The modified consumption index implies the following demands:

$$C_{DHt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{P_{DHt}} = \frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}$$

$$C_{DFt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{T_{Dt}e_tP^*_{DFt}} = \frac{\theta}{2}\frac{P_tC_t}{T_{Dt}e_tP^*_{DFt}}$$

$$C^*_{DHt} = \frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}} = \frac{\theta}{2}\frac{P_t\,C_t}{T^*_{Dt}P_{DHt}}$$

$$C_{Dt} = \theta\frac{P_tC_t}{P_{Dt}}$$

$$C_{Nt} = (1-\theta)\frac{P_tC_t}{P_{Nt}}$$

$$c_t(h) = \left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}$$

2. Optimal price setting for differentiated good

The home firm maximizes

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_D}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(p_t(h)-\frac{W_t}{\alpha_D}\right)\left(\frac{T_t^*p_t(h)}{e_t}\Big/\left(\frac{T_t^*P_{DH,t}}{e_t}\right)\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

or $E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(p_t(h)-\frac{W_t}{\alpha_D}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}\left(C_{DH,t}+C^*{}_{DH,t}\right)\right]$,

implying the price setting rule

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*{}_{DH,t}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*{}_{DH,t}\right)\right]}.$$

Substitute in demands from above

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}}\right)\right]},$$

and substitute in for μ_t and exchange rate

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\right]}.$$

Use $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$ from the main text:

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}.$$

The foreign firm counterpart is:

$$P_{DFt}^* = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt} + C_{DF,t}\right)\frac{W_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt} + C_{DF,t}\right)\right]}.$$

Substitute in for in μ_t^* mu and exchange rate:

$$P_{DFt}^* = \frac{\phi}{\phi - 1} \frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*} + \frac{\mu_t}{T_{Dt}e_t P_{DFt}^*}\right)\right]}$$

$$P_{DFt}^* = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}$$

So the home price index can be written:

$$P_t = 2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right] P_{DHt}{}^{\theta/2}\left(T_{Dt}e_t P_{DFt}^*\right)^{\theta/2} P_{NHt}{}^{1-\theta}.$$

Use $P_{NHt} = \frac{W_t}{\alpha_N} = \frac{\kappa\mu_t}{\alpha_N}$ to write the price index in terms of exogenous variables:

$$P_t = 2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right] P_{DHt}{}^{\theta/2}\left(T_{Dt}e_t p_t^*(f)\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}$$

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}e_t\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}.$$

3. Labor

Given the homogenous second sector, the easiest way to derive equilibrium labor is from the household budget constraint, which under balanced trade, implies labor income equals total nominal expenditure minus profits from the home differentiated sector.

Write the household budget constraint:

$$W_t l_t + \pi_t = P_t C_t,$$

where π is profits of home differentiated goods firms, used in the firm maximization problem above to determine price setting. Use labor supply condition to substitute out wage: $\kappa\mu_t l_t + \pi_t = \mu_t$, and use this to compute the term in welfare including labor, $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home differentiated good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right)\right].$$

Use $C_{DHt} + C^*_{DHt} = \frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}$ from price setting derivation

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + P_{DHt}\frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

$$\text{So } \kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$$

Note that there is no μ_t left in this term, so the labor term in the welfare condition will have no bearing on the optimal monetary policy under our specifcation.

4. Home optimal policy, Nash

Write home welfare, and express as a function of exogenous variables.

$$W_t = E_{t-1} \ln C_t - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - \ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right) - \theta\ln\left(\frac{\phi}{\phi-1}\right) - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)$$
$$-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}\right) + \ln\mu_t - \ln\mu_t^* + \ln\alpha_N^* - \ln\alpha_N - \ln T_{N,t}\right] - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right) - (1-\theta)E_{t-1}\left(\ln\kappa + \ln\mu_t - \ln\alpha_N\right)$$
$$-1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

Differentiate welfare with respect to the home monetary policy variable μ:

$$\frac{1}{\mu} - \frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} - \left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu} = 0$$

$$\mu = \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)}$$

Conjecture the solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$,

which is easily verified by substituting this in the equation immediately above.

e) Foreign optimal policy, Nash

The foreign price index is:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}/e_t\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

Foreign labor:

$$\kappa E_{t-1}\left[l_t^*\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Welfare:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$
$$W_t^* = E_{t-1}\left[\ln\mu_t\right] - E_{t-1}\left[\ln P_t^*\right] - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right] - E_{t-1}\left[\ln\left(2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right] - \ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right) - \theta\ln\left(\frac{\phi}{\phi-1}\right) - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right)$$
$$-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}^*\right) + \ln\mu_t^* - \ln\mu_t - \ln\alpha_N^* + \ln\alpha_N + \ln T_{Nt}\right] - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right) - (1-\theta)E_{t-1}\left(\ln\kappa + \ln\mu_t^* - \ln\alpha_N^*\right)$$
$$1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Which is directly analogous to home, with solution $\mu_t^* = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$

5. Cooperative policy

Take derivative of sum of home and foreing welfare with respect to μ_t

$$\frac{1}{\mu} - \frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} - \left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu} + \frac{\theta}{2}\frac{1}{\mu} - \frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} = 0$$

$$\mu = \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}.$$

Conjecture solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$, which is easily verified

We note that the cooperative solution is same as Nash in this case.

Now take the derivative with respect to μ_t^*:

$$\frac{1}{\mu_t^*}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}-\left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu}+\frac{\theta}{2}\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\theta\frac{1}{\mu_t^*}-\theta\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\mu_t^*=\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\mu_t^*\right]}{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)}$$

Conjecture same solution as before: $\mu_t^* = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$, verified.

This is also same as Nash solution above.

6. Defining conditions under which home is both producer and net importer of homogeneous good

We can easily compute home consumption of the non-differentiated good:

$$C_{Nt}=(1-\theta)\frac{P_t C_t}{P_{Nt}},$$

where $P_{Nt}=e_t T_{N,t}P_{Nt}^*$ and $P_{Nt}=\frac{W_t}{\alpha_N}=\frac{\kappa\mu_t}{\alpha_N}$, $P_{Nt}^*=\frac{W_t^*}{\alpha_N^*}=\frac{\kappa\mu_t^*}{\alpha_N^*}$.

