# Trade and Firm Financing

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**Abstract**
This paper studies how financial frictions pose a barrier to export entry by altering the firm's long-term capital structure, and thereby affecting the ability to finance sunk entry costs. Our focus on long-term firm financing stands in contrast with the emphasis in recent trade literature on the financing of short-term working capital as a barrier to export entry. We provide evidence that U.S. firms engaged in export tend to have leverage ratios higher than non-exporting firms in terms of long-term debt, but not in terms of short-term debt. To explain this fact and understand its implications, we marry a corporate finance model of capital structure, featuring an endogenous choice between equity and long-term debt financing, with a trade model featuring heterogeneous firms. The model of optimal capital structure indicates that in the long run, exporting firms will prioritize reducing the cost of long-term capital, used to pay sunk costs, over relaxing a short-term working capital constraint, which could be used to scale up production.

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## 1. Introduction

The trade collapse during the global financial crisis of 2007-9 served to highlight the importance of financial considerations in shaping international trade.[1] It has been argued in the trade literature that access to short-term external financing as working capital to cover the costs of production and shipping is an important barrier to participation in exporting. (See, for example, Amiti and Weinstein, 2011; Chaney, 2016; Manova, 2013; Manova, Wei and Zhang, 2015; Minetti and Zhu, 2011.) In particular, Kohn et al. (2016) presents this type of financial friction as an alternative to the more standard sunk export entry costs emphasized as barriers to export entry in earlier trade literature. In international markets, sunk costs typically represent the expenditure on establishing a commercial and distribution network abroad (see for example, Das et al., 2007; Alessandria and Choi, 2014).

This paper raises the question of how firms go about financing the payment of these one-time sunk entry costs. It argues that financial frictions that affect long-term firm financing, used to pay sunk entry costs, may be even more consequential than those affecting short-term working capital used to finance current production. Intuitively, if external financing is needed for working capital to pay for labor or shipping costs before sales are realized later in a given period, then the role for external financing likely is even more relevant when a one-time sunk entry cost needs to be paid long in advance of the long-term stream of future profits that motivates the entry decision. However, the existing trade literature studying financial frictions generally models only the external financing of short-term working capital recurring each period, not the financing of the long run, one-time sunk cost. While sunk entry costs typically are present in these models, no mention is made of how firms pay the sunk entry cost.[2]

[1] See Amiti and Weinstein (2011), Bricongne et al. (2012), Paravisini et al. (2015), and Chor and Manova (2012), etc. For a detailed literature review on the empirical evidence, see Bems et al. (2013).
[2] Even though Kohn et al. (2016) posit a general theoretical model with sunk cost in the working capital constraint, this is only a device to nest their two cases; the models they simulate either include a sunk cost or

As motivation for emphasizing long-term firm financing, we present original empirical evidence, using financial data of public U.S. manufacturing firms which distinguishes between long-term debt with maturities of greater than a year and shorter-term debt like working capital. Not only is it immediately clear that the average firm's long-term debt is multiple times larger than the short-term debt, but we present a new stylized fact that long-term debt is even more important for exporters than for non-exporters. In particular, panel regressions indicate that exporting firms tend to have a higher leverage ratio of long-term debt relative to overall firm value, while this is not true for short-term debt. About two-thirds of the higher leverage ratio of exporters can be attributed to their larger firm size. This is not entirely surprising, as the corporate finance literature previously has documented that larger firms tend to be more leveraged in their capital structures, while the trade literature has documented that exporters tend to be larger in size than non-exporters.[3] About one-third of the extra leverage of exporters remains after we control for firm size.

The theoretical contribution of the paper is to develop a model to explain the empirical evidence above, and then lay out implications for how financial constraints limit firms' participation in the export market by affecting long-term firm financing.[4] We thus study an environment with both short-term debt for working capital and long-term debt that can be used to finance sunk cost outlays. We marry a corporate finance model featuring a capital structure choice between equity and long-term debt, with a trade model featuring heterogeneous firms and export entry.

In particular, the model includes a short-term working capital constraint, where access

financial frictions, never both together. Further, we will argue that the financing of sunk cost should be viewed as a combination of equity and long-term debt, not financed out of short-term working capital.

[3] See Xu, 2012; Kurshev and Strebulaev, 2015.

[4] The model builds on the closed economy model of Bergin, Feng and Lin (2018a), but studies heterogeneous firms, as well as an open economy setting with financing related to export activity.

to intratemporal debt must be secured with collateral, mainly in the form of firm equity. This intratemporal debt pays no interest and must cover the variety of production costs incurred within each period, such as wage costs and per-period fixed costs of domestic production, as well as trade costs. Firms also face a capital structure decision regarding long-term firm financing. Firms have the ability to issue intertemporal (interest bearing) debt and sell firm equity. The choice between debt and equity financing has consequences for real economic activity in our model, as tax benefits of debt suspend the Modigliani-Miller (MM) theorem.[5] So firms face a tradeoff between long-term and short-term debt, where long-term debt is a cheaper form of long-term firm financing due to tax advantages, but it also lowers the amount of equity collateral available to secure short-term working capital.

An entry condition specifies that firms enter the domestic market if the firm value equals or exceeds a one-time sunk entry cost; likewise for entering the export market, if the value of exporting exceeds a one-time sunk export entry cost. Given that the entry condition equates entry cost to firm value, and the firm financing decision divides this firm value into debt and equity, these conditions together describe the means by which the marginal firm finances payment of the sunk entry cost. As shifts in long-term leverage affects the cost of firm financing, it alters the effective cost of financing entry.

We study the long-run implications for participation of the export market by solving for the general equilibrium of the model in steady state.[6] The model provides a ready

[5] This is consistent with developments in the corporate finance literature, where there is ample empirical evidence of failure in Modigliani-Miller (see Rajan and Zingales (1995); Berger and Udell (1998); Hovakimiana, Hovakimian and Tehranianc (2004)), as well as significant research focused on the microeconomic implications of capital restructuring between debt and equity (see Strebulaev and Whited (2011) for a survey).

[6] We find that endogenizing the capital structure actually makes it significantly easier to solve for general equilibrium, as it allows us to apply the standard approach of Melitz (2003) to aggregate over heterogeneous firms even in the presence of firm financial constraints. This is an approach that could be useful to others in the literature. For example, Kohn et al (2016) solve for firm behavior under heterogeneity and financial constraints in partial equilibrium, taking wages, interest rates and demands as given. Kahn and Thomas (2013) require significantly more elaborate numerical methods to solve for general equilibrium implications in their

explanation for the empirical finding above that exporting firms are more leveraged. As the size of firm sales grows with the firm-specific productivity level, profits and hence firm value grow proportionately. But the presence of fixed costs implies the need for working capital to finance production costs grows less than proportionately with size. So there is less benefit from firms issuing equity as collateral to procure working capital, and the firm raises leverage to benefit from the tax benefits of debt. This logic applies in particular to exporters, since the entry condition implies more productive and hence larger firms with greater firm value self-select into exporting. To further explain why exporters choose more leverage than non-exporters even after controlling for larger firm size, the model appeals to the idea that exporter status confers firms with access to additional sources of collateral other than equity.

One lesson from the calibrated model is that it is optimal for exporters to prioritize the cost of long-term firm financing over relaxing the short–term working capital constraint. Even for exporters that are large and hence have easier access to short-term working capital, optimal capital structure indicates that these firms should not use this advantage to further relax their working capital constraint and scale up production closer to the unconstrained optimal, but rather to restrain equity collateral so as to keep the tightness of their working capital constraint to be the same as non-exporters. The motivation is to reap the tax benefits of long-term debt relative to equity.

A corollary to the finding above is that the distinction emphasized in recent literature, whereby exporting is associated with greater working capital needs, should be largely neutralized by an endogenous capital structure in long-run equilibrium. The optimality condition implied by our capital structure problem indicates that as one exogenously raises the working capital requirements for export sales relative to domestic sales, firms will

model with financial frictions and heterogeneity.

optimally choose to adjust their capital structure to provide extra equity collateral to compensate, and this will nearly offset any effect on the level of sales from higher working capital requirements. Nonetheless, higher working capital requirements are found to have a significant dampening effect on trade, but through a mechanism very different from past literature. Rather than reducing the intensive margin of trade per firm, it reduces trade at the extensive margin: the capital structure adjustment to the short term financial constraint raises the cost capital in long-term firm financing, and hence raises the cost of financing the sunk entry cost of exporting.

Our paper is related to the empirical literature studying the importance of financial frictions in shaping international trade. It is motivated in part by early work such as Greenaway et al. (2007), which argued exporting firms appear to be financially healthier than non-exporters, in that they had lower leverage and greater liquidity. Provocatively, their tests reject the idea that firms with greater financial strength ex-ante tend to become exporters, but rather find that the distinctive financial position of exporters appears to be the consequence of export status. We believe this points to the need for the trade literature to understand firm leverage as an endogenous decision by firms, and how this decision can be affected by export activity.[7]

This paper is closely related to Kohn et al. (2016), which examines the role of financial frictions on working capital in shaping new exporter dynamics in a partial equilibrium model, and finds that financial frictions reduce the benefit from trade liberalization. We differ in introducing capital structure choice for long-term firm financing, and in arguing that financing of sunk entry costs is an important barrier to entry in the long run.[8] Similarly,

[7] The fact we distinguish between short term and long-term finance allows us to be consistent with the finding of Greenaway et al. (2007) as a special case, but also to go beyond it in scope. Our finding that short term leverage is smaller for exporters echoes their finding, which only focuses on short term debt. However, we find that overall leverage is greater for exporters than non-exporters, and this is driven by the fact that long term leverage is much higher, and this dominates the overall result.

[8] Our result focusing on the long run is complementary rather than contradictory to that of Kohn et al. (2016),

Caggese and Cunat (2013) find in a dynamic model that financing constraints deter firms from entering export markets directly due to the binding financing constraint and indirectly due to the precautionary motive to avoid increasing their bankruptcy risk, and hence reduce the aggregate productivity gains induced by trade liberalization. Again we differ, in that their model precludes any borrowing to finance sunk entry costs. Along similar lines, Chaney (2016) finds that more productive firms that generate large liquidity from their domestic sales, and wealthier firms that inherit a large amount of liquidity, are more likely to export. We differ from these in studying the capital structure of firms that have entered the export market.[9]

The next section of the paper presents empirical work supporting our stylized fact. Section 3 presents the benchmark model and section 4 some analytical insights from it. Section 5 uses numerical solutions to demonstrate the model can explain the key facts, along with sensitivity analysis to explore the mechanism. Section 6 uses additional sensitivity analysis to uncover novel implications for how financial frictions interact with endogenous capital structure to pose a barrier to export entry. Section 7 concludes.

## 2. Empirical Motivation

We use a panel dataset which covers the financial data of public U.S. manufacturing firms

which focus on transition dynamics. For example, if we augment our model with adjustment costs for changing dividends, in order to introduce meaningful dynamics, the short-run of the model would be consistent with the result of Kohn et al., and retain the long-run implications we study. However, our methodology for solving for general equilibrium is not readily extended to solving for dynamics.

[9] Our paper is also related to the many empirical studies that investigate the role of financial frictions and financial shocks in shaping international trade. For example, Amiti and Weinstein (2011), Paravisini et al. (2012), Bricongne et al. (2012), and Chor and Manova (2012) find a causal effect of credit disruptions depressing firm exports subject to tighter credit constraints. This paper differs, both in studying the effect of exporting on firm finance, rather than the other way around, and in studying the steady state effect of export participation in normal times, rather than the effects on exporting of a financial shock. We also are related to theoretical work in Russ and Valderrama (2012); however, they study a different financial choice, between alternative forms of debt finance, rather than equity versus debt, and they use a model based on fixed versus variable costs of bond financing, rather than a finance model with collateral constraints and endogenous capital structure. Smith and Valderrama (2009) does show a choice of firm financing between equity and bonds (as well as FDI), but it does not study the trade dimensions of interaction with firm heterogeneity and participation in exporting.

from 1975 to 2014 to study the relationships of financial choice, firm size and trade behavior. Financial data come from Compustat. Since our interest focuses on the firms' choice between debt and equity financing, the leverage ratio in the benchmark model is defined as book debt to total assets which is defined as in Baker and Wurgler (2002), where book debt is defined as total asset minus book equity. We also will consider other measures of debt with varying maturities, when computing leverage ratios as shares of total assets: short-term borrowing, long-term debt, debt in current liabilities, and book debt minus short-term borrowing.[10]

Table 1 reports summary statistics for the full sample of firms, and subsamples based on firms' export market participation. Comparison of columns (2) and (3) indicates a clear pattern in terms of leverage among these groups. Exporters tend to be more leveraged than non-exporters in overall book debt, and in the two categories listed as long-term debt and book debt minus short-term debt. However, exporters are less leveraged in measures of debt labeled as short term or current liabilities. We infer that the greater overall leverage of exporters compared to non-exporters is due to longer term debt, and not due to greater short-term debt. This stands in contrast with the usual focus in the trade literature on working capital and trade credit, included in short-term debt, and instead suggests additional focus should be placed on the decisions determining longer-term forms of debt financing. An examining of the underlying data indicates the raw value of short-term debt per firm indeed is larger on average for exporting firms than non-exporting terms. But exporting firms are on average much larger than non-exporting firms, and taken as ratio to total assets, short term debt is smaller for exporters.

[10] Short-term borrowing represents the approximate average aggregate short-term financing outstanding during the company's reporting year, which are usually in the form of lines of credit with banks. Long-term debt represents debt obligations due more than one year from the company's Balance Sheet date or due after the current operating cycle. Debt in current liabilities represents the total amount of short-term notes and the current portion of long-term debt that is due in one year.

Columns (4) and (5) report results for subsamples of newly exporting firms, and continuing exporting firms (where the former are defined as firms that export in a year but not the previous year). The leverage ratio of the two subsets of exporters are very similar to each other, and hence to the full sample of exporters discussed above. For our purposes, this supports the choice to use a model that does not focus on dynamics of new entry for the current issue at hand.

To investigate this pattern more systematically, we estimate panel regressions. The benchmark regression takes the form:

$$Y_{j,t} = \alpha_0 + \alpha_1 size_{j,t} + \alpha_2 D\_\exp_{j,t} + F_j + \phi t + \varepsilon_{j,t} \quad (1)$$

where $Y_{j,t}$ is the leverage ratio of firm $j$, and $size_{j,t}$ is measured as the log of net sales of firm $j$. The regressor $D\_\exp_{j,t}$ indicates if the firm is a net exporter (its foreign income is greater than 0). We include a firm fixed effect $F_j$ to control for the large set of firm-specific characteristics that the corporate finance literature has found to influence a firm's choice of leverage, such as industry in which the firm operates. We assume such firm characteristics do not vary over time. A time fixed effect $t$ is also controlled in the regression.

Estimates of Eq. (1) are presented in Colum (1) of Table 2, indicating that size is significantly and positively correlated with the leverage ratio ($\alpha_1 > 0$, significant at the 1% level). The coefficient estimate indicates that as the firm size doubles, the leverage ratio increases about 1.2 percentage points on average. This is not a surprise, as a standard finding in the large corporate finance literature on firm capital structure is that firm size raises leverage (see Kurshev and Strebulaev (2006), and Xu (2012)). Regression results also indicate that if the firm is a net exporter, it has a higher overall book leverage ratio

($\alpha_2 > 0$, significant at the 1% level). Given that we control for size, this finding indicates that comparing firms with the same size, the leverage ratio will be higher by 1.58 percentage points if the firm is associated with exporting.

So there are two reasons why exporters are more leveraged on average than non-exporters. First, given that exporters tend to be larger than non-exporters (as supported in Table 1), this in itself tends to make exporters more leveraged. But the finding that $\alpha_2 > 0$ indicates there is something beyond size leading to higher leverage for exporters. By combining information from the panel regressions with the mean levels of regressors we can compute that a bit over two-thirds (69.0%) of the higher leverage ratio observed for exporters on average is due to their larger size, and the remaining one-third is due to some other factor associated with export status.[11]

To discuss how the maturities of debt affect the results, we replace the leverage ratio in the benchmark regression by the ratio of short-term borrowing to total assets, the ratio of long-term debt to total assets, the ratio of debt in current liabilities to total assets, and the ratio of book debt minus short-term borrowing to total assets. Results reported in Table 2 indicate that firm size and dummy of net exporter are not significantly correlated with short-term borrowing to total assets (see columns (2) and (4) in Table 1). However, as the terms of debts become longer, the ratios of debt to total assets are consistently positively correlated to size and being a net exporter (see Columns (3) and (5) in Table 1). This finding further supports our choice of model that studies the choice of long-term debt rather than

[11] In particular, by substituting the average log sales from a given year (2014, since this is the final year of the sample) into the regression equation with estimated coefficients (including fixed effect coefficients not reported in Table 2), we compute a leverage ratio of 0.4266 for the average non-exporter in that year. By including the value of the estimated exporter dummy, we compute a leverage ratio of 0.4775 for the average exporter in that year. We then synthetically construct the predicted value for the leverage ratio of a non-exporter with the same size as the average exporter (using the average size of exporters but excluding the exporter dummy) to be 0.4617. This implies that the larger size of the average exporter explains (0.4617 - 0.4266) / (0.4775 – 0.4266) = 68.96% of the higher leverage ratio of the average exporter compared to non-exporter, with the remaining portion of the total gap (0.4775 - 0.4617) / (0.4775 – 0.4266) = 31.04 % explained by the exporter dummy.

shorter-term working capital loans alone.

## 3. Benchmark Model

The model considers a small open economy, where the home country is in financial autarky, but trade is integrated with the rest of world. There are three different sectors: (1) a perfectly competitive final goods sector whose goods will be consumed domestically, (2) a monopolistically competitive intermediate goods sector where some producers are exporters and the rest are non-exporters, (3) a representative household who supplies labor to domestic intermediate firms and finances domestic intermediate firms through equity and bond purchases. For clarity, we present here the equations for the full dynamic model, although our analysis will focus on the steady state solution for this model. (Appendix 1 lists the corresponding equations for the steady state.) This steady state analysis is appropriate, given that the goal of the model is to replicate key features of the cross-sectional distribution of leverage ratios across firms discussed in the empirical section, and given that these leverage ratios were computed empirically as averages over time.

