Modeling the intensity of competition

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Abstract

Within the context of a symmetric duopoly with linear demand and costs, we construct a parameterized family of price-setting games, where the parameter $\gamma \in [0, 2]$ measures the degree or intensity of competition; $\gamma = 0$ corresponds to collusion, a particular value of γ between 0 and 1 corresponds to the Cournot outcome, $\gamma = 1$ corresponds to the Bertrand outcome and, in general, as γ increases the intensity of competition increases. All the games within the parameterized family share the same strategic properties. We also construct a parameterized family of quantity-setting games, where the parameter $\beta \in [0, 2]$ measures the intensity of competition; $\beta = 0$ corresponds to collusion, $\beta = 1$ corresponds to the Cournot outcome and a particular value of β between 1 and 2 corresponds to the Bertrand outcome. As β increases, the intensity of competition increases. As an example of the potential usefulness of this approach, we show that, contrary to the view first put forward by Schumpeter (but later challenged by Arrow), the incentive to introduce a cost-reducing innovation is an increasing function of the intensity of competition (that is, an increasing function of γ in the price-setting case and of β in the quantity-setting case).

Keywords: Cournot game; Bertrand game; price competition; quantity competition; strategic substitute; strategic complement; degree of competition

May 27, 2026

1 Introduction

How can one model the possibility that a given industry may operate under a regime of more intense or less intense competition? In his path-breaking model of imperfect competition (in a homogeneous-product industry) Cournot (1838) showed that increasing the number of firms leads to a decrease in the equilibrium price; since a lower price is the symptom, or manifestation, of competition, one can take the number of firms as a measure of the intensity of competition. However, changing the number of firms alters the nature of the industry. Is it possible to model different degrees of competition for an industry that remains constant in terms of the objective data, namely demand, costs and the number of firms?

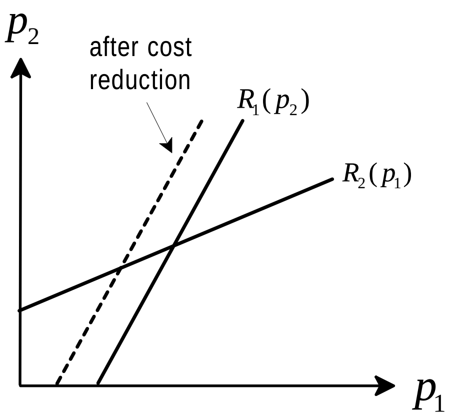
In his critique of Cournot's model, Bertrand (1883) showed that, by replacing the firms' behavior from choosing quantities to choosing prices, the equilibrium price drops from a level above marginal cost to marginal cost. Thus price-competition can be viewed as a more intense form of competition than quantity competition. That price competition leads to lower prices than quantity competition was later confirmed in models where the firms' products are differentiated. Hathaway and Rickard (1979) proved that in a duopoly with general demand and costs at least one Cournot-equilibrium price must be greater than the Bertrand-equilibrium price. Shubik and Levitan (1980) showed that prices are lower in Bertrand competition than in Cournot competition in a duopoly with symmetric linear demand and substitute goods. Singh and Vives (1984) extended this result to a duopoly with asymmetric linear demand and constant marginal cost and showed that Bertrand prices are lower than Cournot prices, not only if the goods are substitutes, but also if they are complements. Cheng (1985) obtained a similar result for a duopoly with general demand using a geometric approach. Okuguchi (1987) further extended the result to the case of oligopoly with linear demand and cost functions when the Jakobian matrix of the demand functions has a dominant negative diagonal. Amir and Jin (2001) and Vives (1985) further investigated the sense in which a Bertrand equilibrium incorporates more intense competition than a Cournot equilibrium.