So $C_{Nt} = (1-\theta)\frac{P_t C_t}{\frac{\kappa\mu_t}{\alpha_N}} = (1-\theta)\frac{\mu_t}{\frac{\kappa\mu_t}{\alpha_N}} = (1-\theta)\frac{\alpha_N}{\kappa}$.

Note this is consant, unaffected by tariffs.

Now compute the level of home production in this sector based on labor allocation. Recall total labor allocation above:

$$l_t = \frac{1}{\kappa} - \frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right).$$

We next subtract labor for the differentiated goods sector: $\frac{C_{DHt} + C^*_{DHt}}{\alpha_D}$.

So the labor allocation for the non-differentiated sector becomes:

$$l_{Nt} = \frac{1}{\kappa} - \frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right) - \frac{C_{DHt} + C^*_{DHt}}{\alpha_D}$$

$$l_t = \frac{1}{\kappa} - \frac{1}{\phi\kappa}\frac{\theta}{2}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T^*_{Dt}}\right] - \frac{1}{\alpha_D}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T^*_{Dt}P_{DHt}}\right)$$

$$l_t = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T^*_{Dt}}\right]$$

$$l_{Nt} = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T^*_{Dt}}\right].$$

So output in the sector may be written:

$$y_{Nt} = \frac{\alpha_N}{\kappa} - \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T^*_{Dt}}\right].$$

We conclude that the condition for where $C_{Nt} > y_{Nt}$ may be written:

$$(1-\theta)\frac{\alpha_N}{\kappa} - \frac{\alpha_N}{\kappa} + \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T^*_{Dt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T^*_{Dt}}\right] > 0$$

$$(-1)+\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>1.$$

Evaluate this condition under perfect foresight, abstracting from risk premium in pricing of the sticky price good:

$$\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>1$$

$$1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}>2$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}>1.$$

Derive the condition under which we also can guarantee positive home production of the non-differentiated good:

$$y_{Nt}=\frac{\alpha_N}{\kappa}-\left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2}+\frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$y_{Nt}=1-\left(\frac{1}{\phi}\frac{\theta}{2}+\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0.$$

Again, evaluate in perfect foresight, abstracting from the risk premium in pricing of sticky-price goods:

$$y_{Nt}=1-\left(\frac{1}{\phi}\frac{\theta}{2}+\frac{\theta}{2}\frac{\phi-1}{\phi}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$y_{Nt} = 1 - \frac{\theta}{2}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta} - 1 .$$

Appendix C: Derivation of Analytical Results, Local Currency Pricing

1. Price setting:

Home good now has distinct prices in home and foreign market, P_{DHt} and P^*_{DHt}, with foreign price in foreign currency.

The home firm maximizes :

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p^*_t(h)-\frac{W_t}{\alpha_t}\right)\left(\frac{T^*_t p^*_t(h)}{T^*_t P^*_{DH,t}}\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p^*_t(h)-\frac{W_t}{\alpha_t}\right)\left(p^*_t(h)/P^*_{DH,t}\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

With respect to $p_t(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{1}{P_{DHt}}\frac{\mu_{t-1}}{\mu_t}\left(P_{DHt}-\frac{W_t}{\alpha_D}\right)\left(C_{DH,t}\right)\right]$$

$$P_{dHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=P_{DHt}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]-E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$(\phi-1)P_{DHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]$$

Maximizing with respect to $p^*_t(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{1}{P^*_{DH,t}}\frac{\mu_{t-1}}{\mu_t}\left(e_t P^*_{DH,t}-\frac{W_t}{\alpha_D}\right)C^*{}_{DH,t}\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]-\frac{1}{P^*_{DH,t}}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$(\phi-1)P^*_{DH,t}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]}$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\right]}.$$

Use the property of a homogeneous sector above: $e_t=\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}$:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\right]}$$

and cancel terms:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu^*_t}{T^*_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha^*_N}{\alpha_N T_{Nt}T^*_{Dt}}\right]}.$$

Analogously for foreign differentiated good:

$$P^*_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}E_{t-1}\left[\mu^*_t\right]$$

$$P_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha^*_N T_{Dt}}\right]}.$$

Non-differentiated prices are the same as in PCP case above. Substitute into the home price index:

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}.$$

Analogously for foreign price index:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt}T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

2. Home equilibrium labor:

Use labor supply condition to substitute out wage:

$$\kappa\mu_t l_t + \pi_t = \mu_t .$$

Use this to compute term for labor required in the welfare fucntion: $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home D good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + e_t P_{DHt}^* C_{DHt}^*\right)\right].$$

Use demands from above: $C_{DH,t} = \frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}}$, $C_{DF,t} = \frac{\theta}{2}\frac{P_t C_t}{T_{Dt}P_{DF,t}}$, $C_{DHt}^* = \frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}} + e_t P_{DHt}^*\frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + e_t\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right].$$

Sub in for exchange rate:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + \frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right]$$

So $$\kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$$

Note this is the same as under PCP pricing.

3. Compute welfare:

Home:

$$W_t = E_{t-1}\ln C_t - E_{t-1}\kappa l_t$$
$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right]$$
$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\mu_t\right]\right) - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]\right) - (1-\theta)E_{t-1}\left[\ln \mu_t\right]$$
$$-E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{1}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

Compute derivative of home welfare with respect to μ_t:

$$\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} - (1-\theta)\frac{1}{\mu_t} = 0$$

$$\theta\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} = 0$$

$$\frac{1}{\mu_t} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} + \frac{1}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}$$

Foreign:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t^*\right] - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t^*\right] - E_{t-1}\left[\ln\left(2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt} T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right]$$

$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t} T_{Nt}^*}\right]\right)$$

This is directly analogous to home. Now W* include only μ_t^* and no μ_t .

So directly analogous optimality condition:

$$\frac{1}{\mu_t^*} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t^*\right]} + \frac{1}{2}\frac{\frac{1}{T_{Dt}^*}}{E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}.$$

Appendix D: Derivation of Analytical Results, Home Dominant Currency Pricing

Home exporters face the PCP problem from above, with the implied price setting rule:

$$P_{DHt} = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right) \frac{\kappa \mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right)\right]},$$

which implies the export price:

$$P_{DHt}^* = \frac{1}{e_t} \frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right) \frac{\kappa \mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right)\right]}$$

where the exchange rate still follows: $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$.