### 3.1 Households preferences and optimization

There is a continuum of homogeneous households who derive utility from consuming the basket of goods ( $C_t$ ) and disutility from labor supply ( $L_t$ ) in each period, and maximize expected lifetime utility,

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \qquad with \qquad U(C_t, L_t) = \frac{C_t^{1-\rho}}{1-\rho} - \kappa \frac{L_t^{1+\psi}}{1+\psi},$$

where $\rho > 0$ is the households' degree of risk aversion, $\beta \in (0,1)$ is the subjective discount factor, and $k, \psi > 0$ are the relative utility weight of labor and the inverse Frisch elasticity of labor supply, respectively.

The households derive income by providing labor services ( $L_t$ ) at the real wage rate ( $w_t$ ), and receiving payments from holding the corporate bond portfolio ( $\tilde{b}_{t-1}$ ) and dividends ( $\tilde{d}_t$ ) from holding the share ( $s_{t-1}$ ) of the equity portfolio of the $N_{t-1}$ existing firms. The households then purchase consumption ( $C_t$ ), and update their corporate bond portfolio holdings at the interest rate of $r_t$ and equity investments at a price of $\tilde{q}_t$ to the $N_{t-1} + Ne_t$ existing firms and pay the lump-sum taxes $T_t$. Here, $Ne_t$ represents the number of new entrants to the intermediate goods sector.

The period budget constraint thus may be written as:

$$C_t + \left(N_{t\text{-}1} + Ne_t\right)\frac{\tilde{b}_t}{1+r_t} + s_t\tilde{q}_t\left(N_{t\text{-}1} + Ne_t\right) + T_t \le w_t L_t + N_{t-1}\tilde{b}_{t-1} + N_{t-1}s_{t-1}\left(\tilde{q}_t + \tilde{d}_t\right),$$

where $T_t = \sum_i^{N_t}\left(\frac{b_{i,t}}{1+r_t(1-\tau)} - \frac{b_{i,t}}{1+(1+r_t)}\right)$ is the lump-sum taxes that are used to finance the tax benefits of bonds for firms.

The households maximize expected lifetime utility subject to the budget constraint, implying the following first-order conditions:

$$U_{C_t} w_t + U_{L_t} = 0, \tag{1}$$

$$\beta\left(1\text{-}\lambda\right)E_{\mathrm{t}}\left[U_{C_{t+1}}\left(1+r_t\right)\right] = U_{C_t}, \tag{2}$$

$$\beta\left(1\text{-}\lambda\right)E_{\mathrm{t}}\left[U_{C_{t+1}}\left(\tilde{q}_{t+1} + \tilde{d}_{t+1}\right)\right] = U_{C_t}\tilde{q}_t \tag{3}$$

where Eq. (1) is the labor-leisure tradeoff condition, Eqs. (2) and (3) are the Euler equations for holding corporate bond portfolio and equity portfolio. $\lambda$ is the probability of the exogenous death shock that hits the intermediate goods producers at the end of each period.

### 3.2 Final goods sector

The final goods sector is perfectly competitive. The final goods ( Y ) are produced using intermediate goods which are domestically produced and imported, with a CES production function described below,

$$Y_t \equiv \left[ \left(Y_t^D\right)^{\frac{\sigma-1}{\sigma}} + \left(Y_t^{X*}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma$ is the substitution elasticity across different varieties, regardless of the origins of the products. $Y_t^{X*}$ is the foreign exports. To simplify the model, we assume the imported bundle is a standardized unit and do not consider variety changes of imports.

$Y_t^D$ is the composite of all domestic products produced by home non-exporters, indexed by $nx$, and by home exporters, indexed by $x$. The production function is a CES aggregator,

$$Y_t^D = \left( \int_{i \in I_t^{nx}} y_{it}^{nx\,\frac{\sigma-1}{\sigma}} di + \int_{i \in I_t^{x}} y_{it}^{xd\,\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where $y_{it}^{nx}$ represents domestic market demand for the good produced by home non-exporter $i$, and $y_{it}^{xd}$ is domestic market demand for good produced by exporter $i$. $I_t^{nx}$ and $I_t^{x}$ represent the sets of all home producing non-exporters and exporters before the death shock hits the economy.

The corresponding price index at Home is thus given by:

$$P_t = \left( \int_{i \in I_t^{nx}} \left(p_{it}^{nx}\right)^{1-\sigma} di + \int_{i \in I_t^{x}} \left(p_{it}^{xd}\right)^{1-\sigma} di + \left(P_t^{x*}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

and the implied relative demand functions for different products are given by

$$y_{it}^{nx} = \left(\frac{p_{it}^{nx}}{P_t}\right)^{-\sigma} Y_t, \quad y_{it}^{xd} = \left(\frac{p_{it}^{xd}}{P_t}\right)^{-\sigma} Y_t, \quad Y_t^{x*} = \left(\frac{P_t^{x*}}{P_t}\right)^{-\sigma} Y_t,$$

where $p_{it}^{nx}$, $p_{it}^{xd}$, and $P_t^{x*}$ are the domestic market price of the product produced by home non-exporter, home exporter and foreign exporter, respectively.

Intermediate firms are heterogeneous in productivity. In particular, in each period $t$, there is a mass $M_t$ of potential intermediate good producers in the Home country with productivity levels over $(0,\infty)$ drawn from a distribution with a cumulative distribution function (CDF) of $G(z)$. Correspondingly, the probability density function (pdf) would be that $g(z)=\frac{\partial G(z)}{\partial z}$. Here we assume that each firm is assigned a productivity level.

Among these potential intermediate goods producers, firms serving the domestic market will have a distribution of productivity levels over $[z_{dt},\infty)$, while firms serving the foreign (world) market, that is, exporters, will have a distribution of productivity levels over $[z_{xt},\infty)$. $z_{dt}$ and $z_{xt}$ are the cut-off productivity level of the marginal domestic producers ($d$) and the marginal exporters ($x$), respectively. Given the exogenous death shock hits the economy with a probability of $\lambda$ at the end of period $t$, the number of home producing firms ($N_t$), home non-exporters ($N_t^{nx}$) and exporters ($N_t^x$) after the shock are

$$N_t = (1\text{-}\lambda)\left(1\text{-}G(z_{dt})\right)M_t, \tag{4}$$

$$N_t^x = (1\text{-}\lambda)\left(1\text{-}G(z_{xt})\right)M_t, \tag{5}$$

$$N_t^{nx} = N_t \text{-} N_t^x. \tag{6}$$

The domestic composite can thus be re-written as

$$Y_t^D = \left( \int_{z_d}^{z_x} y_t^{nx}(z)^{\frac{\sigma-1}{\sigma}} Mg(z)dz + \int_{z_x}^{\infty} y_t^{nx}(z)^{\frac{\sigma-1}{\sigma}} Mg(z)dz \right)^{\frac{\sigma}{\sigma-1}}.$$

The price index and relative demand functions can also be re-written as:

$$P_t = \left( \int_{z_{dt}}^{z_{xt}} \left(p_t^{nx}(z)\right)^{1-\sigma} Mg(z)dz + \int_{z_{xt}}^{\infty} \left(p_t^{xd}(z)\right)^{1-\sigma} Mg(z)dz + \left(P_t^{x*}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{7}$$

$$y_t^{nx}(z_i) = \left(\frac{p_t^{nx}(z_i)}{P_t}\right)^{-\sigma} Y_t, \tag{8}$$

$$y_t^{xd}(z_i) = \left(\frac{p_t^{xd}(z_i)}{P_t}\right)^{-\sigma} Y_t, \tag{9}$$

$$Y_t^{x*} = \left(\frac{P_t^{x*}}{P_t}\right)^{-\sigma} Y_t. \tag{10}$$

The corresponding foreign market demand for domestic exporter ($y_t^{xx}(z_i)$) is given by

$$y_t^{xx}(z_i) = \left(\frac{p_t^{xx}(z_i)}{P_t^*}\right)^{-\sigma} Y_t^* = \left(\frac{p_t^{xx}(z_i)}{P_t}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma} Y_t^*, \tag{11}$$

where $p_t^{xx}(z_i)$ is the foreign market price of the product produced by the home exporter with a productivity level of $z_i$. For the small open economy, $Y^*$ and $P^*$ are treated as exogenous, and will be calibrated below. We also normalize the price of all foreign-produced goods to equal 1.

### 3.3 Intermediate goods sector

Firms in the intermediate goods sector are heterogeneous in productivity, $z_i$. Following Jerman and Quadrini (2009, 2012), we assume that firms use debt ($b_t(z_i)$) and equity to finance production, where debt is preferred to equity because of a tax advantage. The effective gross interest rate for the debt is $R_t = 1 + r_t(1-\tau)$, where $\tau$ is the tax benefit.

The timeline of the economy is as follows. Each period starts with two aggregate state variables: the technology level ($A_t$), and the financial condition ($\xi_t$). (We will describe ($\xi_t$) in more detail in the next section.) At the beginning of each period, the economy consists of $N_{t-1}$ incumbent firms, among which $N_{t-1}^x$ are incumbent exporters. There are also $Ne_t$ new entrants who enter the domestic market and $Ne_t^x$ new exporters who enter the foreign market. At this point the incumbents and new firms hire labor, issue corporate bonds and stocks, and produce goods. Workers also supply labor and make consumption and financial investment decisions over these $N_{t-1}+Ne_t$ firms, and the goods and labor markets clear.

At the end of each period after all markets have cleared, the exogenous death shock hits each incumbent and new firm with probability $\lambda$. Because the death shock occurs at the end of each period, $N_t$ firms survive in the market after the death shock,

$$N_t = (1-\lambda)(N_{t-1} + Ne_t), \quad (12)$$

$$N_t^x = (1\text{-}\lambda)(N_{t-1}^x + Ne_t^x). \quad (13)$$

Additionally, in each period an existing firm has to pay a fixed cost $w_t f_t^d$ in order to produce domestically, and an extra fixed cost $w_t f_t^x$ if the firm also produces for the export market, while a new entrant must pay a sunk entry cost $K_t^E$ to enter the domestic market and an additional sunk entry cost $K_t^{EX}$ if the firm wants to access the export market.

### 3.3.1 Firm enforcement constraint

The labor market requires that firms must make factor payments to the worker at the beginning of each period before the realization of revenue. In addition to the inter-temporal debt, $b_t^k(z_i)$, $k=x,nx$ as described above, a firm has to borrow an intra-period loan to pay a certain portion of the labor cost in advance ($\phi^d$ for domestic production and $\phi^x$ for export production), that is, $loan_t^{nx}(z_i) = \phi^d w_t l_t^{nx}(z_i) + \phi^d w_t f_t^d$ for non-exporter $i$ and

$loan_t^x(z_i) = \phi^d w_t(l_t^{xd}(z_i)+f_t^d) + \phi^x w_t(l_t^{xx}(z_i)+f_t^x)$ for exporter $j$. A specification that distinguishes between the working capital requirement of exports and domestic sales is consistent with the empirical evidence documented in Kohn et al. (2016) which shows that exporters face higher working capital needs, reflecting the greater inventory held by exporters and the extended transport period between production and sale. The intra-period loan is repaid at the end of each period and there is no interest.

Because firms may default on their debt repayments, their access to intratemporal loans to use as working capital defined above is restricted by an enforcement constraint:

$$\xi_t E_t\left(m_{t+1}V_{t+1}^{nx}\left(b_t^{nx}(z_i)\right)\right) \geq \phi^d\left(w_t l_t^d(z_i)+w_t f_t^d\right)$$

for nonexporter *i*, and

$$\xi_t E_t\left(m_{t+1}V_{t+1}^{x}\left(b_t^{x}(z_j)\right)\right) + \gamma_t^{xx}\frac{p_t^{xx}(z_j)y_t^{xx}(z_j)}{P_t} \geq \phi^d\left(w_t l_t^{xd}(z_j)+w_t f_t^d\right) + \phi^x\left(w_t l_t^{xx}(z_j)+w_t f_t^x\right) \quad (14)$$

for exporter *j*. The primary source of collateral available to firms in securing their working capital is the end-of-period firm value, $E_t\left(m_{t+1}V_{t+1}^k\left(b_t^k(z_i)\right)\right)$, $k = x, nx$, where $m_{t+1} = \beta(1-\lambda)U_{C_{t+1}}/U_{C_t}$ is the discount factor, as the firms are essentially owned by the household through equity purchases. In this case, $E_t\left(m_{t+1}V_{t+1}^k\left(b_t^k(z_i)\right)\right)$ is the ex-dividend market value of the firm, that is, the end-of-period equity value which excludes the dividend of period $t$. As argued in Jermann and Quadrini (2009), profits from current period domestic sales are excluded as collateral, since they are regarded as too liquid and too easily divertible in the case of default.

In our setting, this enforcement constraint stipulates that lenders are willing to lend only if the liquidation value in case of default is at least sufficient to cover the loaned amount. Here, the lenders can liquidate the firms' end-of-period value $E_t\left(m_{t+1}V_{t+1}^k\left(b_t^k(z_i)\right)\right)$ and export productions, but suffer liquidation losses ($\xi_t$ smaller than

1). The potentially time varying variable $\xi_t$ captures "liquidity" of firm assets. It can be shown that the end-of-period firm value is the same as the firm's equity value ($q_t^k\left(z_i\right)$) as the latter is defined as the expected discounted value of dividend payouts starting from period $t+1$. The end-of-period firm value is typically decreasing in debt issuance, because debt issuance reduces the future payments that can be delivered to the shareholders, holding everything else equal.

Equation 14 includes an additional source of collateral specific to exporters, in the form of a portion of current exports sales: $\gamma_t^{xx}\frac{p_t^{xx}\left(z_j\right)y_t^{xx}\left(z_j\right)}{P_t}$. The idea of an additional source of collateral is not new, and it could take a variety of forms. For example, exporters tend to represent more capital-intensive industries, suggesting greater quantities of physical capital to post as collateral.[12] In the context of our model, it is analytically convenient to associate this additional collateral with current period export sales. Recall that the end-of-period firm value defined above as collateral intentionally excludes current sales, as Jermann and Quadrini (2009) regarded them as too liquid and divertible. But recall also that the motivation for why exporters have greater working capital needs draws on the evidence from Kohn et al. (2016) that exports hold greater inventories and experience longer shipping times between production and sale. We interpret this evidence as additionally supporting the notion that export sales revenue is less liquid and divertible than domestic sales revenue, as it is locked in the form of inventories or on ships during the extended period of transport. The model thus allows for the possibility that a portion of the export sales can be used as collateral, where this portion is parameterized by $\gamma_t^{xx}$.

[12] Similarly, Manova (2013) suggests that exporters could use the capital represented by their sunk entry cost as a source of collateral. In the context of our model, such interpretations would have a disadvantage, in that these forms of firm capital are implicitly incorporated in firm value, and we wish to avoid double counting them as sources of collateral.

Given that equity collateral excludes current sales, we avoid double counting. This specification is a fitting counterpart to the assumption that exporters have greater working capital needs ( $\phi^x > \phi^d$ ): if exporters need trade credit to cover the extended time waiting for shipping and delivery of certain goods, then it seems fitting that firms can pledge some part of these physical goods as collateral to secure this trade credit.

### 3.3.2 Firm production and pricing

Each firm produces a unique variety, requiring only one factor, labor:

$$y_t^{nx}(z_i) = A_t z_i l_t^{nx}(z_i),\ y_t^{xd}(z_i) = A_t z_i l_t^{xd}(z_i),\ \frac{y_t^{xx}(z_i)}{1-\tau_x} = A_t z_i l_t^{xx}(z_i) \tag{15}$$

where $A_t$ is the aggregate productivity common to all firms, $\tau_x$ is the iceberg cost for firms engaged in exports.

Firm dividends ( $d_t^k(z_i), k = x, nx$ ) are given by:

$$d_t^{nx}(z_i) = \frac{p_t^{nx}(z_i) y_t^{nx}(z_i)}{P_t} - w_t\left(l_t^{nx}(z_i) + f^d\right) - \left(b_{t-1}^{nx}(z_i) - \frac{b_t^{nx}(z_i)}{R_t}\right) \tag{16}$$

for a non-exporter ( $nx$ ), and

$$\begin{aligned} d_t^x(z_i) = {} & \frac{p_t^{xd}(z_i) y_t^{xd}(z_i)}{P_t} + \frac{p_t^{xx}(z_i) y_t^{xx}(z_i)}{P_t} - w_t\left(l_t^{xd}(z_i) + l_t^{xx}(z_i)\right) \\ & - b_{t-1}^x(z_i) + \frac{b_t(z_i)}{R_t} - w_t\left(f_t^d + f_t^x\right) \end{aligned} \tag{17}$$

for an exporter ( $x$ ).

The optimization problem involves choosing the price of an individual variety, $p_t^k(z_i), k = nx, x$ , the dividend payout, $d_t^k(z_i)$ , and the new debt, $b_t^k(z_i)$ , to maximize the cum-dividend market value of the firm, $V_t^k(b_{t-1}(z_i))$, that is, the beginning-of-period firm value which includes dividend:

$$V_t^k\left(b_{t-1}^k\left(z_i\right)\right)=\max_{p_t^k(z_i),d_t^k(z_i),\ b_t^k(z_i)}\left(d_t^k\left(z_i\right)+E_t(m_{t+1}V_{t+1}^k\left(b_t^k\left(z_i\right)\right)\right),\quad k=nx,x\,, \tag{18}$$

subject to the enforcement constraint, (Eq. 14), the demand for individual variety, (Eq. 8) for non-exporter and (Eqs. 10 and 11) for exporter , the production technology (Eq. 14), and the dividend equation, (Eq. 15) for non-exporter and (Eq. 16) for exporter.