Thus, one can view price competition as a more intense regime of competition than quantity competition. This observation has been used to study whether more intense competition is associated with a stronger or weaker incentive to innovate. A traditional line of reasoning, associated with Schumpeter (1943), is that market concentration (thus, less intense competition) is a stimulus to innovation. This view was challenged by Arrow (1962), who showed that the gain from a cost-reducing innovation is higher for a firm in a perfectly competitive industry than for a monopolist. Under the assumption of a homogeneous product, Delbono and Denicolò (1990) showed that the incentive to introduce a cost-reducing innovation is greater for a Bertrand competitor than for a Cournot competitor. On the other hand, Bester and Petrakis (1993) considered the case of differentiated products and obtained a mixed result: if the degree of differentiation is "large", the incentive to introduce a cost-reducing innovation is higher for the Cournot competitor, while if the degree of differentiation is "small", then the incentive is higher for a Bertrand competitor. Several more recent papers extended the analysis of the different incentives to innovate under the Bertrand and Cournot regimes.[1]

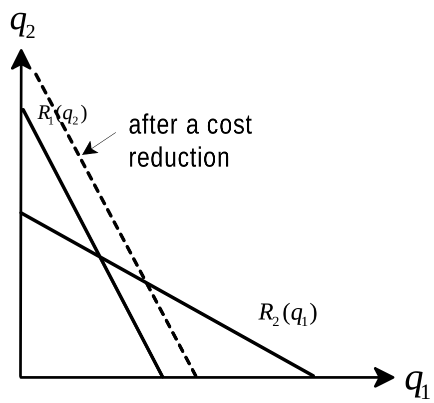
A comparison between Bertrand competition and Cournot competition, however, is not merely a comparison between two different intensity levels of competition, because the two games have different strategic properties. For example, using the terminology introduced by Bulow et al. (1985), when demand and costs are linear, prices in the Bertrand game are strategic complements (that is, the reaction functions are increasing), while quantities in the Cournot game are strategic substitutes (that is, the reaction functions are decreasing). As a consequence, the introduction of a cost reduction by one firm leads to a heightening of competition in the Bertrand regime while it leads to a lessening of competition in the Cournot regime. This is illustrated in Figure 1. Part (*a*) refers to the Bertrand game: a reduction in the marginal cost of Firm 1 is reflected in a leftward shift of Firm 1's reaction function, meaning that the best reply of Firm 1 to any price p_2 of Firm 2 is a lower price than before. Since prices are strategic complements, the optimal reaction of Firm 2 is to respond with a lower price itself, thus leading to an equilibrium with lower prices than in the initial situation. Part (*b*) of Figure 1 refers to the Cournot game: a reduction in the marginal cost of Firm 1 is reflected in a rightward shift of Firm 1's reaction function, meaning that the best reply of Firm 1 to any quantity q_2 of Firm 2 is a higher quantity than before. Since quantities are strategic substitutes, the optimal reaction of Firm 2 is to respond with a lower quantity, thus leading to an equilibrium where Firm 1 gains market share.

Thus if the Cournot model yields a stronger incentive for Firm 1 to pursue a cost reduction than the Bertrand model, it is not clear whether this is due to the fact that the Cournot model reflects less intense competition (i.e. higher prices) or to the different strategic nature of the two games. In order to separate the

[1]See, for example, Belleflamme and Vergari (2011), Bonanno and Haworth (1998), Boone (2001), Chang and Ho (2014), Schmutzler (2013), Symeonidis (2003), Weiss (2003).



(*a*) The reaction functions in the Bertrand game



(*b*) The reaction functions in the Cournot game

Figure 1: The different strategic nature of the Bertrand and Cournot games

intensity-of-competition effect from the strategic effect, it would be desirable to construct a parameterized family of games – all with the same strategic properties – where the parameter can be interpreted as expressing the degree or intensity of competition. This is done in Sections 3 (for a family of price-setting games) and in Section 4 (for a family of quantity-setting games), within the contexts of a symmetric duopoly with linear demand and costs; the case of a homogeneous product is also analyzed in Section 4. In Section 5 we use these parameterized families of games to revisit the issue of whether more intense competition is associated with a stronger or weaker incentive to introduce a cost-reducing innovation. Section 6 concludes.

2 Symmetric duopoly

As sarting point we consider a duopoly where both firms have the same constant marginal cost $m \geq 0$ and the demand functions are symmetric:

$$\begin{aligned} q_1 &= a - bp_1 + cp_2 \\ q_2 &= a + cp_1 - bp_2 \end{aligned} \tag{1}$$

with $a > 0$, $b > c > 0$ and $\frac{a}{b-c} > m$. Since $c > 0$, the goods are substitutes.