Foreign exporters follow the price setting rules derived for the LCP case above:

$$P_{Ft}^* = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} E_{t-1}\left[\mu_t^*\right]$$

$$P_{Ft} = \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} \frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}.$$

These prices imply the home price index:

$$P_t = 2^\theta \left[\left(\frac{1-\theta}{\theta}\right)^{\theta - 1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right] \left(\frac{\phi}{\phi - 1} \frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right) \frac{\kappa \mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N} \frac{1}{T_{N,t} T_{Dt}^*}\right)\right]}\right)^{\theta/2} \left(T_{Dt} \frac{\phi}{\phi - 1} \frac{\kappa}{\alpha_D^*} \frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2} \left(\frac{\kappa \mu_t}{\alpha_N}\right)^{(1-\theta)}$$

Note that this is a hybrid of cases above, since the home good prices are PCP, while home import prices are LCP.

The foreign price index is:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

This too is a hybrid case, since the foreign good prices are LCP, while foreign imports of home goods are PCP.

Home labor supply is the same as the PCP case:

$$\kappa E_{t-1}\left[l_t\right] = 1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right],$$

and foreign the same as the LCP case:

$$\kappa E_{t-1}\left[l_t\right] = 1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$$

Home welfare may be computed:

$$W_t = E_{t-1}\left[\ln\mu_t\right]-\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right)-\theta\ln\left(\frac{\phi}{\phi-1}\right)-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right)$$
$$+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}\right)-\ln\alpha_D^*\right]-\frac{\theta}{2}\ln\left(E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]\right)$$
$$-\left(1-\theta\right)E_{t-1}\left(\ln\kappa+\ln\mu_t-\ln\alpha_N\right)-1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

Note that the home policy variable, μ, interacts with both home and foreign tariffs, T_{Dt} and T_{Dt}^*, inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will involve both tariffs. Note also that the foreign policy variable, μ_t^*, does not appear in this welfare computation for home.

Foreign welfare is

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right]-E_{t-1}\left[\ln\left(\begin{array}{c}2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\\ \left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\end{array}\right)\right]-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

Note that the foreign policy variable, μ_t^*, does not interact with either home or foreign tariffs, T_{Dt} and T_{Dt}^*, inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will not involve either tariff.

The optimality conditions for both Nash and cooperative problems are

$$\mu_t = \frac{1}{2}\left(\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)} + \frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{\frac{1}{T_{Dt}}} \right)$$

for home policy, and

$$\mu_t^* = E_{t-1}\left[\mu_t^*\right]$$

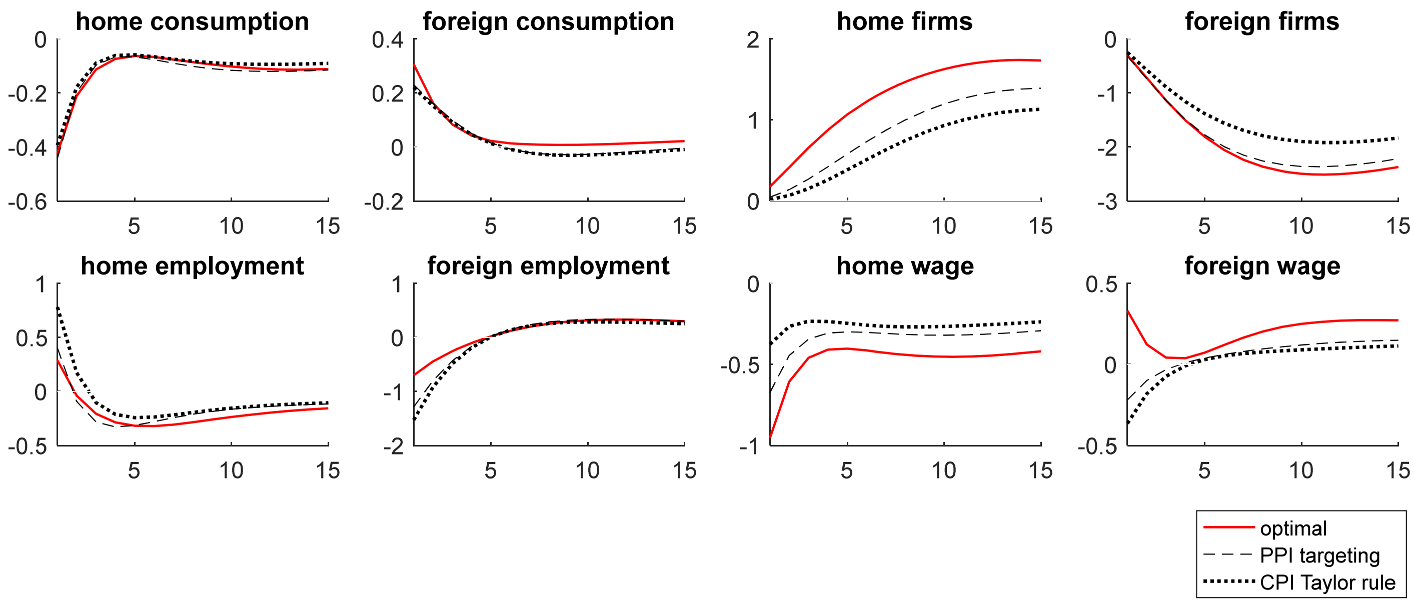
for foreign.