The optimization implies the following pricing rules

$$\frac{p_t^{nx}\left(z_i\right)}{P_t}=\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\left(1+\phi^d\mu_t^{nx}\left(z_i\right)\right), \tag{19}$$

$$\frac{p_t^{xd}\left(z_i\right)}{P_t}=\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\left(1+\phi^d\mu_t^{x}\left(z_i\right)\right), \tag{20}$$

$$\frac{p_t^{xx}\left(z_i\right)}{P_t}=\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\frac{1+\phi^x\mu_t^x\left(z_i\right)}{\left(1\text{-}\tau_x\right)\left(1+\gamma_t^{xx}\mu_t^x\left(z_i\right)\right)}, \tag{21}$$

$$\mu_t^k\left(z_i\right)=\frac{1/R_t-E_t m_{t+1}}{\xi_t E_t m_{t+1}}\qquad k=nx,x\,. \tag{22}$$

where $\mu_t^k\left(z_i\right)$ is the Lagrange multiplier associated with the enforcement constraint.

From Eq. (22), it can be seen that $\mu_t^{nx}=\mu_t^x=\mu_t$, independent of firm productivity. This is because $\mu_t$ is the shadow price of the intra-period loan on firm value, and measures the relative cost of the two types of external financing, that is, the bond financing ($R_t$) to equity financing ($1/E_t m_{t+1}$), adjusted by the financial market condition ($\xi_t$). It can also be seen that, holding everything else constant, a worsening financial market condition (falling $\xi_t$) increases the tightness of the financial constraint (rising $\mu_t$).

Note, in steady state, we have $m=\frac{1}{1+r}$ from Eq. (2). Given that $R=1+r\left(1\text{-}\tau\right)$, Eq. (22) shows that, first, in steady state it is always the case that $\mu>0$, as bond financing is cheaper due to its tax advantage, and firms prefer cheaper bond financing to more expensive equity financing. This suggests that the enforcement constraint is binding and a firm borrows up to the limit. If the tax benefit is higher, firms would like to issue more bonds such that $\mu$ is higher; this makes the borrowing constraint even tighter.

### 3.4 The entry condition for the marginal firms

Each period, there are a mass of potential entrants in the Home market. New firms must pay the entry cost, $K_t^E$, to enter domestic market, and pay additional entry cost, $K_t^{EX}$, to enter foreign market. This implies that only firms whose values net off entry costs are non-negative will become non-exporters or exporters, which gives the following entry conditions,

$$\frac{p_t^{nx}(z_D)y_t^{nx}(z_D)}{P_t} - w_t\left(l_t^{nx}(z_D) + f^d\right) - \left(b_{t-1}^{nx}(z_D) - \frac{b_t^{nx}(z_D)}{R_t}\right) + E_t(m_{t+1}V_{t+1}^{nx}\left(b_t^{nx}(z_D)\right) - K_t^E = 0 \quad (23)$$

for becoming a non-exporter, and

$$\begin{aligned}&\frac{p_t^{nx}(z_X)y_t^{nx}(z_X)}{P_t} - w_t\left(l_t^{nx}(z_X) + f^d\right) - \left(b_{t-1}^{nx}(z_X) - \frac{b_t^{nx}(z_X)}{R_t}\right) + E_t(m_{t+1}V_{t+1}^{nx}\left(b_t^{nx}(z_X)\right) - K_t^E \\ &= \frac{p_t^{xd}(z_X)y_t^{xd}(z_X)}{P_t} + \frac{p_t^{xx}(z_X)y_t^{xx}(z_X)}{P_t} - w_t\left(l_t^{xd}(z_X) + l_t^{xx}(z_X) + f^d + f^x\right) - \left(b_{t-1}^{x}(z_X) - \frac{b_t^{x}(z_X)}{R_t}\right) + E_t(m_{t+1}V_{t+1}^{x}\left(b_t^{x}(z_X)\right) - \left(K_t^E + K_t^{EX}\right)\end{aligned}$$

for becoming an exporter.

The two entry conditions for the marginal firms generate the cut-off productivity levels, $z_{dt}$ and $z_{xt}$, for being a non-exporter and an exporter, respectively. Thus, firms with productivity level in the range of $[z_{dt}, z_{xt})$ are non-exporters, and firms with productivity level in $[z_{xt}, \infty)$ are exporters.

With a little transformation of the entry condition to export market, we have the following entry condition

$$\left(\frac{p_t^{xx}(z_x)y_t^{xx}(z_x)}{P_t} - w_t l_t^{xx}(z_x)\right) - w_t f^x + \frac{b_t^x(z_x) - b_t^{nx}(z_x)}{R_t} + \left(q_t^x(z_x) - q_t^{nx}(z_x)\right) = K_t^{EX}. \quad (24)$$

Eq. (24) says that the profit earned from foreign market plus the additional value of bond and equity issuance for being an exporter must equal the sunk entry cost to the foreign market, $K_t^{EX}$.

### 3.5 Aggregation and equilibrium

For the numerical analysis, we make an assumption that firms' idiosyncratic assigned productivity follows a Pareto distribution with a cumulative distribution function (CDF) of $G(z_t)$ where $G(z)=1-z^{-\theta}$. Correspondingly, the probability density function (pdf) would be that $g(z)=\theta z^{-\theta-1}$.

Given the production function of the domestic composite, it is convenient to define the average productivity level for all producing firms in Home country, $\tilde{z}_t$, the average productivity level for the non-exporters, $\tilde{z}_t^{nx}$, and the average productivity level for the exporters, $\tilde{z}_t^{x}$, as follows, respectively,

$$\tilde{z}_t = \left( \frac{1}{1\text{-}G(z_{dt})} \int_{z_{dt}}^{\infty} z_t^{\sigma-1} dG(z) \right)^{\frac{1}{\sigma-1}} \tag{25}$$

$$\tilde{z}_t^{nx} = \left( \frac{1}{G(z_{dt})-G(z_{xt})} \int_{z_{dt}}^{z_{xt}} z_t^{\sigma-1} dG(z) \right)^{\frac{1}{\sigma-1}} \tag{26}$$

and

$$\tilde{z}_t^{x} = \left( \frac{1}{1-G(z_{xt})} \int_{z_{xt}}^{\infty} z_t^{\sigma-1} dG(z) \right)^{\frac{1}{\sigma-1}}. \tag{27}$$

The market clearing condition for the labor market is then given by

$$L_t = \frac{N_t^{nx}\tilde{L}_t^{nx} + N_t^{x}\left(\tilde{L}_t^{xd} + \tilde{L}_t^{xx}\right) + \left(N_t^{nx} + N_t^{x}\right) f^{d} + N_t^{x} f^{x}}{1-\lambda} \tag{28}$$

where $\tilde{L}_t^{nx}$ is the labor demand of the non-exporter with the average productivity level of $\tilde{z}_t^{nx}$; $\tilde{L}_t^{xd}$, and $\tilde{L}_t^{xx}$ are the labor demand for domestic and export production of the exporter with the average productivity level of $\tilde{z}_t^{x}$. They are respectively given by

$$\tilde{l}_t^{nx} = \frac{\tilde{y}_t^{nx}}{A\tilde{z}_t^{nx}}, \tag{29}$$

$$\tilde{l}_t^{xd} = \frac{\tilde{y}_t^{xd}}{A\tilde{z}_t^x}, \tag{30}$$

$$\tilde{l}_t^{xx} = \frac{\tilde{y}_t^{xx}}{A\tilde{z}_t^x\left(1\text{-}\tau_x\right)}. \tag{31}$$

The final goods market clearing condition is

$$Y_t = Ne_t K_t^E + Ne_{Xt} K_t^{EX} + C_t. \tag{32}$$

In addition, as the intermediate firms are fully owned by the investor, the equity share is thus normalized at $s_t = 1$ for all $t$.

Suppose balanced trade:

$$N_t^x \tilde{p}_t^{xx} \tilde{y}_t^{xx} = P_t^{X^*} Y_t^{X^*}. \tag{33}$$

Equilibrium is a sequence of the 66 endogenous variables summarized in Appendix Table 1.

## 4. Analytical results

This section derives some key analytical results, to provide intuition for how the model explains the stylized fact in the empirical section. In particular, we focus on the analysis of leverage ratio in steady-state, and examine how firm size affect the firms' external financing decision. We provide here the main results and discuss their interpretations, leaving most derivations to Appendix 2.

In the model, the leverage ratio of a firm is defined as

$$leverage\left(z\right) = \frac{total\ debt}{total\ asset} = \frac{IntraLoan\left(z\right)+b\left(z\right)}{IntraLoan\left(z\right)+q\left(z\right)+b\left(z\right)}.$$

A little transformation implies that

$$leverage(z)=\frac{1}{\dfrac{1}{\dfrac{IntraLoan(z)}{q(z)}+\dfrac{b(z)}{q(z)}}+1}. \tag{34}$$

Eq. (34) shows that the leverage ratio of a firm is determined positively by the two ratios, $\frac{IntraLoan(z)}{q(z)}$ and $\frac{b(z)}{q(z)}$, which are the short-term and long-term debt-equity ratios respectively. Below we will study how firm size affect the firms' external financing decision by looking at the response of equity prices, short- and long-term debts to firm productivity changes.

### 4.1 Non-exporters

From the enforcement constraint, Eq. (14), we see the intra-period loan is given by

$$IntraLoan(z)=\phi^d w\left(l^{nx}(z_i)+f^d\right).$$

Hence, for non-exporters, the short-term debt-equity ratio is a constant, that is, $\frac{IntraLoan(z)}{q(z)}=\xi$, and Eq. (34) becomes

$$leverage(z)=\frac{1}{\dfrac{1}{\xi+\dfrac{b(z)}{q(z)}}+1}. \tag{35}$$

Therefore, in order to examine how firm size affects non-exporters' external financing decision, we should look at its effect on firm equity prices, $q(z)$, the long-term debt financing, $b(z)$, and the long-term debt-equity ratio, $\frac{b(z)}{q(z)}$. We now analyze each of these three terms in turn.

**(1) Equity price**

From the enforcement constraint, for an individual non-exporter $i$, we have that

$$\xi q^{nx}(z_i)=\phi^d w\left(l^{nx}(z_i)+f^D\right)$$

Combing the production function, Eq. (15), the pricing equation (19), and the market demand, Eq. (8), and taking the derivatives of the equity price with respect to firm productivity gives

$$\frac{\partial q^{nx}(z_i)}{\partial z_i} = \frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma} Y\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma-1}{z_i}\right), \tag{36}$$

and
$$\frac{\partial^2 q^{nx}(z_i)}{\partial (z_i)^2} = \frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma} Y\left(\frac{w}{A}\right)^{1-\sigma}(\sigma-1)(\sigma-2)(z_i)^{\sigma-3}.$$

Hence, $\frac{\partial q^{nx}(z_i)}{\partial z_i} > 0$ if $\sigma > 1$, and further $\frac{\partial^2 q^{nx}(z_i)}{\partial (z_i)^2} > 0$ if $\sigma > 2$. These conditions can be easily satisfied.

We notice that the impact of productivity on equity prices works through the component of variable production cost, by affecting the labor demand, rather than the component of fixed costs. When firm productivity rises, firm equity value increases more for larger firms than for smaller firms, as firm sales increase more for larger firms when the substitution elasticity across varieties is greater than 2, that is, $\sigma > 2$. This is because, from the pricing equation (19), the higher productivity of larger firms allows them to charge lower prices, enjoying larger market demand. Rising productivity will further amplify this advantage, allowing larger firms to procure an even larger market demand.

The impact also relies on a few fundamental parameters, such as the external financing needs of working capital ($\phi^d$), the substitution elasticity across varieties ($\sigma$), the mean level of credit market condition ($\xi$), given the tightness of financial constraints ($\mu$), the aggregate market demand ($Y$), the wage level ($w$) and the aggregate technology level ($A$).

**(2) Bond position**

From the dividend equation, (16), we have that

$$b^{nx}(z_i) = \frac{R}{R-1}\left[\frac{p^{nx}(z_i)}{P}y^{nx}(z_i) - w\left(l^{nx}(z_i)+f^d\right) - d^{nx}(z_i)\right].$$

Substituting the enforcement constraint, Eq. (14), and the firm value function, Eq. (18), we have that

$$b^{nx}(z_i)=\frac{R}{R-1}\left[\frac{p^{nx}(z_i)}{P}y^{nx}(z_i)-\left(\frac{\xi}{\phi}+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^{nx}\left(z_i\right)\right]. \tag{37}$$

Then taking the derivatives of bond position with respect to firm productivity, by combining the pricing equation (19) and the market demand, Eq. (8), yields

$$\frac{\partial b^{nx}(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{\xi}{\phi^d}\left\{\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)-\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\right)\right\}\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i},$$

and
$$\frac{\partial^2 b^{nx}(z_i)}{\partial\left(z_i\right)^2}=\frac{R}{R-1}\frac{\xi}{\phi^d}\left\{\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)-\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\right)\right\}\frac{\partial^2 q^{nx}\left(z_i\right)}{\partial\left(z_i\right)^2}.$$

To ensure $\frac{\partial b^{nx}(z_i)}{\partial z_i}>0$ and $\frac{\partial^2 b^{nx}(z_i)}{\partial\left(z_i\right)^2}>0$, we need $\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)>1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}$.

This can be easily satisfied. Note, the left-hand-side, $\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)$, is the price mark-up, capturing the effect of changing $z_i$ on firm sales, while the right-hand-side captures the effect of changing $z_i$ on firm production costs (the $1$) plus the dividend payout costs (the $\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}$).

This implies that the larger the firm productivity is, the more the long-term debt the firm chooses. Additionally, when firm productivity rises, firm long-term debt positions will increase more for larger firms than for smaller firms, as long as firm sales increase more than its production costs plus dividend payouts, which is guaranteed as debt is cheaper than equity and preferred by firms. Further, we see that the impact on bond position is increasing with the impact on equity prices.

**(3) Debt-to-equity ratio**

Now we are ready to look at the long-term debt-equity ratio included in Eq. (35). Let $LR_1^{nx}(z_i)$ denote this long-term ratio, thus $LR_1^{nx}(z_i)=\frac{b^{nx}(z_i)}{q^{nx}\left(z_i\right)}$. Using Eq. (37), we then have

$$LR_1^{nx}(z_i)=\frac{R}{R-1}\left[\frac{\frac{p^{nx}(z_i)}{P}y^{nx}(z_i)}{q^{nx}(z_i)}-\left(\frac{\xi_q}{\phi}+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\right], \tag{38}$$

which shows the effect of changing $z_i$ on the long-term debt-equity ratio is fully through the sales-equity ratio, $\frac{\frac{p^{nx}(z_i)}{P}y^{nx}(z_i)}{q^{nx}(z_i)}$. That is, $\frac{\partial LR_1^{nx}(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i}$.

Taking derivative of the sales-equity ratio w.r.t. $z_i$ shows that[13]

$$\frac{\partial LR_1^{nx}(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{wf^D}{q^{nx}(z_i)^2}\frac{\partial q^{nx}(z_i)}{\partial z_i}. \tag{39}$$

Thus, $\frac{\partial LR^{nx}(z_i)}{\partial z_i}>0$ if $\sigma>1$. This implies that rising $z_i$ leads to rising leverage ratio. It can also be seen that the effect of productivity on bond-to-equity ratio is linear in the effect of productivity on equity prices.

More importantly, we see from this result that fixed costs are essential to deriving our desired result of a leverage ratio that rises with firm size. In the absence of fixed costs to the domestic market, the leverage ratio is the same for all non-exporters, regardless of productivity level. This is because the bond value and equity value rise together with productivity. But the presence of fixed costs means that the leverage ratio rises, since the bond value rises by a multiple of the equity value. And this rise in the relative bond value is a function of variables that tighten the collateral constraint for all firms, such as $\mu$, $\phi^d$ and $R$.

### 4.2 Exporters

The equations for exporters are analogous, but with additional parameters governing the working capital needs of exporters ($\phi^x$), and the collateral value of exporter's accounts

[13] See Appendix 2 for the detailed derivations.

receivable ( $\gamma^{xx}$ ). With the presence of the accounts receivable used as collateral, $\gamma^{xx}\frac{p^{xx}\left(z_j\right)y^{xx}\left(z_j\right)}{P}$, the short-term debt-equity ratio is no longer a constant. Therefore, in order to examine how firm size affect exporters' external financing decision, we should look at its effect on firm equity prices, $q\left(z\right)$, the long-term debt financing, $b\left(z\right)$, the short-term debt-equity ratio, $\frac{IntraLoan\left(z\right)}{q\left(z\right)}$, and the long-term debt-equity ratio, $\frac{b\left(z\right)}{q\left(z\right)}$. See Appendix 2 for details.

Let $LR_1^x(z_i)$ and $SR_1^x(z_i)$ denote the long-term and short-term ratios respectively, thus $LR_1^x(z_j)=\frac{b^x(z_j)}{q^{nx}\left(z_j\right)}$, and $SR_1^x(z_i)=\frac{IntraLoan\left(z_i\right)}{q\left(z_i\right)}$. Using Eq. (40), we have the long-term debt-equity ratio, given by

$$LR^x(z_i)=\frac{R}{R-1}\left[\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\frac{sales^{xx}(z_i)}{q^x(z_i)}-\frac{w\left(f^d+f^x\right)}{q^x(z_i)}-\left(\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\right], \quad (40)$$

which shows the effect of changing $z_i$ on the long-term debt-equity ratio is through the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}$, the export sales-equity ratio, $\frac{sales^{xx}(z_i)}{q^x(z_i)}$, and the fixed costs-equity ratio, $\frac{w\left(f^d+f^x\right)}{q^x(z_i)}$.

A few steps of calculations show that

$$\frac{\partial LR^x(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{1}{q^x\left(z_i\right)^2}\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{w}{\xi}\left[\begin{array}{l}\left(\phi^d f^D+\phi^x f^X\right)\left(\begin{array}{l}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)\\+\left(f^D+f^X\right)\left(\begin{array}{l}\phi^d\frac{\sigma-1}{\sigma}\left(1+\phi^d\mu\right)^{-\sigma}Y\\+\left(\phi^x\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)\end{array}\right] \quad (41)$$

Thus, $\frac{\partial LR^x(z_i)}{\partial z_i}>0$ if $\sigma>1$ and $\frac{\gamma^{xx}}{\phi^x}<\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}<1$, where the last inequality places bounds on the effective price markup, inclusive of financing frictions.[14]

Additionally, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D = f^X = 0$, the leverage ratio is the same for all exporters, regardless of productivity level. As with non-exporters, the presence of fixed costs are essential to deriving the result of a leverage ratio rising in firm size. However, for exporters, it is the sum of fixed costs of production and exporting that matter.

Similarly, for the short-term debt ratio:

$$\frac{\partial\left(SR^x(z_i)\right)}{\partial z_i} = \left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{1}{q^x(z_i)^2}\frac{w}{\xi}\left(\phi^d f^d + \phi^x f^x\right)\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma\text{-}1}(1\text{-}\tau_x)^{\sigma\text{-}1}\left(\frac{P}{P^*}\right)^{-\sigma} Y^*\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\frac{\phi^x}{\sigma}+\gamma^{xx}\right)$$

Thus, $\frac{\partial\left(SR^x(z_i)\right)}{\partial z_i} > 0$ if $\sigma > 1$. Again, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D = f^X = 0$, the short-term leverage ratio is the same for all exporters, regardless of productivity level.

## 5. Quantitative Analysis of the Leverage Ratio

This section presents quantitative results by calibration and numerical solution. The first part explains our choices of parameterization, and the second part demonstrates that the model can replicate the facts uncovered in the empirical section. The third part identifies key elements of the mechanism by conducting sensitivity analysis, and the fourth section discusses how to interpret the economic significance of this mechanism.

### 5.1 Parameter values

We take trade-related parameters from the recent trade and firm dynamics literature. Following Ghironi and Melitz (2005), the Pareto distribution parameter and substitution

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14 In steady state $\frac{p^{xx}(z_i)}{P} = \frac{\sigma}{\sigma-1}\frac{1+\phi^x\mu}{\left(1+\gamma^{xx}\mu\right)}\frac{w}{(1\text{-}\tau_x)Az}$, so $\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu} < 1$, which implies there is a positive markup of price over marginal cost.

elasticity are set to be $\theta=3.8$ and $\sigma=3.8$, respectively, and the exogenous death shock probability is set at $\lambda=0.025$. Following Bergin, Feng and Lin (2018b), we set iceberg trade cost at $\tau_X = 0.16$.

We set the weight of the disutility of labor in the period utility function at $\kappa = 3.409$. The inverse of the labor supply elasticity is set at $\psi = 0.5$, following Hall (2009). We take the values of working capital requirements for domestic and export productions from Kohn et al. (2016) by setting $\phi^d = 0.53$, and $\phi^x = 1$. We follow Jermann and Quadrini (2012) to set the tax benefit parameter at $\tau=0.35$. We set $\beta = 0.99$ to coincide with a quarterly frequency. Risk aversion is set at $\rho = 2$ (Arellano, Bai and Kehoe, 2012). We set the enforcement constraint parameter $\xi$ to match the average leverage ratio in our firm-level data.[15] Similarly, we choose the exporter enforcement constraint parameter $\gamma^{xx}$ to match the average in our firm level data for exporters for the ratio of debt to total assets. Sensitivity analysis to follow will demonstrate the effect of alternative values of this parameter.

As this is a small open economy model, the rest-of-world income and price level are exogenous. We follow literature in setting the rest of world income at 5 times that of the small open economy (see Feenstra et al., 2018). And we set the price level in rest of world so that, in combination of the size of the foreign market and the iceberg trade cost defined above, the share of home exports in GDP equals 0.26, which is the average value for OECD countries in 2011-16 according to World Bank data.

We set the sunk cost of exporting, $K^{EX}$, so to match the standard fact that 22% of firms engage in export. The domestic sunk cost, $K^{E}$, is set high enough to assure that no firms are so small as to have negative leverage in our experiments. In the benchmark case

[15] This is nearly equivalent to Kohn et al. (2016) sets their enforcement constraint parameter to match the ratio of debt to collateral in their data, and it is similar to how Jermann and Quadrini (2012) choose their corresponding parameter value, producing a steady state ratio of aggregate debt over GDP which matches aggregate data.

we assume no endogenous exit from exporting, which implies no fixed costs of exporting ($f^X = 0$), though we will consider sensitivity analysis to alternative calibrations. We set the fixed cost of production, $f^d$, to match the range in leverage ratio for non-exporting firms in our sample. Recall from the analytical result above, that it is this term that leads to higher leverage for larger firms. The parameters used in the benchmark model are listed in Table 3. Again, sensitivity analysis to follow will demonstrate the effect of alternative values to this key parameter.

## 5.2 Benchmark results

This section uses numerical solution to demonstrate that the model can replicate the fact uncovered in the empirical section. The objective is to generate a cross-sectional distribution of leverage ratios among firms that is consistent with the empirical finding that exporters have higher leverage ratios than non-exporters. Given that the empirical fact was based upon leverage ratios computed as averages over time, our strategy is to solve numerically for the deterministic steady state of the model, which gives us a stationary distribution of leverage ratios.

Figure 1a plots the cross-sectional distribution of key firm variables across the heterogeneous firms in steady state. The first row shows that firm debt and equity both grow exponentially with the firm productivity index, as does the level of overall firm production. If we plot the logs of debt and equity against the log of real sales, as in rows 2 and 3, not only do most variables show a nearly linear relationship, but it also facilitates comparison with our empirical results, where regressions used the log of real sales as a regressor. The first important observation is that, while equity and bond issues both grow in firm size, bond issue grows more quickly. As a result, when we plot measures of the leverage ratios, either the ratio of bond to equity ($\frac{b(z)}{q(z)}$), or the total loan divided by total firm asset ($LR1$) which is defined as $LR1 = \frac{loan(z)+b(z)}{q(z)+loan(z)+b(z)}$, these ratios grow in firm size. So the model reflects the first of our two key empirical findings: larger firms have higher leverage ratios.

Figure 1b isolates the plot of the leverage ratio, to view this feature more clearly. One notes that there is a concavity in the leverage ratio plotted against log sales. Of course, in part this concavity comes from the fact that there is an upper bound of 1 on the ratio of $\frac{loan(z)+b(z)}{q(z)+loan(z)+b(z)}$. But concavity is present even in the b/q ratio which has no upper bound. The reason is that the fixed cost, which we explain below is the source of positive slope of leverage in size (see Eqs. (39) and (41)), becomes less important as a share of firm value as firm size grows.

The second important feature to note in the leverage ratio is the jump for the marginal exporter relative to non-exporters. Given that we plot leverage against firm size, it is easy to verify visually that this jump in leverage is greater than that which would occur just for the larger size of exporting firms. This too reflects our empirical finding, that exporting firms tend to have leverage ratios higher than non-exporting firms in terms of long-term debt, even after controlling firm size.

A simple cross–sectional regression on the simulated data confirms that our model can replicate the two main facts from the empirical section. We collect the set of leverage ratios plotted in Figure 1b for our set of firms, and regress these data on the log of sales for each firm as a measure of firm size, as well as on an indicator of whether the firm chooses to be an exporter in our model equilibrium. The coefficient on firm size nearly exactly matches that of the empirical regression, 0.0119, reflecting the empirical finding that leverage rises with size. This outcome is not surprising, of course, as it reflects the fact that we calibrated the fixed cost of production to match this empirical implication.

The regression coefficient for the effect of export status on leverage is 0.0145, which is close to the value 0.0158 reported in table 2 for a regression using the actual data. This coefficient is the product of the parameter governing the use of export sales as collateral, $\gamma^{xx}$, which was calibrated to match the average leverage of exporters in our full sample in Table 1. The regression on simulated data indicates that this calibration also facilitates matching the effect of export status on leverage in the regression analysis.

## 5.3 Exploring the mechanism

In this section we explain the main mechanism driving our result for leverage ratios, and present evidence of its workings. The essence of the capital structure choice in this model is the tradeoff firms face between the benefits of equity versus bond issuance, when allocating overall firm value into these two components: bonds have the advantage of tax breaks, while equity provides collateral that relaxes the working capital constraint.

In our model, the presence of fixed costs of production in the working capital constraint means that this tradeoff differs for firms of different size. Profits and hence overall firm value grow in proportion to the size of a firm's sales; however, the need for working capital grows less than proportionally with sales due to the fixed component. As a result, a larger firm can allocate a smaller fraction of firm value to equity in order to secure the working capital required to permit desired production, and can allocate a larger share of firm value to bonds to reap the tax benefits. This mechanism also helps explain higher leverage ratios for exporters: like the rest of the trade literature with heterogeneous firms and free entry, our model implies that exporting firms systematically tend to be more productive and larger, since only firms with sufficiently high productivity can afford to pay the one-time sunk cost of export entry.

The essential role in our mechanism played by the fixed cost component in working capital is shown in Eqs. (39) and (41), and can be seen clearly by comparing simulations with alternative values for the fixed cost, $f^d$. Figure 2 shows that when this fixed cost is eliminated from the model ( $f^d = 0$), the leverage ratio of all non-exporters now is uniform (and equals the value that would be the limit in Figure 1b for firms of the largest size within the group); similarly, the leverage ratio is uniform for all exporters (given there is no additional fixed cost for exporters in the benchmark calibration). As the fixed cost is raised progressively, the gradient of leverage in firm size gets progressively larger. For small firms, where the per-period fixed production cost common to all firms is large relative to firm value, the needs for working capital to finance the fixed production costs are large relative to firm value. So for these small firms, a larger share of firm value needs to take the form of equity, rather than long-term bonds, to be used as collateral to secure these working capital loans. As simulations in the figure consider cases with progressively higher fixed production costs, the small firms become even more reliant upon equity as collateral, and hence the leverage ratios become progressively smaller. But as one considers firms that are larger because they are at the higher end of the productivity distribution, one sees that firm profits and hence firm value grow faster than working capital needs, to the degree production costs take the form of fixed costs. So in Figure 2, leverage of the smallest firms starts at a higher level for smaller $f^d$, and so the slope of leverage ratio in size is flatter as leverage converges to the limit for the largest firm (which in the limit is unaffected by fixed cost, since the cost is sufficiently small relative to the value of the largest firm).

It is not a surprise that the model can replicate this fact, inasmuch as the value of $f^d$ was calibrated to replicate this property of the data. But the first contribution of this model to the literature lies in finding that fixed production costs can in fact generate this property. While it is a fundamental fact in corporate finance that larger firms tend to be more leveraged, no paper to our knowledge, either in trade or corporate finance, has used the presence of fixed costs as an explanation for this result. Approaching the problem from the perspective of trade, with a focus on firm heterogeneity in terms of productivity and hence size, suggests fixed cost as a natural explanation to this empirical regularity.

Figure 3 shows that including additional fixed costs specific to export ( $f^x$>0) work in the same manner as domestic production fixed costs. Assuming these export fixed costs are included in working capital, they increase the marginal exporter's need for working capital relative to non-exporters of the same size. This implies they need a higher ratio of equity to long-term debt, in order to have sufficient collateral to procure working capital. This implies that export fixed costs could augment the explanation of the first fact, rising leverage in size, but that it damages the ability to explain the second fact, higher leverage for exporters compared to non-exporters (after controlling for size). We focus on this second fact next in our discussion.

Recall that our stylized fact has a second part to explain: while two-thirds of the higher leverage ratio of exporters is due to their larger size, the remaining one-third of higher leverage of exporters exists even after controlling for firm size. The calibrated model attributes this part of the empirical result to the parameter $\gamma^{xx}$, representing additional sources of collateral available to exporters not available to non-exporters. The logic is related to that of the size effect above. If exporters have additional sources of collateral, this would tend to reduce their reliance on equity for collateral, freeing them to respond to tax incentives to allocate a larger share of firm value to bonds rather than equity, hence a higher leverage ratio.

Figures 4a and 4b make clear that the ability of the model to replicate the second fact derives entirely from the ability to post export shipments as collateral ( $\gamma^{xx}$ > 0). Figure 4a shows that when exporters have no special source of collateral ( $\gamma^{xx}$ =0), exporters all have a lower leverage ratio than non-exporters. This results from the fact that the benchmark calibration implies exporters have greater needs for working capital ( $\phi^x > \phi^d$ ), so they choose more equity as a share of overall firm value in order to secure the necessary working capital.

Even more informative is Figure 4b, which shows the case where exporters have neither special sources of collateral ( $\gamma^{xx} = 0$) nor special needs for working capital ( $\phi^x = \phi^d$ ), compared to non-exporters. It now becomes clear that all exporters and non-exporters alike lie on the same curve: leverage rises in a concave fashion with firm sales alone; exporters have higher leverage just because they have more sales. Given that our empirical evidence indicates that about two thirds of the higher leverage of the average exporter is due just to their larger size compared to the average non-exporter, this graph shows that the model can explain the large majority of exporter leverage with a completely standard trade model specification, without adding in a special source of exporter collateral. However, extensive model experiments confirm that without such an extra source of collateral, the best the model can do is place the exporters on the same leverage-size curve as non-exporters, not above it as implied by the regressions results. We conclude that no manipulation of the standard menu of trade model features, neither those governing iceberg, fixed nor sunk costs of trade, can explain this second part of the empirical result.[16]

It is also interesting to note from Figure 4b that the presence of export iceberg costs does not visibly lower the leverage ratio of exporters relative to non-exporters, despite the fact they raise the working capital needs due to higher production costs per unit of sale.[17]

[16] We also experimented with a model specification that replaced exports sales with the sunk entry cost as collateral for exporters in equation (14). In addition to the disadvantage of double counting of collateral already included in firm equity (see discussion in section 3.3.1), this specification also implied that the benefit of export status is stronger for smaller exporters near the export margin, and becomes inconsequential for large exporters. It thus inverts the logic of the fixed costs in the working capital constraint, and hence negates the contribution of size to higher leverage among exporters. Simulation results for this alternative model are available from the authors upon request.

[17] We conducted the experiment, but do not show the additional figure as it makes no visible difference.

This results from the fact that export prices are marked up to compensate for the higher production costs, which raises profits and hence firm value and hence collateral per unit of production. This means that the relationship of leverage to actual export sales that reach the export market (as opposed to production per se) is unaffected by the presence of export costs of this type.

## 5.4 Interpreting the mechanism for firms' financing strategy

We conclude this discussion of our main result by interpreting its implications for the tradeoff firms face between short-term and long-term debt. Given that the fixed cost implies the need for working capital grows less than proportionately with firm sales and hence firm value, one might imagine that a large firm would take advantage of this position to enjoy the benefit of a looser working capital constraint and scale up production closer to the unconstrained optimal level. One might even imagine that a sufficiently large firm would have enough firm value relative to the sum of fixed and variable costs that the working capital constraint would no longer bind, and the firm could achieve the unconstrained optimal level of production. But our main result shows that this will not occur even for large firms; instead large and financially strong firms choose to reduce the share of equity in firm value, while not relaxing the working capital constraint.

The underlying reason for the choice not to loosen the working capital constraint can be seen in the Euler equation arising from the capital structure optimality problem (Eq. 22), which indicates in steady state that $\xi\mu(z_i) = (1/R - m)/m$. In this equation $\mu$ is the Lagrange multiplier on the working capital constraint and measures the degree of tightness. Literally, it is the shadow value of one unit of equity as collateral, by relaxing the working capital constraint and allowing higher production and profits. So, multiplied by the collateral share parameter, $\xi$, the left-hand side of the Euler equation measures the shadow value of one unit of equity as collateral. The right-hand side of the equation is the difference between the return on bonds and the household discount factor, which represents the tax benefit of bonds over equity. Of course, in a more standard model setting with no distinction between bonds and equity in terms of collateral value of equity or tax benefit of bonds, the Euler equation would require the interest rate equal the reciprocal of the stochastic discount factor; but in our setting the difference is the tax benefit of bonds. Note that in our model this difference on the right-hand side of the equation is constant for all firm productivity indexes, as it is determined by the marginal utilities of the households that supply both debt and equity financing. This implies that the tightness of the short-term borrowing constraint on the left-hand side of the equation is the same for all firms, regardless of size or export status.

It is this Euler condition which generates our leverage ratio result. So if one were to consider firms progressively higher in the productivity and size distribution, hypothetically keeping capital structure fixed, the fixed cost component in working capital would imply that the working capital constraint would become looser. But the Euler equation says it is not optimal to keep this outcome, and mandates a fall in the share of equity relative to sales and hence firm value, which implies an increase in tightness of the working capital constraint to the same level as for all other firms. In other words, as firm size grows, the marginal benefit of equity as collateral falls, but the marginal cost in terms of lost tax benefits is constant.

We conclude that optimal capital structure in our model implies that larger firms will not take advantage of their financial strength to relax the short-term working capital constraint in order to scale up production closer to their unconstrained optimum. It is optimal instead to sacrifice access to short-term debt in order to scale up long-term debt, and reap the associated tax benefits. In other words, it is optimal to transform financial strength in terms of short-term financing in order to reduce the overall cost of capital in terms of long-term firm financing.

## 6. Financial frictions as a barrier to exporter entry

This section uses the model to demonstrate that, in the presence of endogenous capital structure, financial constraints present a barrier to export entry in a way quite different from that found in preceding work. Preceding literature studying financial frictions as barriers to exporting has noted that exporters tend to need more short-term working capital, such as financing during the extended time of transshipment and holding of inventories. In particular, Kohn et al. (2016) has emphasized that exporters consequently are more financially constrained than non-exporters, especially when newly exporting; this limits their ability to scale up production optimally for the export market, and thereby poses a financial barrier to exporting.

Our model featuring endogenous capital structure produces a different picture of how financial frictions pose a barrier to exporting. First, the model indicates that higher working capital requirements do not restrict the scale of production in long run equilibrium, as adjustment in the optimal capital structure should compensate by providing extra collateral. Second, this implies that such financial frictions limit trade not primarily through the intensive margin of exports per firm, but instead by affecting the extensive margin by restricting firm entry. Third, this further implies that the effects on firm entry work not by limiting the benefits of entry in terms of sales and profits, but instead by raising the difficulty of financing the sunk cost of entry.