Appendix E. Discussion of Simulation Results for LCP and Foreign Dominant Cases

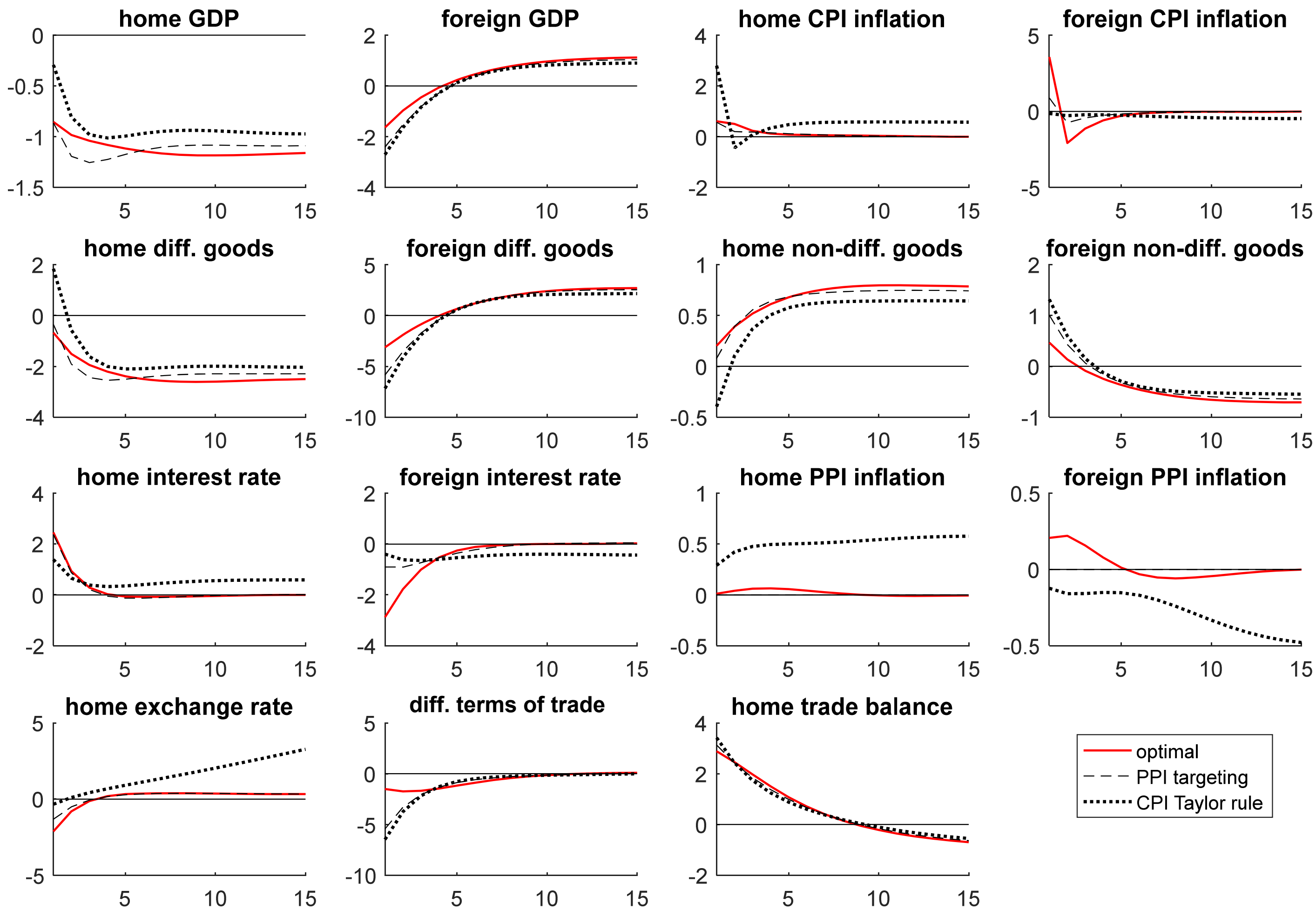
Appendix Figure 3 shows impulse responses for an environment with local currency pricing (LCP) in both countries. In this case, consistent with our analytical section, the home interest is lower than that implied by the PPI targeting rule, implying a relative expansion, and home PPI inflation as sharply positive. While the analytical result does not indicate any response in foreign monetary policy stance, in the richer environment of the simulation model, the optimal foreign response to the home tariff is mildly contractionary—the interest rate is above that implied by the PPI target, and there is a moderate fall in PPI inflation.

We also analyze the case in which the dominant currency is issued by the country targeted by tariff, i.e., the foreign country. The dynamic responses are shown in Appendix Figure 4. The analytical result in the simplified model implied that optimal policy in the country with the dominant currency in this case should resemble that under PCP. In the case of the present simulation, this prescription implies the foreign (dominant) country should expand in response to the home (non-dominant) tariff. Indeed, Figure 4 shows a sharp foreign expansion, with interest rate falling more than that implied by PPI targeting, and with a substantial PPI inflation. While the analytical result did not call for any policy response from the non-dominant country (home country in this case), the simulation in Figure 4 shows a mild contraction, with interest rate slightly above that of the PPI targeting rule, and a small reduction in PPI inflation.

Appendix Figure 1. Additional Impulse responses for benchmark case: home tariff on differentiated imports

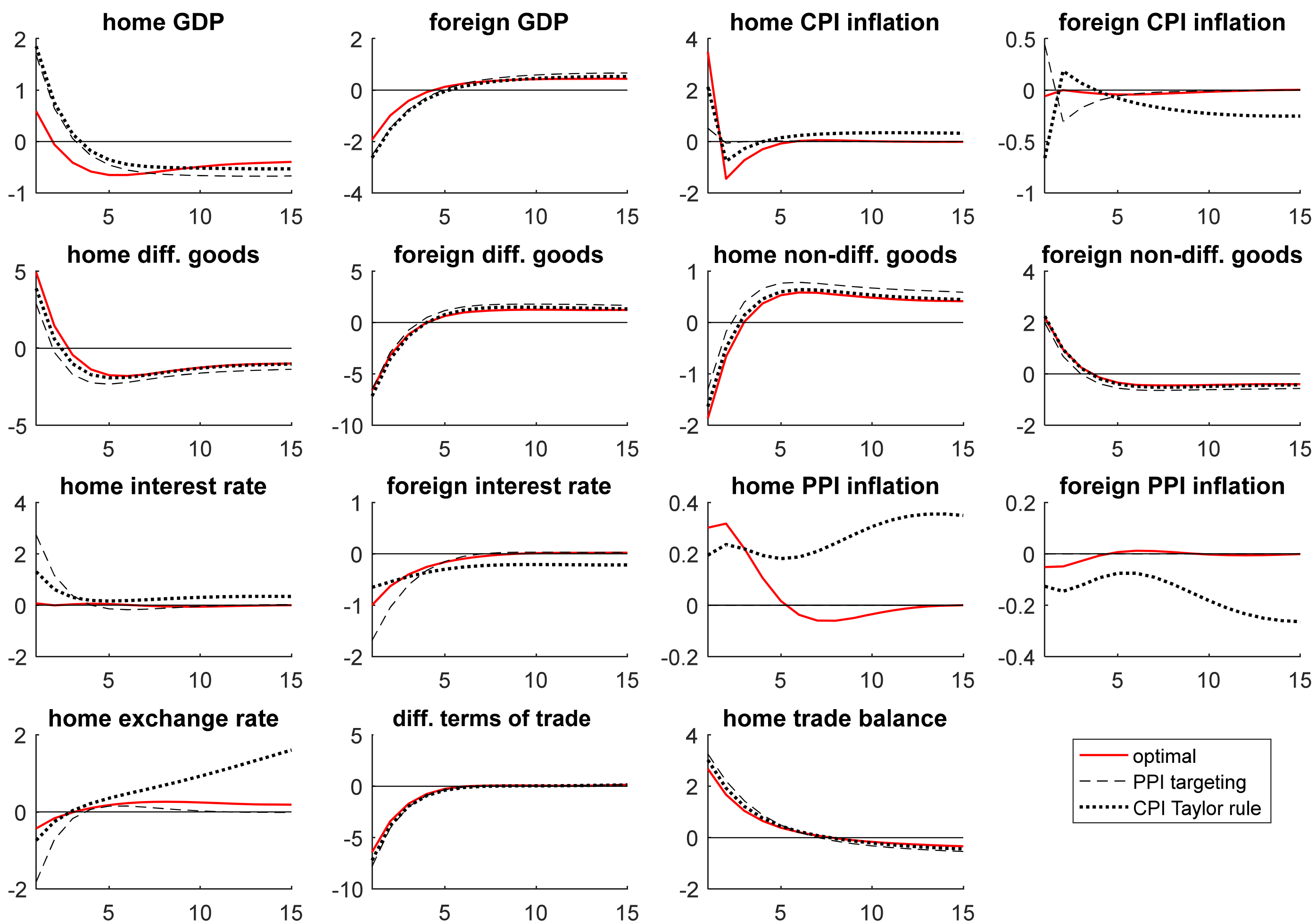


Appendix Figure 2. Impulse responses to a rise in home tariff on differentiated imports, lower trade elasticity (3.8)



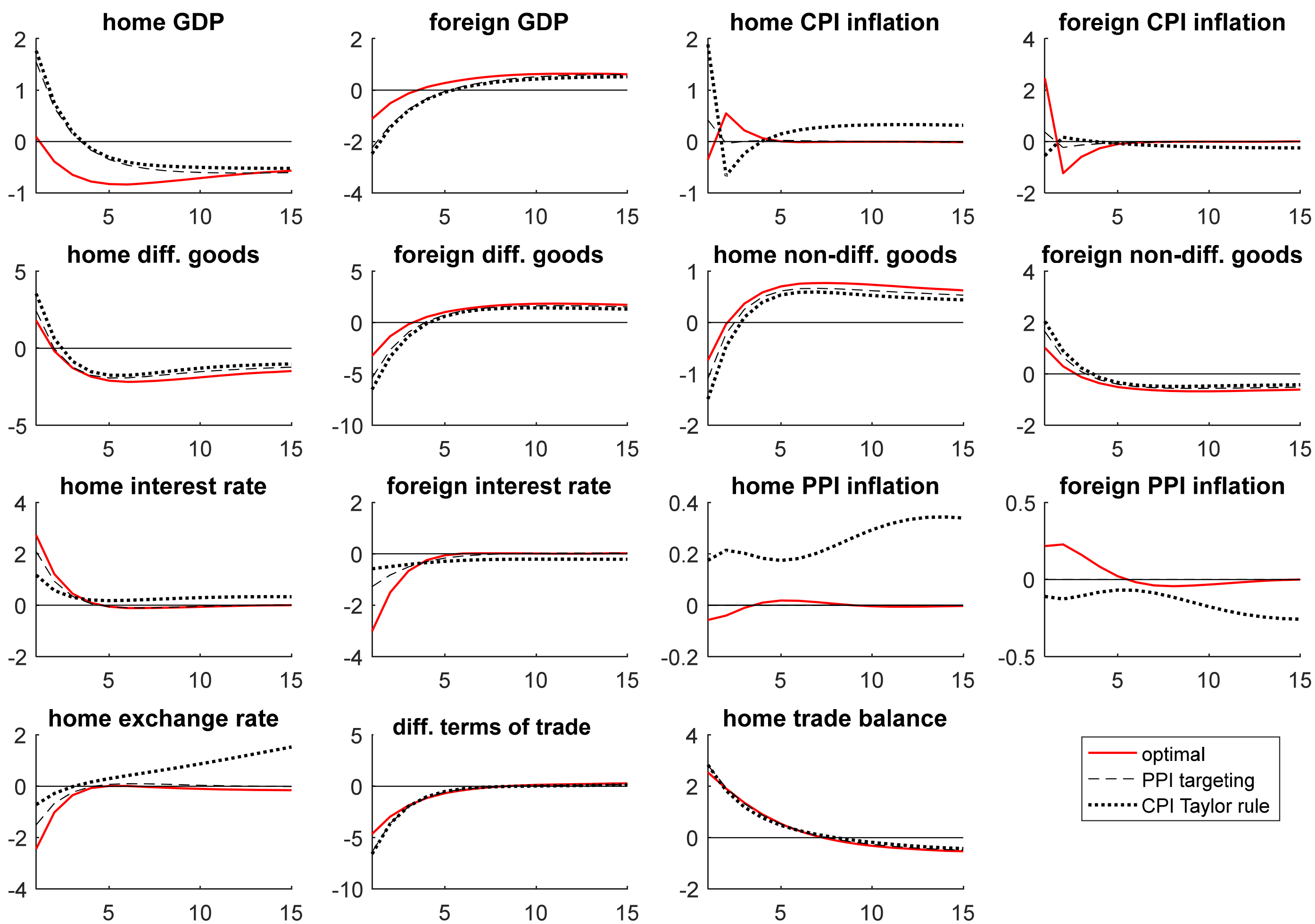
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 3. Impulse responses to a rise in home tariff on differentiated imports; LCP price stickiness in both countries