Before proceeding, we wish to be clear that our result should be viewed as complementary rather than contradictory to that of Kohn et al, (2016). Our analysis focuses on the long-term equilibrium, when endogenous capital structure has had an opportunity to adjust fully, rather than the dynamics of the short to medium term that they study. Far from disputing the finding in Kohn et al. (2016) of higher working capital requirements for exporters, we adopt their empirical estimates in calibrating the working capital constraint in our quantitative model.[18]

We begin by demonstrating through model simulations the effects of higher working capital requirements for exporters, showing this expresses itself primarily by affecting their long-term leverage ratios and the extensive margin, rather than the scale of production for a given firm. The way to measure higher working capital requirements in our model is in terms of the parameter specification $\phi^x > \phi^d$. A direct comparison of Figures 4a and 4b, discussed above separately, already provides an implicit indication that assuming exporters have higher needs for working capital ($\phi^x > \phi^d$) implies large changes in the leverage ratio for exporters. Figure 4a assumes a higher working capital requirement for exporters ($\phi^x$ =1) than does Figure 4b ($\phi^x$=0.53), and indicates a correspondingly lower leverage ratio for exporters. This implication can be seen even more clearly in Figure 5, which plots leverage ratios for various values of $\phi^x$. As this parameter is raised from 0.5 to 1, the leverage ratios for all exporting firms fall together by approximately a quarter (22 percentage points, from around 0.67 to around 0.45). The logic for this capital structure decision mirrors that in the preceding section: as the working capital required for exporting progressively rises (rises $\phi^x$), exporting firms choose to trade away the tax benefits of

[18] We also wish to emphasize that our conclusion, that an endogenous capital structure can adjust to neutralize higher short-term working capital requirements in steady state for exporters, is in no way inconsistent with the large literature studying the trade collapse of 2007-10 as a temporary tightening of the working-capital constraint. The latter literature studies the dynamics implied a financial shock to the short-run working capital requirement, and relate to the short run, while our study of the steady state is relevant to the long run. The two cases can be fully compatible if we were to augment our model with adjustment costs to changing capital structure, such as a cost of changing dividend payouts, to reflect the fact that firms cannot instantly and costlessly adjust the capital structure to offset financial shocks. But because our objective is to explain the long-run cross sectional distribution of leverage ratios among heterogeneous firms, we study a model of the long-run steady state and abstract from some aspects of short-run dynamics.

long-term debt in order to accumulate equity as collateral for securing working capital.

In contrast, it may be surprising that imposing higher working capital needs on exporters has only small effects on the level of sales for exporting firms. Figure 6 plots the distribution of sales across firm indexes, and how this varies with alternative values of $\phi^x$. While the figure plots six different lines for the six different values of $\phi^x$, these lines are almost indistinguishable. That is, relative to the cross-sectional variation of export sales, there is very little effect on the level of these sales for a given firm when varying degrees of extra working capital requirements are imposed on exporters. This is surprising, since when working capital requirements are increased in models with exogenous capital structure, such as Kohn et al. (2016), this tightens the collateral constraint and forces a reduction in the use of labor and hence sales. Further, they find this drop in export sales can even be amplified as firms substitute away from export sales to domestic sales, given the lower relative working capital requirements. However, endogenous capital structure has the potential to fundamentally change this result in the long-run equilibrium. Firms now have a choice of responding to higher working capital needs, either by reducing sales or by adjusting capital structure, the latter to raise the level of equity to compensate for the greater needs for collateral. The figure shows that the optimal choice for firms in our model is to significantly adjust capital structure to offset most of the effect on production.

Figure 7 permits a more concise comparison of the effects of $\phi^x$ on capital structure and export sales. We choose a particular firm with an index that is near the margin of exporting, but which is consistently an exporter for all values of $\phi^x$ that we consider. We report the values of a number of variables for this particular form for a range of cases of $\phi^x$. Again we see that the leverage ratio falls by fully one third (from 0.67 to 0.45). If leverage is measured as the ratio of bonds to equity, this ratio falls 60% (from 1.9 to 0.75). In contrast, export sales of this firm fall by a much smaller 8% (log real export sales falls from -0.155 to -0.23).

We next confirm that the reason the effect on exports is small is due to the presence of endogenous capital structure. Ideally, we would like to compare to a counterfactual case where the capital structure decision is suspended, and leverage ratio held constant. But as explained earlier, we are not able to solve for a general equilibrium in this case, since we rely upon implications of endogenous capital structure to make it possible for us to apply the Meltiz (2003) strategy of aggregation over heterogeneous firms, even in the context of firms whose production levels are constrained by a financial friction. Nonetheless, we can get a partial equilibrium analytical approximation to this counterfactual case by simple examination of the exporter working capital constraint. Given that this constraint is binding (and assuming for simplicity that export sales cannot be used as collateral, $\gamma^{xx}$=0), we know that

$$\xi_t E_t\left(m_{t+1} V_{t+1}^x\left(b_t^x\left(z_j\right)\right)\right)=\phi^d\left(w_t l_t^{xd}\left(z_j\right)+w_t f_t^d\right)+\phi^x\left(w_t l_t^{xx}\left(z_j\right)+w_t f_t^x\right).$$

When there is a doubling of $\phi^x$ from 0.5 to 1, there are two main ways for the firm to respond: reducing the use of inputs for production on the right hand side of the constraint, or raising the firm collateral on the left hand side (via capital restructuring). In our benchmark numerical results, the substantial rise in firm value (fall in leverage ratio) due to capital restructuring therefore prevents the need for a large fall in export production.

However if we imagine that firm value were held constant with no capital restructuring, then the adjustment in firm export sales for a doubling of $\phi^x$ would be very large. A doubling of $\phi^x$ would require that the quantity $w\left(l^{xx}\left(z_j\right)+f^x\right)$ be cut in half in order to compensate and satisfy the constraint. Given that part of these costs are fixed costs that cannot be reduced by reduction in the scale of exports, this would require a fall in labor costs and hence a fall in output that would be larger than the rise in $\phi^x$. So we should see a fall in export sales by more than 50%. The fact that our benchmark simulation produces a much smaller fall in exports, of 8%, is due to the ability of capital restructuring to insulate exports sales. Appendix 2 (section 10) presents some analysis to understand the economic logic determining the degree to which the intensive margin of exports is offset by capital restructuring.

We note here briefly that the experiments reported in Figures 5-7 also implicitly provide a concrete example of the point made more abstractly at the end of section 5.3. While the discussion in the paragraphs above is worded in terms of a rise in $\phi^x$ and weakening financing position of exporters, the narrative of the experiments and graphs can easily be inverted to tell a story showing the effects of a fall in $\phi^x$ and strengthening financial position. In other words, as a firm is made financially stronger in terms of lower working capital needs, it will not take advantage of this position to loosen the short-term working capital constraint and scale up production closer to the unconstrained optimum. The graphs now make clear that the equilibrium level of firm production barely changes. Instead there is a large rise in the leverage ratio as the firm prioritizes tax benefits as a way to lower the long-term cost of capital.

The fact that capital structure adjustment appears to insulate the level of sales per firm from higher working capital requirement indicates that it buffers the effect on trade's intensive margin, the scale of exporter sales per firm. But this does not mean that overall exports are unaffected by the higher exporter working capital requirement, as we find evidence of large effects on the extensive margin trade. Figure 8 shows that the share of home firms that export falls as $\phi^x$ rises, from a share of 0.26 to around 0.22, that is a fall of over 15%. The focus of the impact on the extensive rather than intensive margin of trade distinguishes our work from recent work studying financial frictions in the form of the working capital constraint, and it is somewhat surprising, since the working capital constraint directly constrains the level of production and sales for exporters, and hence would normally be expected to have a direct effect on the intensive margin.

The channel by which the extensive margin is affected is also strikingly different from the related literature. The export entry condition (24) indicates that new exporters enter up to the point that the sunk cost of entry equals the benefit of entry, where the latter can be written as the difference in overall firm value as an exporter versus non-exporter, or equivalently as the present discounted value of the stream of future profits from export sales. The numerical results above indicate that the higher working capital requirement for exporters does not affect entry by lowering the level of sales and hence profits in this entry condition. Instead, it works via the capital restructuring, in that the shift from debt to equity long-term financing raises the cost of capital for the firm, which raises the cost of financing the sunk entry cost in the entry condition.

The linkage between capital structure and entry can be summarized by noting that capital structure affects firm value, and firm value is one side of the firm entry equation. Given that firm value equals the discounted sum of discounted profits, we can write the export entry condition (24) in terms of firm values:

$$V_t^x\left(b_{t\text{-}1}^x\left(z_i\right)\right)-V_t^{nx}\left(b_{t\text{-}1}^{nx}\left(z_i\right)\right)+\pi_t^{xx}\left(z_i\right)=K_t^{EX}.$$

Next, we note that the capital structure decision allocates firm value into the sum of debt and equity (plus current period profits since firms start producing in the initial period of entry):

$$V_t^k\left(b_{t\text{-}1}^k\left(z_i\right)\right)=q_t^k\left(b_t^k\left(z_i\right)\right)+\frac{b_t^k\left(z_X\right)}{R_t}+\pi_t^k\left(z_X\right),\quad k=nx,x.$$

These two statement imply we can rewrite the entry condition as

$$\left(q_t^x\left(z_x\right)-q_t^{nx}\left(z_x\right)\right)+\frac{b_t^x\left(z_x\right)-b_t^{nx}\left(z_x\right)}{R_t}+\pi_t^{xx}\left(z_X\right)=K_t^{EX}.$$

As discussed in Bergin, Feng and Lin (2018a), when firms respond to changes in the working capital constraint by lowering the leverage ratio and raising equity collateral, the fact that equity is a more costly form of long-term firm financing than debt due to the latter's tax advantages, means that firm value is lower for a given firm and given level of production. Intuitively, if one dollar of debt is retired by issuing new equity, that is, reallocated between the equity and bond components of the equation above, this capital restructuring raises equity value by less than a dollar. This means that the capital structure response to the working capital constraint lowers firm value, meaning that a range of firms that otherwise would have had firm value sufficient to justify export entry now will have firm value too low, and they will no longer become exporters.

To recap, numerical experiments indicate that endogenous capital structure fundamentally changes the way that the standard financial frictions in the working capital constraint act as a barrier to export entry. In contrast with recent work, they do not primarily constrain the scaling up of production and hence profits needed to justify exporting. Rather, the need to compensate for the working capital constraint tightness by raising collateral raises the long-term cost of capital for the firm, which is needed to pay the sunk entry cost. As a result the cost of financing the payment of the sunk entry cost becomes a steeper barrier to entry.

## 7. Conclusions

This paper studies the financing of entry costs as a barrier to export entry. While recent literature has emphasized financing of working capital as a barrier export entry, one might expect financing is even more relevant for one-time sunk entry costs, which need to be paid long before the benefits of entry are realized. Motivation for emphasizing long-term financing is taken from evidence we present that exporting firms tend to be more leveraged than non-exporting firms in terms of long-term debt as a share of overall firm value, but less leveraged in terms of short-term forms of debt like working capital debt, again as a ratio to overall firm value.

We marry a corporate finance model of capital structure, featuring an endogenous choice between equity and long-term debt, with a trade model featuring heterogeneous firms and export entry. The model considers the differing means of financing a range of costs that firms face, including variable, fixed and one-time sunk costs. This model emphasizes the linkages between short term financing of working capital, and the costs of long-term firm finance.

The model provides a ready explanation for our empirical fact above, that exporting firms are more leveraged due in part to greater size. As the size of firm sales grows with the firm-specific productivity level, profits and hence firm value grow proportionately. But the need for working capital to finance production costs grows less than proportionately, because the fixed costs do not increase with sales volume.

One lesson offered by the model is that an optimal capital structure for exporting firms will prioritize reducing the cost of capital used to pay sunk costs over relaxing a short-term working capital constraint. While exporters tend to be larger and they may be in a stronger position to secure working capital than non-exporters, the model indicates it is not optimal for them to use this advantage to further relax their working capital constraint and further scale up production closer to the unconstrained optimal. Rather, it is optimal to reduce equity collateral so as to raise the tightness of their working capital constraint to be the same as non-exporters, in order to reap the tax benefits of long-term debt relative to equity.

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## Table 1. Summary Statistics

| | (1) total sample | (2) not exporting | (3) exporting | (4) newly exporting | (5) continuing exporting |
|---|---|---|---|---|---|
| means: | | | | | |
| log of size | 4.793 | 4.221 | 6.609 | 5.897 | 6.797 |
| Book Leverage ratio | 0.425 | 0.419 | 0.445 | 0.448 | 0.445 |
| Short-term borrowing to total assets | 0.054 | 0.055 | 0.047 | 0.046 | 0.047 |
| Long-term debt to total assets | 0.152 | 0.150 | 0.161 | 0.160 | 0.161 |
| Debt in current liabilities to total assets | 0.057 | 0.061 | 0.043 | 0.047 | 0.042 |
| Book debt minus short-term borrowing to total assets | 0.358 | 0.351 | 0.391 | 0.383 | 0.394 |
| standard deviation of size (within) | 0.855 | 0.810 | 0.554 | 0.504 | 0.517 |
| Observations | 74830 | 56909 | 17921 | 3738 | 14183 |

note: we drop firms appearing only once in the sample.

## Table 2: Panel Regressions

| | (1) Book Leverage ratio | (2) Short-term borrowing to total assets | (3) Long-term debt to total assets | (4) Debt in current liabilities to total assets | (5) Book debt minus short-term borrowing to total assets |
|---|---|---|---|---|---|
| Size | 0.0118*** | 0.00249 | 0.0132*** | 0.00124*** | 0.00797*** |
| | (0.000746) | (0.00189) | (0.000621) | (0.000358) | (0.00221) |
| Exporter | 0.0158*** | -0.00432 | 0.00473** | 0.00255* | 0.0219*** |
| | (0.00217) | (0.00467) | (0.00181) | (0.00104) | (0.00545) |
| N | 74641 | 24842 | 74618 | 74619 | 24842 |

Table 3. Benchmark Parameterization

| Description | | |
| --- | --- | --- |
| | | From literature |
| Pareto distribution parameter | | $\theta = 3.8$ |
| Substitution elasticity in the consumption | | $\sigma = 3.8$ |
| Probability of death shock | | $\lambda = 0.025$ |
| Iceberg trade cost | | $\tau_X = 0.16$ |
| Tax benefit | | $\tau = 0.35$ |
| household discount factor | | $\beta = 0.99$ |
| household relative risk aversion | | $\rho = 2$ |
| Weight of labor disutility in utility function | | $\kappa = 3.409$ |
| Inverse of labor supply elasticity | | $\psi = 0.5$ |
| Working capital requirement, domestic sales | | $\phi^d = 0.53$ |
| Working capital requirement, export sales | | $\phi^x = 1$ |
| Rest-of-world income | | $Y^* = 5Y$ |
| Rest-of-world export price | | $P^{x*} = 1$ |
| | To Match | Targeted Moments |
| Fixed costs (export production) | $f^x = 0$ | No endogenous exit |
| Fixed costs (domestic production) | $f^d = 0.00225$ | Range in leverage ratio for non-exporting firms |
| Export Entry costs | $K^{EX} = 8.845$ | 22% of firms engaging in export |
| Domestic Entry costs | $K^E = 2.85$ | No firms having negative leverage |
| Enforcement parameter for export sales | $\gamma^{xx} = 0.27$ | Average ratio of debt to collateral for exporters |
| Enforcement parameter | $\xi = 0.055$ | Average ratio of debt to collateral for all firms |
| Rest-of-world price level | $P^* = 0.51$ | Exports 26% of GDP |

Figure 1a. Numerical solution of cross-sectional distributions of key variables across firms

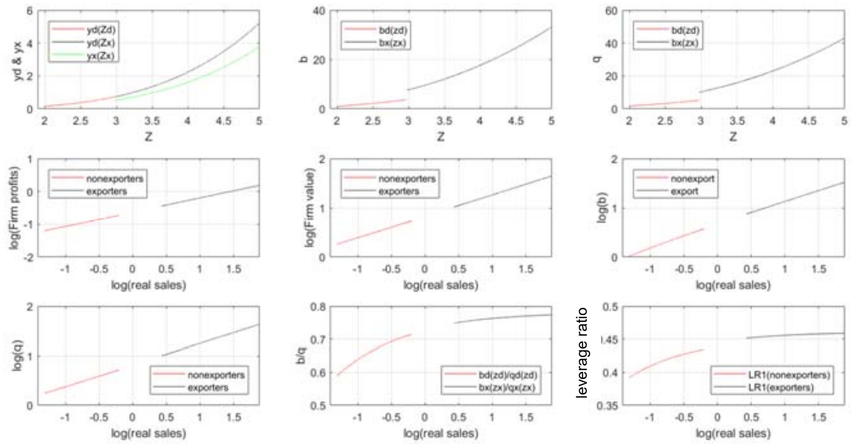



Figure 1b: Distribution of leverage ratios for benchmark case, enlarged

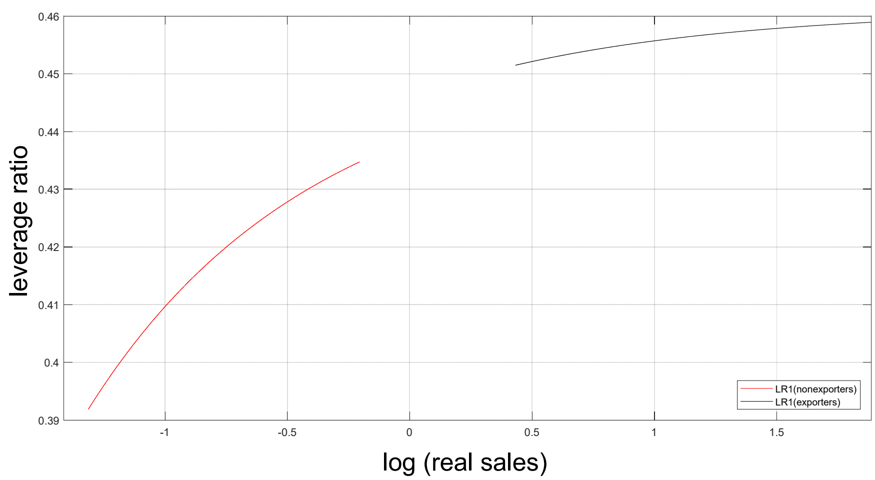


Figure 2: Leverage Ratio for various domestic fixed costs ( $f_d$ )