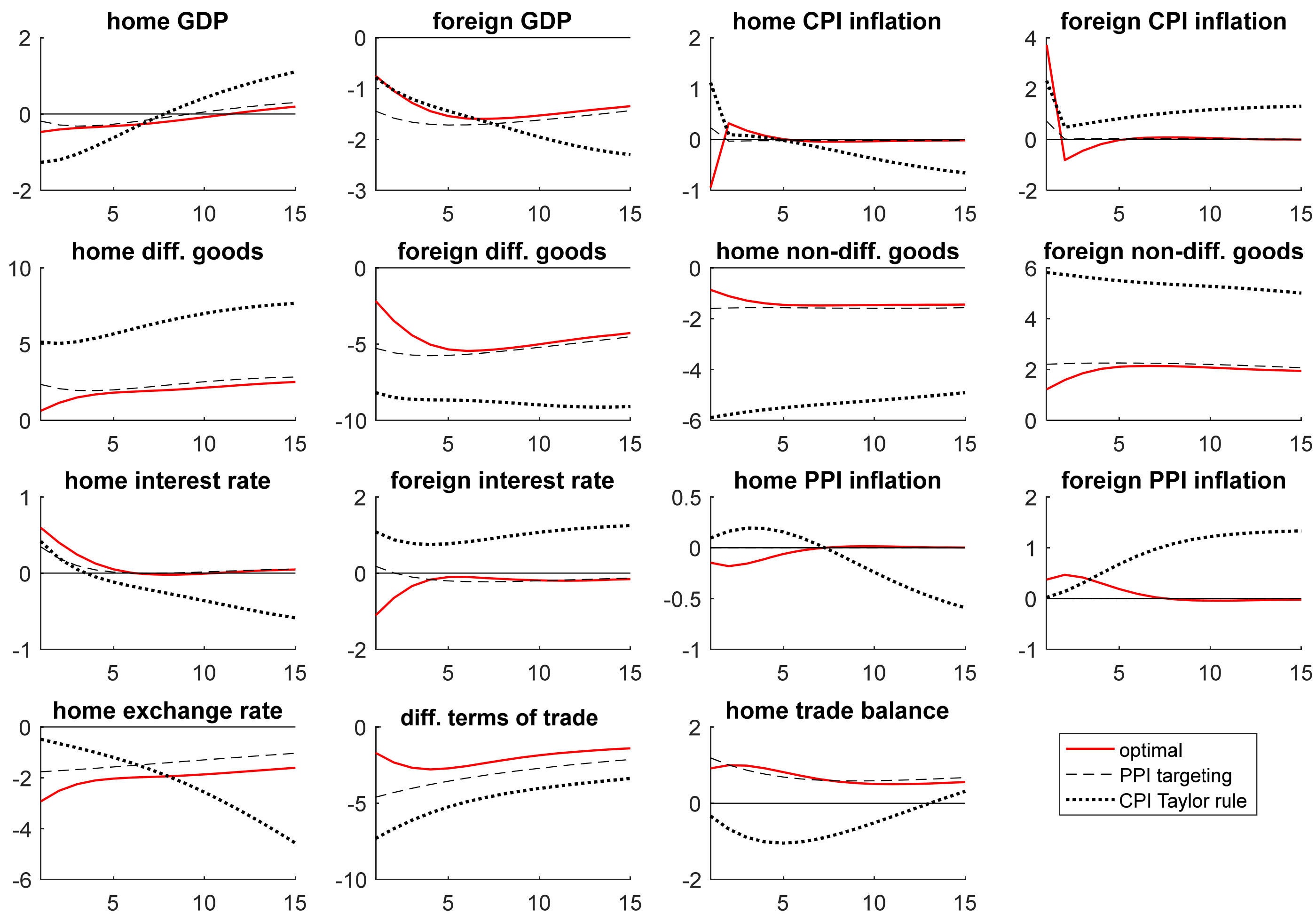
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 4. Impulse responses to a rise in home tariff on differentiated imports; foreign currency dominant



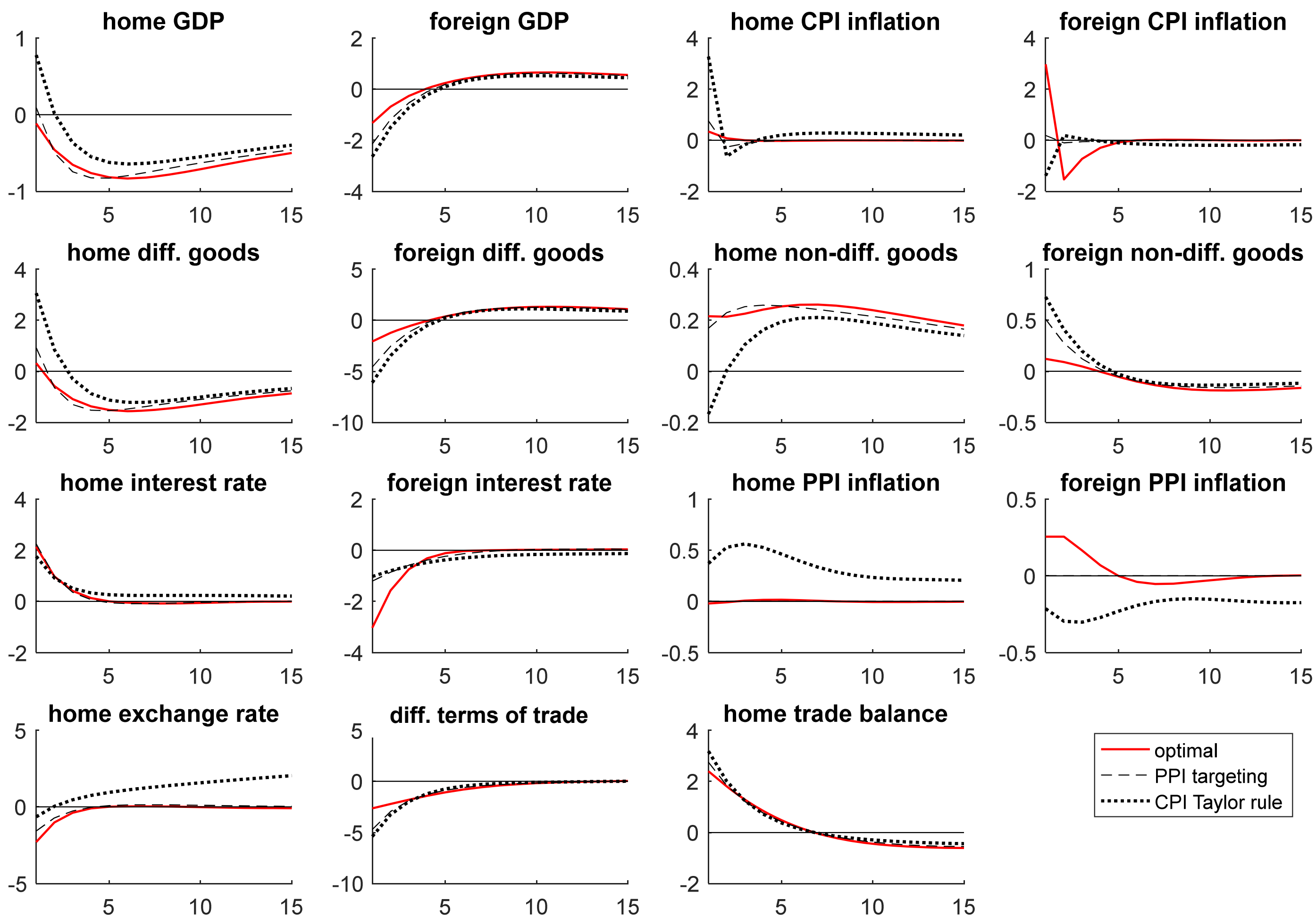
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 5. Impulse responses to a more persistent tariff on home differentiated imports