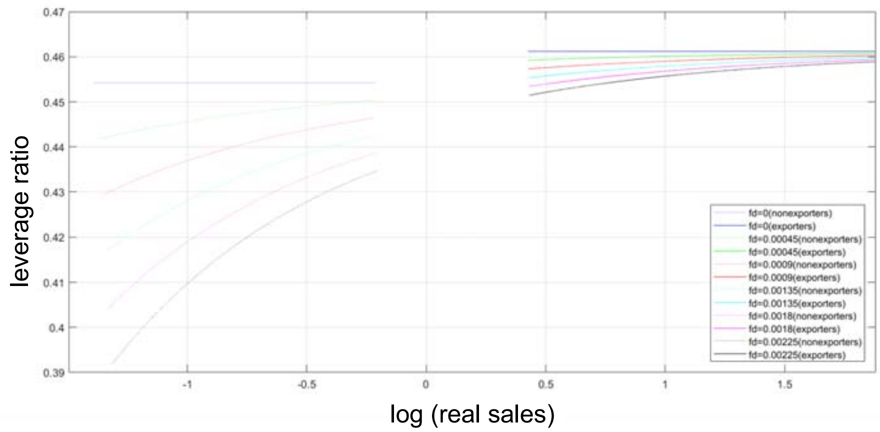



Figure 3: Leverage Ratio for Model with fixed cost of exporting ( $f_x = 0.003$ )

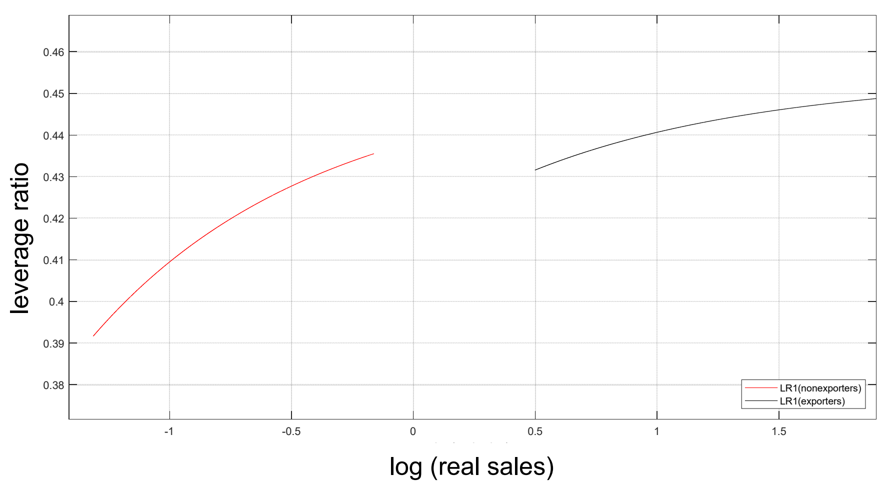


Figure 4a: Leverage Ratio with no collateral value of export inventories ( $\gamma^{xx} = 0$)

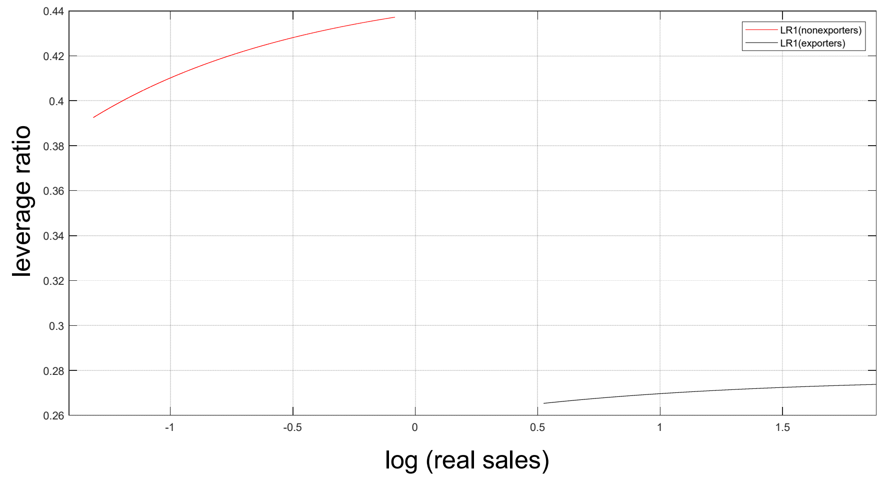



Figure 4b: Leverage Ratio with neither extra working capital requirements nor collateral for exporters ( $\phi^{x} = 0.53$, $\gamma^{xx} = 0$)

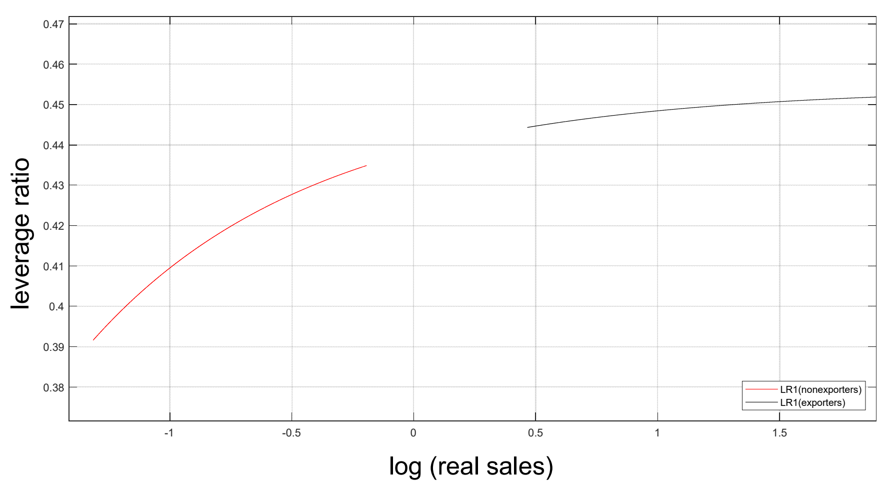


Figure 5: Leverage Ratio for various working capital requirements for exporters ($\phi^x$)

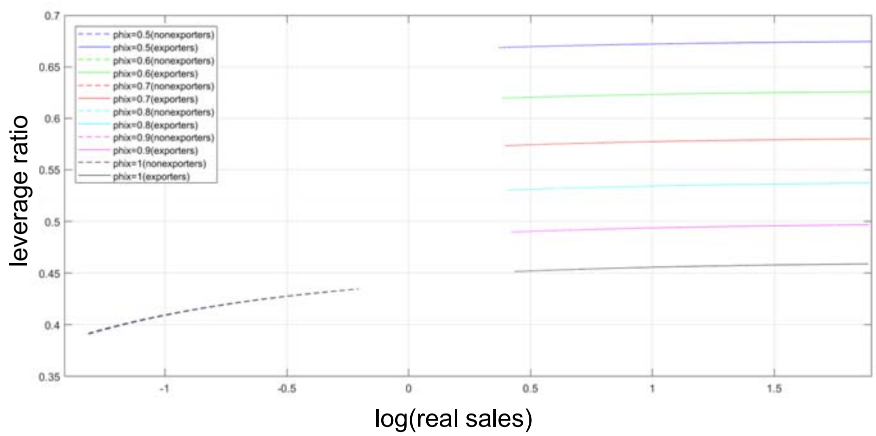



Figure 6: Firm-level sales for various working capital requirements for exporters ($\phi^x$)

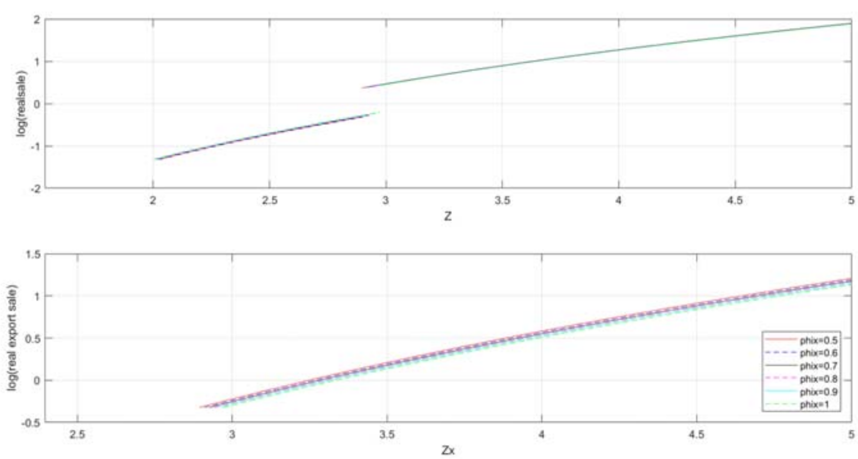


Figure 7: Leverage and other variables for various working capital requirements for exporters ($\phi^x$) for the exporter z=3.0739 (the intensive margin)

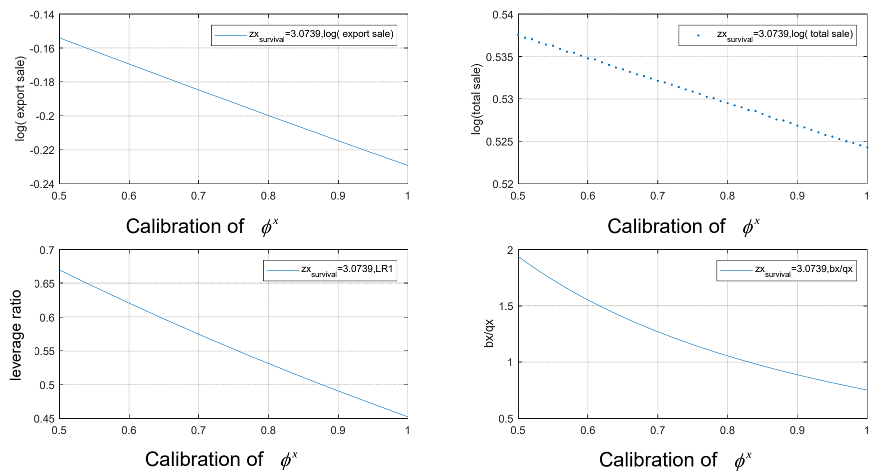
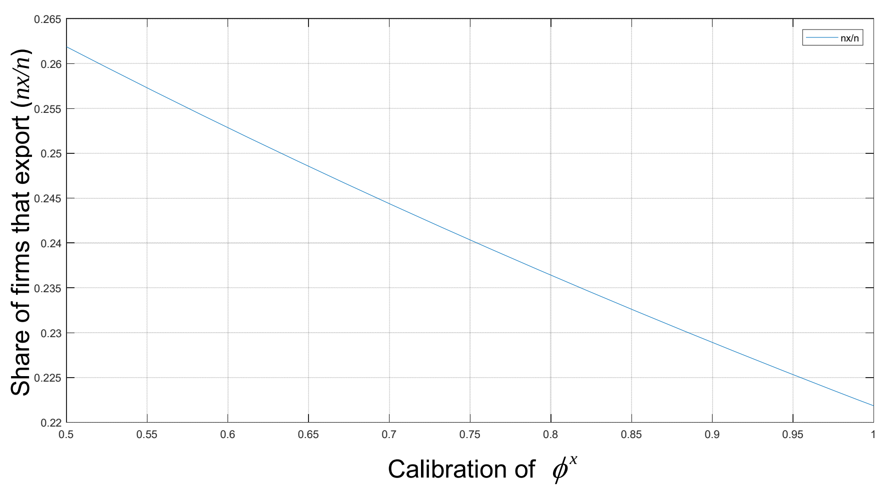



Figure 8: Export participation for various working capital requirements for exporters ($\phi^x$)



Appendix 1: Equations for small open economy.

Appendix Table 1

| Equations | Variables |
| --- | --- |
| 1. Households | $C \ \ w \ \ L$ |
| 1) $C^{-\rho} w = \kappa L^{\psi}$ | $r \ \ \tilde{d} \ \ \tilde{q}$ |
| 2) $\beta(1-\lambda)=\frac{1}{1+r}$ | |
| 3) $\tilde{q}(\tilde{z})=\frac{\beta(1-\lambda)}{(1-\beta(1-\lambda))}\tilde{d}(\tilde{z})$ | |
| 2. Final Goods | $\tilde{Y}^{xx} \ P^{*} \ \ \tilde{P}^{xx} \ Y^{*}$ |
| 4) $\tilde{Y}^{xx}=\left(\frac{\tilde{P}^{xx}}{P^{*}}\right)^{-\sigma} Y^{*}$, | $\tilde{Y}^{nx} \ \tilde{P}^{nx} \ Y \ \ P$ |
| 5) $\tilde{Y}^{nx}=\left(\frac{\tilde{P}^{nx}}{P}\right)^{-\sigma} Y$, | $\tilde{Y}^{xd} \ \ \tilde{P}^{xd}$ |
| 6) $\tilde{Y}^{xd}=\left(\frac{\tilde{P}^{xd}}{P}\right)^{-\sigma} Y$, | $\tilde{Y}^{x*} \ \ p^{x*}$ |
| 7) $\tilde{Y}^{x*}=\left(\frac{p^{x*}}{P}\right)^{-\sigma} Y$ | $\tilde{P}^{d} \ N^{x} \ N^{nx} \ N$ |
| 8) $P^{1-\sigma}=\frac{N^{x}}{1\text{-}\lambda}\left(\tilde{P}^{xd}\right)^{1-\sigma}+\frac{N^{nx}}{1\text{-}\lambda}\left(\tilde{P}^{nx}\right)^{1-\sigma}+\left(p^{x^{*}}\right)^{1-\sigma}$ | |
| 9) $N\left(\tilde{P}^{d}\right)^{1-\sigma}=N^{x}\left(\tilde{P}^{xd}\right)^{1-\sigma}+N^{nx}\left(\tilde{P}^{nx}\right)^{1-\sigma}$ | |
| 3. Intermediate | |
| 3.1 Average Nov-exporters | |
| 10) $\tilde{V}^{nx}=\tilde{d}^{nx}+\tilde{q}^{nx}$ | $\tilde{V}^{nx} \ \tilde{d}^{nx} \ q_{t}^{nx}$ |
| 11) $\tilde{q}^{nx}=\frac{\beta(1-\lambda)}{(1-\beta(1-\lambda))}\tilde{d}^{nx}$ | $\tilde{L}_{t}^{nx} \ \tilde{b}_{t}^{nx}$ |
| | $\tilde{z}^{nx} \ \tilde{\mu} \ \ R \ \ z_{x} \ z_{d}$ |

12) $\tilde{b}^{nx} = \frac{R}{R-1}\left(\frac{\tilde{P}^{nx}\tilde{Y}^{nx}}{P} - w\,\tilde{L}^{nx} - w\,f^{d} - \tilde{d}^{nx}\right)$

13) $R = 1 + r\,(1-\tau)$

14) $\tilde{l}^{nx} = \frac{\tilde{y}^{nx}}{A\tilde{z}^{nx}}$

15) $\xi\,\tilde{q}^{nx} = \phi^{d} w\left(\tilde{L}^{nx}(z_i) + f^{d}\right)$

16) $\tilde{\mu} = \frac{1/R - m}{\xi m}$

17) $\frac{\tilde{P}^{nx}}{P} = \frac{\sigma}{\sigma-1}\frac{w}{A\,\tilde{z}^{nx}}\left(1+\phi^{d}\tilde{\mu}\right)$

18) $\left(\tilde{z}^{nx}\right)^{\sigma-1} = \frac{\theta\left(z_X^{\ \sigma-\theta-1} - z_D^{\ \sigma-\theta-1}\right)}{(\sigma-\theta-1)\left(z_D^{\ -\theta} - z_X^{\ -\theta}\right)}$

3.2Average exporter

$\tilde{V}^{X}$ $\tilde{d}^{X}$ $\tilde{q}^{X}$

$\tilde{L}^{xd}$ $\tilde{L}^{xx}$ $\tilde{b}^{x}$

$\tilde{z}^{x}$ $\tilde{L}^{x}$

19) $\tilde{V}^{x} = \tilde{d}^{x} + \tilde{q}^{x}$

20) $\tilde{q}^{x} = \frac{\beta\,(1-\lambda)}{\left(1-\beta\,(1-\lambda)\right)}\tilde{d}^{x}$

21) $\tilde{b}^{x} = \frac{R}{R-1}\left[\frac{\tilde{P}^{xd}\tilde{Y}^{xd}}{P} + \frac{\tilde{P}^{xx}\tilde{Y}^{xx}}{P} - w\,\left(\tilde{L}^{xd} + \tilde{L}^{xx}\right) - w\,\left(f^{d} + f^{x}\right) - \tilde{d}\right.$

22) $\tilde{l}^{xd} = \frac{\tilde{y}^{xd}}{A\tilde{z}^{x}}$

23) $\tilde{l}^{xx} = \frac{\tilde{y}^{xx}}{A\tilde{z}^{x}\left(1\text{-}\tau_x\right)}$