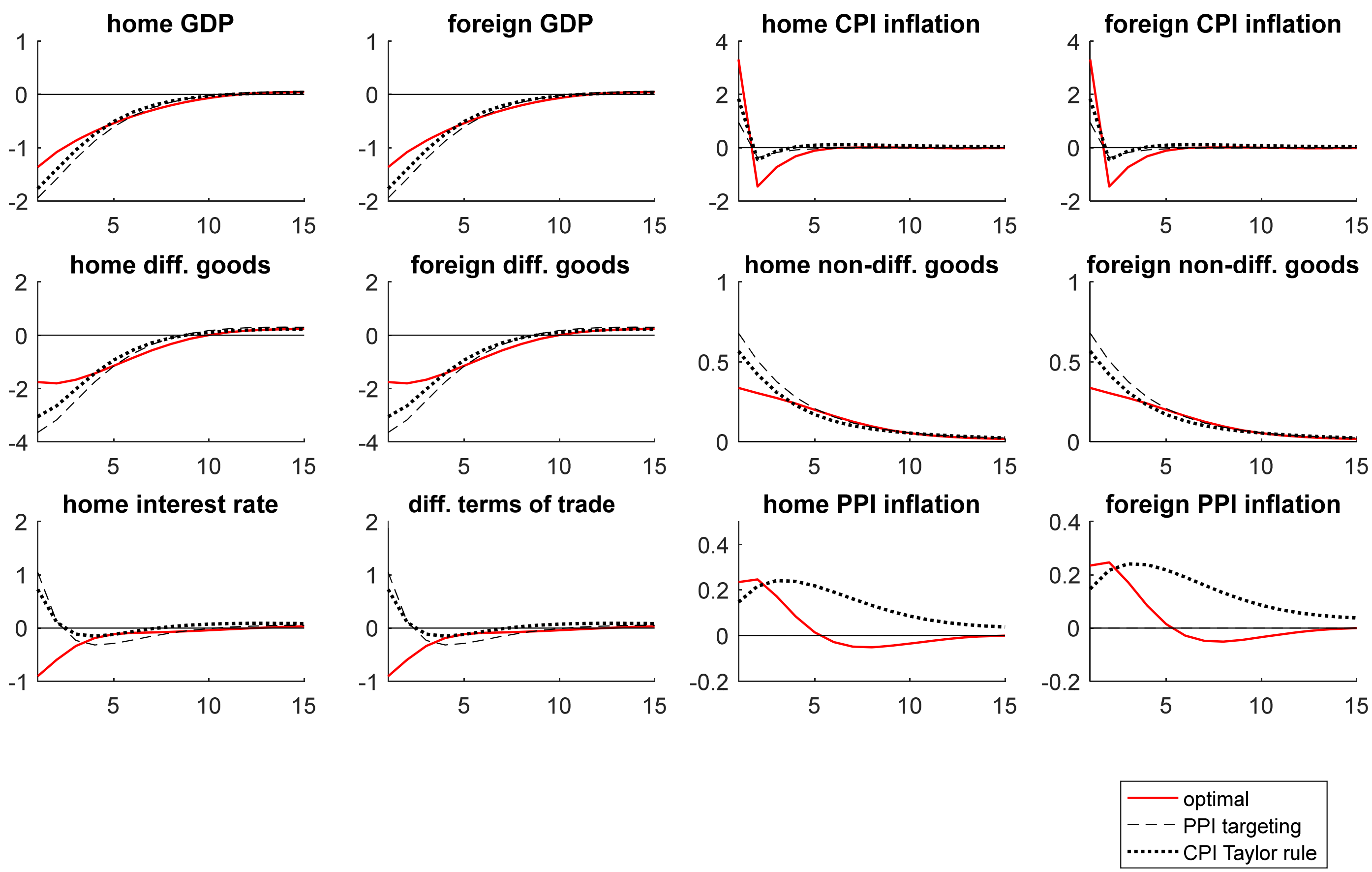
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 6. Impulse responses to home tariff on differentiated imports, nontraded non-diff. goods