24) $\tilde{L}^{x} = \tilde{L}^{xd} + \tilde{L}^{xx}$

25) $\xi\,\tilde{q}^{x} + \gamma^{xx}\frac{\tilde{p}^{xx}\tilde{y}^{xx}}{P} = \phi^{d}\left(w\tilde{l}^{xd} + w\,f^{d}\right) + \phi^{x}\left(w\tilde{l}^{xx} + w\,f^{x}\right)$

| | |
|---|---|
| 26) $\frac{\tilde{P}^{xd}}{P}=\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}^{x}}\left(1+\phi^{d}\tilde{\mu}\right)$ | |
| 27) $\left(\tilde{z}^{x}\right)^{\sigma-1}=\frac{-\theta z_{x}{}^{\sigma-\theta-1}}{\left(\sigma-\theta-1\right)z_{x}{}^{-\theta}}=\frac{-\theta z_{x}{}^{\sigma-1}}{\sigma-\theta-1}$ | |
| 28) $\frac{\tilde{P}^{xx}}{P}=\frac{\sigma}{\sigma-1}\frac{w_{t}}{A\,\tilde{z}^{x}\left(1-\tau_{x}\right)}\frac{1+\phi^{x}\tilde{\mu}}{1+\gamma^{xx}\tilde{\mu}}$ | |
| 3.3Marginal New Entrant | |
| 29) $V^{nx}\left(z_{d}\right)=d_{d}^{nx}+q_{d}^{nx}$ | $V_{d}^{nx}$ $d_{d}^{nx}$ $q_{d}^{nx}$ |
| 30) $b_{d}^{nx}=\frac{R}{R-1}\left[\frac{p_{d}^{nx}y_{d}^{nx}}{p}-w\,\left(l_{d}^{nx}+f^{d}\right)-d_{d}^{nx}\right]$ | $l_{d}^{nx}$ $b_{d}^{nx}$ $y_{d}^{nx}$ $p_{d}^{nx}$ |
| 31) $l_{d}^{nx}=\frac{y_{d}^{nx}}{A\,z_{d}}$ | |
| 32) $y_{d}^{nx}=\left(\frac{p_{d}^{nx}}{P}\right)^{-\sigma}Y$ | |
| 33) $q_{d}^{nx}=\frac{\beta\,\left(1-\lambda\right)}{\left(1-\beta\,\left(1-\lambda\right)\right)}d_{d}^{nx}$ | |
| 34) $\xi q_{d}^{nx}=\phi^{d}w\left(l_{d}^{nx}+f^{d}\right)$ | |
| 35) $\frac{p_{d}^{nx}}{P}=\frac{\sigma}{\sigma-1}\frac{w}{A\,z_{d}}\left(1+\phi^{d}\mu\;\right),$ | |
| 36) $\frac{p_{d}^{nx}y_{d}^{nx}}{P}-w\,\left(l_{d}^{nx}+f^{d}\right)-\left(b_{d}^{nx}-\frac{b_{d}^{nx}}{R}\right)+q_{d}^{nx}-K^{E}=0$ | |
| 3.4 marginal exporter | $V_{x}^{x}$ $d_{x}^{x}$ $q_{x}^{x}$ |
| 37) $V^{x}\left(z_{x}\right)=d_{x}^{x}+q_{x}^{x}$ | $p_{x}^{xd}$ $y_{x}^{xd}$ $p_{x}^{xx}$ $y_{x}^{xx}$ |
| 38) $b_{x}^{x}=\frac{R}{R-1}\left[\frac{p_{x}^{xd}y_{x}^{xd}}{P}+\frac{p_{x}^{xx}y_{x}^{xx}}{P}-w\,\left(l_{x}^{xd}+l_{x}^{xx}\right)-w\,\left(f^{d}+f^{x}\right)-d_{x}^{x}\right]$ | $b_{x}^{x}$ $l_{x}^{xd}$ $l_{x}^{xx}$ |

| | |
|---|---|
| 39) $l_x^{xd} = \dfrac{y_x^{xd}}{A\, z_x}$ | $V_x^{nx}\ d_x^{nx}\ q_x^{nx}$<br>$p_x^{nx}\ y_x^{nx}\ l_x^{nx}\ b_x^{nx}$ |
| 40) $l_x^{xx} = \dfrac{y_x^{xx}}{A\, z_x\left(1-\tau_x\right)}$ | |
| 41) $y_x^{xd} = \left(\dfrac{p_x^{xd}}{P}\right)^{-\sigma} Y$ | |
| 42) $y_x^{xx}\left(z_i\right) = \left(\dfrac{p_{xt}^{xx}}{P^*}\right)^{-\sigma} Y^* = \left(\dfrac{p_x^{xx}}{P}\right)^{-\sigma}\left(\dfrac{P}{P^*}\right)^{-\sigma} Y^*$ | |
| 43) $\xi q_x^x + \gamma^{xx}\dfrac{p_x^{xx} y_x^{xx}}{P} = \phi^d w\left(l_x^{xd} + f^d\right) + \phi^x w\left(l_x^{xx} + f^x\right)$ | |
| 44) $\dfrac{p_x^{xd}}{P} = \dfrac{\sigma}{\sigma-1}\dfrac{w}{A\, z_x}\left(1+\phi^d \mu\right)$ | |
| 45) $\dfrac{p_x^{xx}}{P} = \dfrac{\sigma}{\sigma-1}\dfrac{w}{A\, z_x}\dfrac{1+\phi^x \mu}{\left(1\text{-}\tau_x\right)\left(1+\gamma^{xx}\mu\right)}$ | |
| 46) $\dfrac{p_x^{nx} y_x^{nx}}{P_t} - w\left(l_x^{nx} + f^d\right) - \left(b_x^{nx} - \dfrac{b_x^{nx}}{R}\right) + q_x^{nx} - K^E$<br>$= \dfrac{p_x^{xd} y_x^{xd}}{P} + \dfrac{p_x^{xx} y_x^{xx}}{P} - w\left(l_x^{xd} + l_x^{xx} + f^d + f^d\right) - \left(b_x^{nx} - \dfrac{b_x^{x}}{R}\right) + q_x^x - \left(K^E + K^{EX}\right)$ | |
| or: $\left(\dfrac{p_x^{xx} y_x^{xx}}{P} - w\, l_x^{xx} - w\, f^x\right) + \dfrac{b_x^x - b_x^{nx}}{R} + \left(q_x^x - q_x^{nx}\right) = K^{EX}$ | |
| 47) $q_x^x = \dfrac{\beta\left(1-\lambda\right)}{\left(1-\beta\left(1-\lambda\right)\right)} d_x^x$ | |
| 48) $q_x^{nx} = \dfrac{\beta\left(1-\lambda\right)}{\left(1-\beta\left(1-\lambda\right)\right)} d_x^{nx}$ | |
| 49) $\dfrac{p_x^{nx}}{P} = \dfrac{\sigma}{\sigma-1}\dfrac{w}{A\, z_x}\left(1+\phi^d \mu\right)$ | $Ne\ \ Ne^x$ |
| 50) $d_x^{nx} = \dfrac{p_x^{nx} y_x^{nx}}{P} - w\, l_x^{nx} - b_x^{nx} + \dfrac{b_x^{nx}}{R} - w\, f^d$ | |

| | |
|---|---|
| 51) $V_x^{nx} = d_x^{nx} + q_x^{nx}$<br>52) $l_x^{nx} = \frac{y_x^{nx}}{A\, z_x}$<br>53) $y_x^{nx} = \left(\frac{p_x^{nx}}{P}\right)^{-\sigma} Y$<br>54) $\xi q_x^{nx} = \phi^d w \left(l_x^{nx} + f^d\right)$<br>4. Aggregation<br>55) $N^x \tilde{p}^{xx} \tilde{y}^{xx} = P^{X^*} Y^{X^*}$<br>56) $Y = Ne\, K^E + Ne^x K^{EX} + C$<br>57) $L = \frac{N^{nx} \tilde{L}^{nx} + N^x\left(\tilde{L}^{xd} + \tilde{L}^{xx}\right) + \left(N^{nx} + N^x\right) f^d + N^x f^x}{1-\lambda}$<br>58) $N = N^{nx} + N^x$<br>59) $N = (1-\lambda)(N + Ne)$<br>60) $N = (1-\lambda)(1-G(z_d))M = (1-\lambda) z_D^{-\theta}$<br>61) $N^x = (1-\lambda)(N^x + Ne^x)$<br>62) $N^x = (1-\lambda)(1-G(z_x))M = (1-\lambda) M\, z_x^{-\theta}$<br>63) $N\, \tilde{d} = N^{nx} \tilde{d}^{nx} + N^x \tilde{d}^x$<br>64) $P^* = 0.51$<br>65) $p^{x*} = 1$<br>66) $Y^* = 5Y$ | |

Appendix 2: Analytical Results for Other Variables

**Non-exporters**

**(1) Sales**

Combining the pricing equation (19) and the market demand, Eq. (8), we have

$$sales = \frac{p^{nx}(z_i)}{P} y^{nx}(z_i) = \left( \frac{\sigma}{\sigma-1} \frac{w}{Az_i} \left(1+\phi^d \mu\right) \right)^{1-\sigma} Y \cdot$$

Taking the first-order derivative and using the first-order derivative of equity price w.r.t. to $z_i$, that is , Eq. (36), give

$$\frac{\partial sales^{nx}(z_i)}{\partial z_i} = \frac{\sigma}{\sigma-1}\left(1+\phi^d \mu\right) \frac{\xi}{\phi^d} \frac{\partial q^{nx}(z_i)}{\partial z_i}.$$

It shows that, first, sales is increasing in productivity when $\sigma > 1$, which is also the condition for equity price to increase in productivity; second, the marginal effect of productivity on firm sale for a non-exporter is linear in the marginal effect of productivity on its equity price.

In particular we find a linear relationship between the log level of sales and the log level of productivity,

$$\log\left(sales^{nx}\right) = (1-\sigma)\log\left( \frac{\sigma}{\sigma-1} \frac{w}{A}\left(1+\phi^d \mu\right) \right) + \log(Y) + (\sigma-1)\log(z_i),$$

which implies that

$$\frac{\partial \log\left(sales^{nx}(z_i)\right)}{\partial \log(z_i)} = (\sigma-1),$$

given the aggregate market condition.

**(2) Sales-equity ratio**

In section 4.1, Eq. (38) shows that the marginal effect of $z_i$ on the long-term debt-equity ratio essentially relies on the sales-equity ratio. So here we show how changing productivity affects the sales-equity ratio formally.

Taking the first-order derivative of the sales-equity ratio w.r.t. $z_i$, we have

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i} = \frac{1}{q^{nx}(z_i)} \frac{\partial sales^{nx}(z_i)}{\partial z_i} - \frac{sales^{nx}(z_i)}{\left(q^{nx}(z_i)\right)^2} \frac{\partial q^{nx}(z_i)}{\partial z_i},$$

or

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i} = \frac{\partial q^{nx}(z_i)}{\partial z_i} \frac{1}{\left(q^{nx}(z_i)\right)^2} \left( \frac{\frac{\partial sales^{nx}(z_i)}{\partial z_i}}{\frac{\partial q^{nx}(z_i)}{\partial z_i}} q^{nx}(z_i) - sales^{nx}(z_i) \right).$$

Given the marginal effect of productivity on equity price, Eq. (36), we have the following equation,

$$sales^{nx}(z_i)=\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i}\frac{\xi}{\phi^d}\frac{z_i}{\sigma-1}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right),$$

which, combining with the marginal effect of productivity on sales,

$$\frac{\partial sales^{nx}(z_i)}{\partial z_i}=\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{\xi}{\phi^d}\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i},$$

gives

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i}=\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{1}{\left(q^{nx}(z_i)\right)^2}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\frac{\xi}{\phi^d}\left(q^{nx}(z_i)-\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i}\frac{z_i}{\sigma-1}\right).$$

Using the enforcement constraint, Eq. (14),

$$q^{nx}(z_i)=\frac{\phi^d}{\xi_q}\left(\frac{\sigma}{\sigma-1}\left(\frac{1+\phi^d\mu_t}{1+\gamma_t^{nx}\mu_t}\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}+\frac{\phi^d wf^D}{\xi_q}$$

and the marginal effect of productivity on equity price, Eq. (36),

$$\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i}=\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma-1}{z_i}\right),$$

we have

$$q^{nx}(z_i)-\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{z_i}{\sigma-1}=\left(\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}+\frac{\phi^d wf^D}{\xi}\right)-\frac{\phi^d}{\xi}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y\left(\frac{w}{Az_i}\right)^{1-\sigma}=\frac{\phi}{\xi}wf^D$$

Hence,

$$\frac{\partial\left(sales^{nx}(z_i)/q^{nx}(z_i)\right)}{\partial z_i}=\frac{\partial q^{nx}(z_i)}{\partial z_i}\frac{1}{\left(q^{nx}(z_i)\right)^2}\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)wf^D.$$

**(3) Product quantity**

Combining the pricing equation (19) and the market demand, Eq. (8), we have

$$y^{nx}(z_i)=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\left(1+\phi^d\mu\right)\right)^{-\sigma}Y.$$

This implies

$$\frac{\partial y^{nx}\left(z_i\right)}{\partial z_i}=\frac{Az_i}{w}\frac{\sigma}{\sigma-1}\frac{\xi_q}{\phi}\frac{\partial q^{nx}\left(z_i\right)}{\partial z_i}.$$

A similar linearity is found in the relation between the log level of product quantity and the log level of productivity, given by

$$\log\left(y^{nx}(z_i)\right)=\log\left(\left(\frac{\sigma}{\sigma-1}\left(\frac{1+\phi^d\mu_t}{1+\gamma_t^{nx}\mu_t}\right)\frac{w}{A}\right)^{-\sigma}Y\right)+\sigma\log\left(z_i\right)$$

which implies

$$\frac{\partial\log\left(y^{nx}\left(z_i\right)\right)}{\partial\log\left(z_i\right)}=\sigma.$$

### (4) Profit

Given $\pi^{nx}(z_i)=\frac{p^{nx}(z_i)}{P}y^{nx}(z_i)-\left(wl^{nx}(z_i)+wf^{nx}\right)$, and combining the pricing equation (19) and the market demand, Eq. (8), and the enforcement constraint, Eq. (14), we have

$$\pi^{nx}(z_i)=sale^{nx}(z_i)-\frac{\xi_q q^{nx}(z_i)}{\phi^d}.$$

Taking the first-order derivative thus gives

$$\frac{\partial\pi^{nx}(z_i)}{\partial z_i}=\frac{\partial q^{nx}}{\partial z_i}\frac{\xi}{\phi^d}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)-1\right).$$

The first item in the bracket captures the rising sales effect associated with the rising $z_i$, while the second item captures the rising production cost associated with rising productivity.

### (5) Firm value

Given that $V^{nx}(z_i)=q^{nx}(z_i)+d^{nx}(z_i)=\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^{nx}(z_i)$, we have

$$\frac{\partial V^{nx}(z_i)}{\partial z_i}=\left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\frac{\partial q^{nx}(z_i)}{\partial z_i}.$$

## Exporters

The equations for exporters are analogous, but with additional parameters government the working capital needs of exporters, and the collateral value of exporter accounts receivable.

### (1) Sales

Given that $sales^{x}(z_i)=sales^{xd}(z_i)+sales^{xx}(z_i)$, $sales^{xd}(z_i)=\frac{p^{xd}(z_i)}{P}y^{xd}(z_i)$, and $sales^{xx}(z_i)=\frac{p^{xx}(z_i)}{P}y^{xx}(z_i)$, we have

$$\frac{\partial sales^{xd}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\left(1+\phi^d\mu\right)\right)^{1-\sigma}Y,$$

$$\frac{\partial sales^{xx}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*,$$

$$\frac{\partial sales^{x}(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\left(1+\phi^d\mu\right)^{1-\sigma}Y_t+\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right).$$

Note, the presence of the asymmetric working capital needs ($\phi^d\neq\phi^x$) and the reliance on trade credit ( $\gamma^{xx} \neq 0$ ) does not allow us to represent $\frac{\partial sales^{x}(z_i)}{\partial z_i}$ as a function of $\frac{\partial q^{x}(z_i)}{\partial z_i}$, as we did for the non-exporter.

As for the non-exporter, we find a linear relationship between the log level of sales and the log level of productivity,

$$\log\left(sales^{x}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w}{A}(1+\mu)\right)+\log\left(\left(1+\phi^{d}\mu\right)^{1-\sigma}Y+\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\right)+(\sigma-1)\log\left(z_i\right),$$

$$\log\left(sales^{xd}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\left(1+\phi^{d}\mu\right)\right)+\log\left(Y\right)+(\sigma-1)\log\left(z_i\right),$$

$$\log\left(sales^{xx}\right)=(1-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t}\right)+\log\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\right)+(\sigma-1)\log\left(z_i\right),$$

which implies that $\frac{\partial\log\left(sales^{xd}\left(z_i\right)\right)}{\partial\log\left(z_i\right)}=\frac{\partial\log\left(sales^{xx}\left(z_i\right)\right)}{\partial\log\left(z_i\right)}=\frac{\partial\log\left(sales^{x}\left(z_i\right)\right)}{\partial\log\left(z_i\right)}=(\sigma-1)$.

**(2) Sales-equity ratio**

In section 4.1, Eq. (41) shows that the marginal effect of $z_i$ on the long-term debt-equity ratio essentially relies on the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}$, and the export sales-equity ratio. So here we show how changing productivity affects these ratios formally.

The domestic and export sales-equity ratios are respectively given by

$$\frac{sales^{xd}(z_i)}{q^{x}(z_i)}=\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}\left(1+\phi^{d}\mu\right)^{1-\sigma}Y\frac{\left(z_i\right)^{\sigma-1}}{q^{x}\left(z_i\right)},$$

and

$$\frac{sales^{xx}(z_i)}{q^{x}(z_i)}=\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}\frac{\left(z_i\right)^{\sigma-1}}{q^{x}\left(z_i\right)}.$$

Taking the first-order derivative of these ratios w.r.t. $z_i$, we have

$$\frac{\partial\left(\frac{sales^{xd}(z_i)}{q^{x}(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^{x}\left(z_i\right)^{2}}\left(\phi^{d}f^{d}+\phi^{x}f^{x}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\left(1+\phi^{d}\mu\right)^{1-\sigma}Y,$$

and

$$\frac{\partial\left(\frac{sales^{xx}(z_i)}{q^{x}(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^{x}\left(z_i\right)^{2}}\left(\phi^{d}f^{d}+\phi^{x}f^{x}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\left(\frac{1+\phi^{x}\mu}{(1\text{-}\tau_{x})\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^{*}}\right)^{-\sigma}Y^{*}.$$

The marginal effect of $z_i$ on the total export sales-equity ratio is given correspondingly by

$$\frac{\partial\left(\frac{sales^x(z_i)}{q^x(z_i)}\right)}{\partial z_i}=\frac{w}{\xi q^x(z_i)^2}\frac{\sigma-1}{z_i}\left(\phi^d f^D+\phi^x f^X\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\left(\left(1+\phi^d\mu\right)^{1-\sigma}Y+\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right).$$

Again, we see the presence of the asymmetric working capital needs ($\phi^d\neq\phi^x$) and the reliance on trade credit ($\gamma^{xx}\neq 0$) prevents us from representing $\frac{\partial sales^x(z_i)}{\partial z_i}$ as a function of $\frac{\partial q^x(z_i)}{\partial z_i}$, as we did for the non-exporter.

**(3) Product quantity**

Combining the pricing equations (20)-(21) and the market demand, Eqs. (9) and (11), we have

$$\begin{aligned} y^x(z_i) &= y^{xd}(z_i)+\frac{1}{1\text{-}\tau_x}y^{xx}(z_i) \\ &=\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)\right)^{-\sigma}Y_t+\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t z_i}\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\end{aligned}.$$

This implies

$$\frac{\partial y^x(z_i)}{\partial z_i}=\frac{\sigma}{z_i}\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{-\sigma}\left(\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)^{-\sigma}Y_t+\left(\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\right).$$

Further, we have

$$\log\left(y^x(z_i)\right)=(-\sigma)\log\left(\frac{\sigma}{\sigma-1}\frac{w_t}{A_t}\right)+\log\left(\begin{aligned}&\left(\frac{1+\phi^d\mu_t^x(z_i)}{1+\gamma_t^{xd}\mu_t^x(z_i)}\right)^{-\sigma}Y_t\\ &+\left(\frac{1+\phi^x\mu_t^x(z_i)}{(1\text{-}\tau_x)\left(1+\gamma_t^{xx}\mu_t^x(z_i)\right)}\right)^{-\sigma}\left(\frac{P_t}{P_t^*}\right)^{-\sigma}Y_t^*\end{aligned}\right)+\sigma\log(z_i)$$

and

$$\frac{\partial\log\left(y^x(z_i)\right)}{\partial\log(z_i)}=\sigma.$$

**(4) Profit**

Given $\pi^X(z_i)=\frac{p^D(z_i)}{P}y^D(z_i)+\frac{p^X(z_i)}{P}y^X(z_i)(1\text{-}\tau_x)-w\left(l^D(z_i)+l^X(z_i)+f^D+f^X\right)$,and combining the pricing equations (20)-(21) and the market demand, Eqs. (9) and (11), we have

$$\frac{\partial\pi^x(z_i)}{\partial z_i}=\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y+\left(1-\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\right)$$

and

$$\frac{\partial\left(\frac{\partial \pi^x(z_i)}{\partial z_i}\right)}{\partial \gamma^{xx}} = \frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma} Y^*(\sigma-1)\mu\frac{\left(\phi^x-\gamma^{xx}\right)\mu}{\left(1+\gamma^{xx}\mu\right)\left(1+\phi^x\mu\right)}.$$

**(5) Firm value**

Given that $V^x(z_i) = q^x(z_i) + d^x(z_i) = \left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^x(z_i)$, we have

$$\frac{\partial V^x(z_i)}{\partial z_i} = \left(1+\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\frac{\partial q^x(z_i)}{\partial z_i}$$

**(6) Equity price**

From the enforcement constraint, for an individual exporter $i$, we have that

$$\xi q^x(z_j) = \phi^d\left(wl^{xd}(z_j)+wf^D\right)+\phi^x\left(wl^{xx}(z_j)+wf^X\right)\text{-}\gamma^{xx}\frac{p^{xx}(z_j)y^{xx}(z_j)}{P}.$$

Combing the production function, Eq. (15), the pricing equations (20)-(21), and the market demand, Eqs. (9) and (11), and taking the derivatives of the equity price with respect to firm productivity gives that

$$\frac{\partial q^X(z_i)}{\partial z_i} = \frac{1}{\xi}\left(\frac{\sigma-1}{z_i}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\left(\begin{array}{l}\left(\phi^d\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y \\ +\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

and

$$\frac{\partial^2 q^X(z_i)}{\partial (z_i)^2} = \frac{1}{\xi}(\sigma-1)(\sigma-2)\left(\frac{\sigma}{\sigma-1}\frac{w}{A}\right)^{1-\sigma}(z_i)^{\sigma-3}\left(\begin{array}{l}\left(\phi^d\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y \\ +\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

Hence, $\frac{\partial q^x(z_i)}{\partial z_i} > 0$ if $\sigma > 1$, and further $\frac{\partial^2 q^x(z_i)}{\partial (z_i)^2} > 0$ if $\sigma > 2$. We also need the condition that $\gamma^{xx} < \phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$. As for non-exporters, when the conditions are satisfied, rising firm productivity will increase firm equity value more for larger exporters than for smaller exporters.

For exporters, the impact of productivity on equity prices works through three channels, the production for domestic sales, the production for export sales and the accounts receivable used as collateral. The first two channels raise firm equity value while the third one reduces it. This is because rising firm productivity is associated with rising sales in both domestic and foreign markets, and hence raises firm equity value. However, rising export sales provide a second type of collateral, that is, the accounts receivable, which reduces firms' reliance on equity as collateral.

Further, we have that

$$\frac{\partial\left(\frac{\partial q^X(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}=\frac{1}{\xi}\left(\frac{\sigma-1}{z_i}\right)\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}(1\text{-}\tau_x)^{\sigma-1}(\sigma-1)\mu\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma-1}\frac{1}{1+\gamma^{xx}\mu}\left(\phi^x\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\left(\frac{P}{P^*}\right)^{-\sigma}Y^*. \quad \text{(A1)}$$

Eq. (37) shows that the rising reliance on accounts receivable as collateral (rising $\gamma^{xx}$) will amplify the effect of productivity on firm equity value, because $\frac{\partial\left(\frac{\partial q^X(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}>0$ when $\gamma^{xx}<\phi^x\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$ which is guaranteed by the condition that $\gamma^{xx}<\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)$.

**(7) Bond position**

From the dividend equation, (17), we have that

$$b^x(z_i)=\frac{R}{R-1}\left(\frac{p^{xd}(z_i)}{P}y^{xd}(z_i)+\frac{p^{xx}(z_i)}{P}y^{xx}(z_i)(1\text{-}\tau_x)-w\left(l^{xd}(z_i)+l^{xx}(z_i)+f^d+f^x\right)-d^x(z_i)\right).$$

Substituting the production function, Eq. (15), the pricing equations (20)-(21), the market demand equations (9)-(11), the firm value function, Eq. (18), , we have that

$$b^x(z_i)=\frac{R}{R-1}\left(\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\frac{p^{xd}(z_i)}{P}y^{xd}(z_i)+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\frac{p^{xx}(z_j)y^{xx}(z_j)}{P_t}-w\left(f^d+f^x\right)-\left(\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)q^x(z_i)\right). \quad \text{(A2)}$$

Note, the presence of trade credit and the asymmetric working capital requirements for exports and for domestic products in the enforcement constraint, prevent us from using the enforcement constraint to substitute the production cost with firm equity value, as we did for the non-exporters.

Then taking the derivatives of bond position with respect to firm productivity, by combining the pricing equations (20)-(21), and the market demand equations (9)-(11), yields

$$\frac{\partial b^x(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\begin{array}{l}\left(\underset{dom\_sales}{1}-\underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}}_{dom\_var\_cost}-\underbrace{\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}}_{equiy\_Needs\_dom\_cost}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\+\left(\underset{exp\_sales}{1}-\underbrace{\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}}_{exp\_var\_cost}-\underbrace{\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)}_{equity\_needs\_exp\_cost}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right).$$

and

$$\frac{\partial^2 b^x(z_i)}{\partial(z_i)^2}=\frac{R}{R-1}(\sigma-1)(\sigma-2)\left(\frac{w}{A}\right)^{1-\sigma}(z_i)^{\sigma-3}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\begin{array}{l}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}-\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{\phi^d}{\xi}\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}-\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left(\phi^x\frac{\sigma-1}{\sigma}\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)-\gamma^{xx}\right)\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{array}\right)$$

Further, we notice that

$$\frac{\partial\left(\frac{\partial b^x(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}=\frac{R}{R-1}\frac{\sigma-1}{z_i}\left(\frac{w}{Az_i}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\left(\frac{1+\phi^x\mu}{1\text{-}\tau_x}\right)^{1-\sigma}\left(1+\gamma^{xx}\mu\right)^{\sigma\text{-}2}\begin{pmatrix}(\sigma\text{-}1)\mu-(\sigma-1)\frac{\mu\left(1+\gamma^{xx}\mu\right)}{1+\phi^x\mu}\\ -\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\frac{1}{\xi}\left((\sigma-1)\frac{\phi^x}{1+\phi^x\mu}\mu\left(1+\gamma^{xx}\mu\right)-\left(1+\gamma^{xx}\mu\right)-\gamma^{xx}(\sigma\text{-}1)\mu\right)\end{pmatrix}.$$

However, the complicated representations of the first- and second-order derivatives make us difficult to conclude the conditions that ensure $\frac{\partial b^x(z_i)}{\partial z_i}>0$ or $\frac{\partial^2 b^x(z_i)}{\partial\left(z_i\right)^2}>0$ or $\frac{\partial\left(\frac{\partial b^x(z_i)}{\partial z_i}\right)}{\partial\gamma^{xx}}>0$.

**(8) Long-term Debt-to-equity ratio**

Now we are ready to look at the leverage ratio defined in Eq. (34), which relies on two ratios, the long-term debt-equity ratio and the short-term debt equity ratio. Let $LR_1^x(z_i)$ and $SR_1^x(z_i)$ denote the long-term and short-term ratios respectively, thus $LR_1^x(z_j)=\frac{b^x(z_j)}{q^{nx}\left(z_j\right)}$, and $SR_1^x(z_i)=\frac{IntraLoan\left(z_i\right)}{q\left(z_i\right)}$.

Using Eq. (40), we have the long-term debt-equity ratio

$$LR^x(z_i)=\frac{R}{R-1}\left[\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}+\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\frac{sales^{xx}(z_i)}{q^x(z_i)}-\frac{w\left(f^d+f^x\right)}{q^x(z_i)}-\left(\frac{1-\beta(1-\lambda)}{\beta(1-\lambda)}\right)\right], \quad \text{(A3)}$$

which shows the effect of changing $z_i$ on the long-term debt-equity ratio is through the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^{xd}(z_i)}$, the export sales-equity ratio, $\frac{sales^{xx}(z_i)}{q^x(z_i)}$, and the fixed costs-equity ratio, $\frac{w\left(f^d+f^x\right)}{q^x(z_i)}$.

A few steps of calculations show that

$$\frac{\partial LR^x(z_i)}{\partial z_i}=\frac{R}{R-1}\frac{1}{q^x\left(z_i\right)^2}\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{w}{\xi}\left[\begin{matrix}\left(\phi^d f^D+\phi^x f^X\right)\begin{pmatrix}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu}\right)\left(1+\phi^d\mu\right)^{1-\sigma}Y\\ +\left(1-\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{pmatrix}\\ +\left(f^D+f^X\right)\begin{pmatrix}\phi^d\frac{\sigma-1}{\sigma}\left(1+\phi^d\mu\right)^{-\sigma}Y\\ +\left(\phi^x\frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}-\gamma^{xx}\right)\left(\frac{1+\phi^x\mu}{(1\text{-}\tau_x)\left(1+\gamma^{xx}\mu\right)}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\end{pmatrix}\end{matrix}\right]$$

Thus, $\frac{\partial LR^{nx}(z_i)}{\partial z_i} > 0$ if $\sigma > 1$ and $\frac{\gamma^{xx}}{\phi^x} < \frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu} < 1$.

Additionally, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D = f^X = 0$, the leverage ratio is the same for all exporters, regardless of productivity level.

Further, we have $\frac{\partial\left(\frac{\partial LR^x(z_i)}{\partial z_i}\right)}{\partial \gamma^{xx}} > 0$ if

$$\frac{\phi^x - \gamma^{xx}}{\left(1+\gamma^{xx}\mu\right)\left(1+\phi^x\mu\right)}\left(\left(\phi^d f^d + \phi^x f^x\right)\mu + \left(f^D + f^X\right)\right) > f^D + f^X .$$

**(9) Short-term Debt-to-equity ratio**

Given $SR^x(z_i) = \frac{IntraLoan^x(z_i)}{q^x(z_i)}$ and $IntraLoan(z_i) = \phi^d\left(w\, l^{xd}(z_j) + w\, f^D\right) + \phi^x\left(w\, l^{xx}(z_j) + w\, f^X\right)$,

and combining with the production function, Eq. (15), the pricing equations (20)-(21), the market demand equations (9)-(11), we have

$$IntraLoan^x(z_i) = = \phi^d \frac{\sigma-1}{\sigma}\frac{1}{1+\phi^d\mu} sales_t^{xd}(z_j) + \phi^x \frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu} sales_t^{xx}(z_j) + \left(\phi^d w\, f^D + \phi^x w\, f^X\right)$$

and

$$SR^x(z_i) = \frac{\sigma-1}{\sigma}\frac{\phi^d}{1+\phi^d\mu}\frac{sales^{xd}(z_j)}{q^x(z_i)} + \phi^x \frac{\sigma-1}{\sigma}\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\frac{sales^{xx}(z_j)}{q^x(z_i)} + \frac{\phi^d w\, f^D + \phi^x w\, f^X}{q^x(z_i)},$$

which shows the effect of changing $z_i$ on the short-term debt-equity ratio is through the domestic sales-equity ratio, $\frac{sales^{xd}(z_i)}{q^x(z_i)}$, the export sales-equity ratio, $\frac{sales^{xx}(z_i)}{q^x(z_i)}$, and the fixed costs-equity ratio, $\frac{w\left(f^d + f^x\right)}{q^x(z_i)}$, as well as the long-term debt-equity ratio.

A few steps of calculations show that

$$\frac{\partial\left(SR^x(z_i)\right)}{\partial z_i} = \left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{1}{q^x(z_i)^2}\frac{w}{\xi}\left(\phi^d f^d + \phi^x f^x\right)\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma\text{-}1}(1\text{-}\tau_x)^{\sigma\text{-}1}\left(\frac{P}{P^*}\right)^{-\sigma} Y^*\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\frac{\phi^x}{\sigma} + \gamma^{xx}\right)$$

Thus, $\frac{\partial\left(SR^x(z_i)\right)}{\partial z_i} > 0$ if $\sigma > 1$.

Additionally, we see from this result that in the absence of fixed costs to domestic and export markets, $f^D = f^X = 0$, the short-term leverage ratio is the same for all exporters, regardless of productivity level.

Further, we have

$$\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial\gamma^{xx}}=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i}\right)^{1-\sigma}\frac{\sigma-1}{z_i}\frac{1}{q^X\left(z_i\right)^2}\frac{w}{\xi}\left(\phi^d f^d+\phi^x f^x\right)\left(\frac{1+\gamma^{xx}\mu}{1+\phi^x\mu}\right)^{\sigma-1}(1-\tau_x)^{\sigma-1}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*\left(\frac{\phi^x\mu}{1+\phi^x\mu}+\frac{\sigma\gamma^{xx}+1}{1+\gamma^{xx}\mu}\right)\text{ and}$$

$$\frac{\partial\left(SR^x\left(z_i\right)\right)}{\partial\gamma^{xx}}>0 \text{ if } \sigma>1.$$

**(10) Intensive margin response to higher working capital requirement**

The degree to which the intensive margin of exports is offset by capital restructuring can be understood by studying the price setting equation for exports (assuming for simplicity that export sales cannot be used as collateral ($\gamma^{xx}$=0)):

$$\frac{p^{xx}\left(z_i\right)}{P}=\frac{\sigma}{\sigma-1}\frac{w}{Az_i\left(1\text{-}\tau_x\right)}\left[1+\phi^x\mu^x\right].$$

The novel part of our equation is in square brackets, indicating an extra price markup due to the tightness of the collateral constraint. A rise in $\phi^x$ has two channels for raising export prices and hence reducing export sales. First there is a direct effect in that $\phi^x$ appears in the equation, and raises price by multiplying a given $\mu^x$. So this effect is conditional on $\mu^x$ >0, meaning it requires the financial constraint is binding. This effect will be small, given that our steady state value of $\mu^x$ is small: $\mu^x = 0.224$ implies that the 100% rise in $\phi^x$ from 0.5 to 1 raises $\frac{p^{xx}\left(z_i\right)}{P}$ by only about 10% ($\frac{1+\phi^x\mu^x}{1+\phi^d\mu^x}$). This coincides with our numerical results that the rise in $\phi^x$ lowers export sales of a given firm $z_i$ by about 8%, once one factors in the general equilibrium effects on wage and relative international price indexes.

Overall export sales of a firm z can be computed here as: $\frac{p^{xx}\left(z_x\right)y^{xx}\left(z_x\right)}{P}=\left(\frac{p^{xx}\left(z_i\right)}{P}\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*=\left(\frac{\sigma}{\sigma-1}\frac{w}{Az_i\left(1\text{-}\tau_x\right)}\left(1+\phi^x\mu^x\right)\right)^{1-\sigma}\left(\frac{P}{P^*}\right)^{-\sigma}Y^*$ . From our simulations, $\frac{p^{xx}\left(z_i\right)}{P}$ increases about 8.1765% but $\frac{P}{P^*}$ decreases about 3.811% for $\phi^x$ from 0.5 to 1. Here $w$ drops about 1.71% so that $\frac{p^{xx}\left(z_i\right)}{P}$ does not increase by 10%. Given $\sigma$=3.8 and the tiny change in $Y^*$ (-0.3060%), the impact on export sales: (1-3.8)*8.1765%-3.8*(-3.811%)=-8.411%.

The second effect is that, in the absence of capital structure, a rise in $\phi^x$ would make the exporter collateral constraint tighter, raising the value of $\mu^x$. But given that our capital structure optimization implies the optimality condition that $\mu^k(z_i) = \frac{1/R - Em}{\xi Em}$, we know that under capital structure, the tightness of the collateral constraint does not move for different values of $\phi^x$. So this second effect is completely eliminated by capital structure adjustments. The main lesson is that under endogenous capital structure, greater financial restrictions on working capitals need not have the effect that the past literature has assumed. Endogenous capital structure fundamentally changes this result, and can dramatically reduce effects on the intensive margin of export sales.