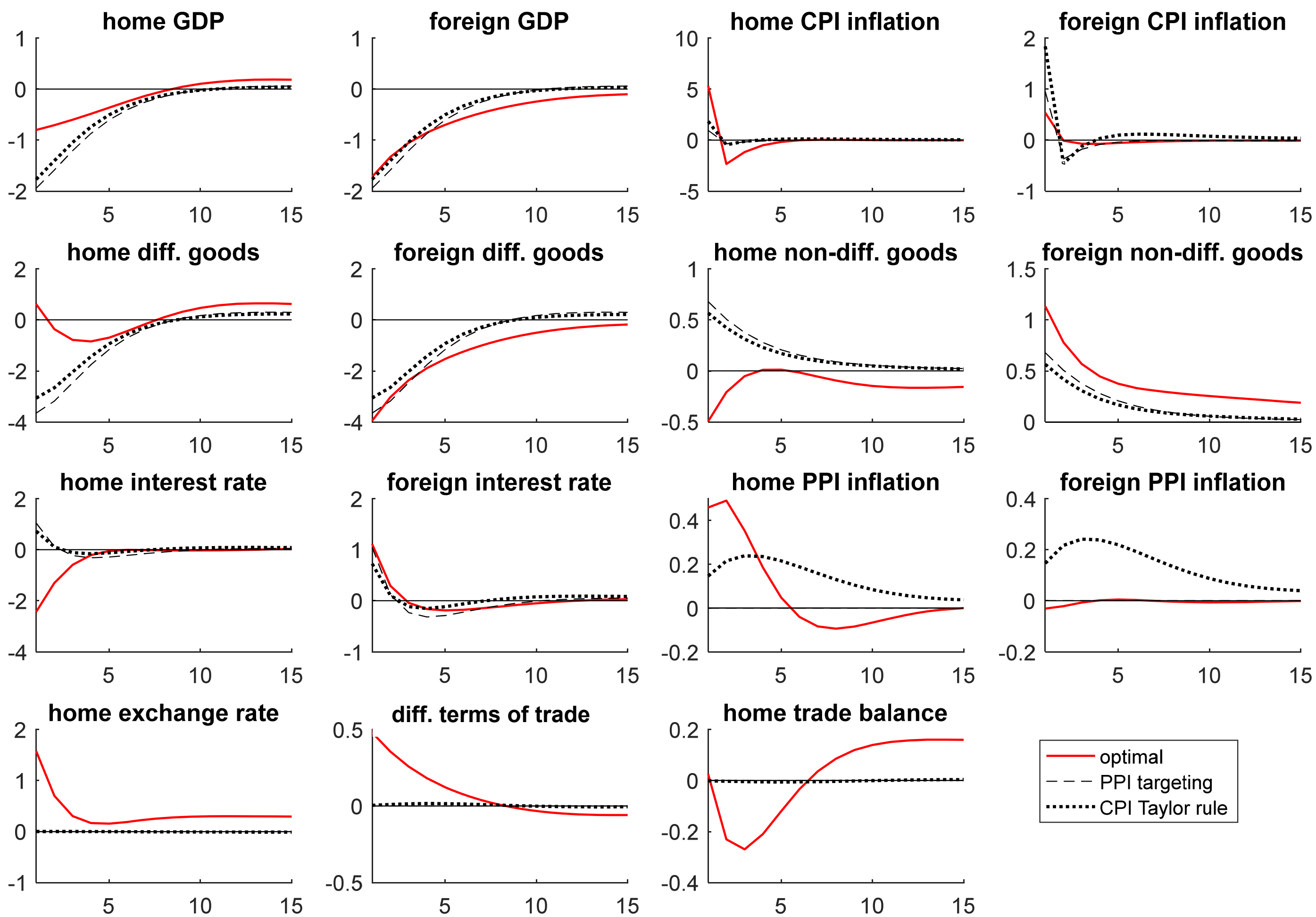
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 7. Impulse responses to a symmetric tariff to differentiated imports in both countries



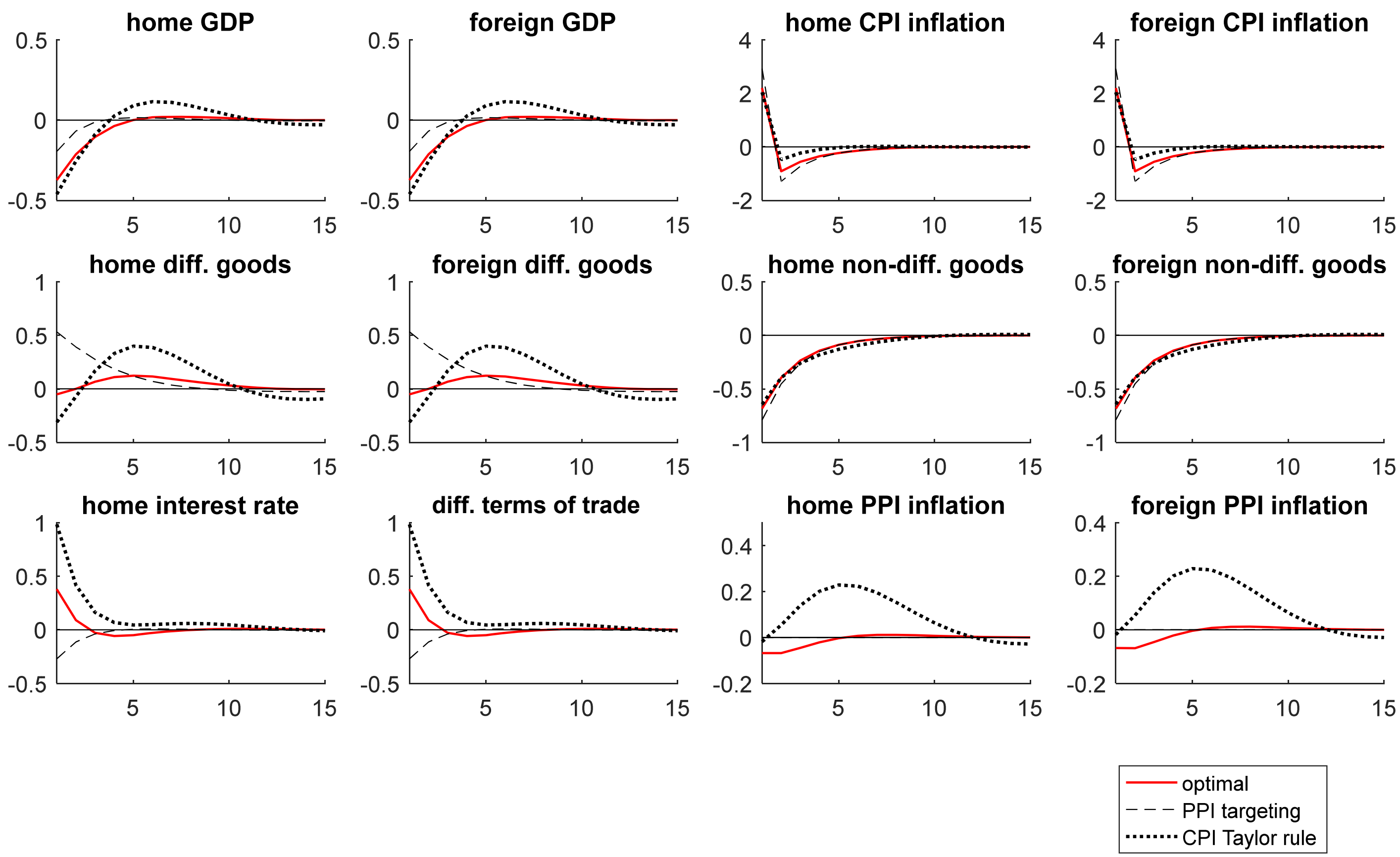
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 8. Impulse responses to a symmetric tariff to differentiated imports in both countries; home currency dominant



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 9. Impulse responses to a symmetric tariff to non-differentiated imports in both countries



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.