2.1 The Bertrand game

In the Bertrand game the payoff functions are

$$\begin{aligned} \Pi_1^B &= (p_1 - m)(a - bp_1 + cp_2) \\ \Pi_2^B &= (p_2 - m)(a + cp_1 - bp_2) \end{aligned} \tag{2}$$

The Bertrand equilibrium is given by the solution to $\frac{\partial \Pi_i^B}{\partial p_i} = 0$ $(i = 1, 2)$, namely

$$p_1^B = p_2^B = \frac{a + bm}{2b - c} \tag{3}$$

Replacing (3) into the demand functions we obtain the corresponding output levels:[2]

$$q_1^B = q_2^B = \frac{[a - m(b - c)]b}{2b - c} \tag{4}$$

[2] Given our assumptions, namely that $b > c > 0$ and $\frac{a}{b-c} > m$, the equilibrium prices and the corresponding quantities are positive.

In the Bertrand game prices are strategic complements. For example, the reaction function of Firm 1 is:

$$R_1^B = \frac{a+mb}{2b} + \frac{c}{2b}p_2 \tag{5}$$

which is increasing in p_2.

2.2 The Cournot game

Inverting (1) with respect to p_1 and p_2 we get the following inverse demand functions:

$$\begin{aligned} p_1 &= \tfrac{a}{b-c} - \tfrac{b}{b^2-c^2}q_1 - \tfrac{c}{b^2-c^2}q_2 \\ p_2 &= \tfrac{a}{b-c} - \tfrac{c}{b^2-c^2}q_1 - \tfrac{b}{b^2-c^2}q_2 \end{aligned} \tag{6}$$

In the Cournot game the payoff functions are

$$\begin{aligned} \Pi_1^C &= q_1(\tfrac{a}{b-c} - \tfrac{b}{b^2-c^2}q_1 - \tfrac{c}{b^2-c^2}q_2) - mq_1 \\ \Pi_2^C &= q_2(\tfrac{a}{b-c} - \tfrac{c}{b^2-c^2}q_1 - \tfrac{b}{b^2-c^2}q_2) - mq_2 \end{aligned} \tag{7}$$

The Cournot equilibrium is given by the solution to $\frac{\partial \Pi_i^C}{\partial q_i} = 0$ $(i = 1,2)$, namely

$$q_1^C = q_2^C = \frac{(b+c)[a-m(b-c)]}{2b+c} \tag{8}$$

Replacing (8) in (6) we get that the prices at the Cournot equilibrium are:

$$p_1^C = p_2^C = \frac{ab+m(b-c)(b+c)}{(b-c)(2b+c)} \tag{9}$$

Note that $p_i^C - p_i^B = \frac{[a-m(b-c)]c^2}{(b-c)(4b^2-c^2)}$ $(i = 1,2)$, which is positive (since $b > c > 0$ and $\frac{a}{b-c} > m$); thus the prices associated with the Cournot equilibrium are higher than the Bertrand equilibrium prices.

In the Cournot game quantities are strategic substitutes. For example, the reaction function of Firm 1 is:

$$R_1^C = \frac{(b+c)[a-m(b-c)]}{2b} - \frac{c}{2b}q_2 \tag{10}$$

which is decreasing in q_2.

2.3 Monopoly

A monopolist would choose p_1 and p_2 to maximize

$$\Pi^M = (p_1 - m)(a - bp_1 + cp_2) + (p_2 - m)(a + cp_1 - bp_2) \tag{11}$$

Solving $\frac{\partial}{\partial p_i}\Pi^M = 0$ ($i = 1, 2$), yields

$$p_1^M = p_2^M = \frac{a + m(b - c)}{2(b - c)} \tag{12}$$

with corresponding quantities

$$q_1^M = q_2^M = \frac{a - m(b - c)}{2} \tag{13}$$

3 Parameterizing the intensity of competition in price-setting games

Consider the one-parameter family of price-setting games where the payoff functions of the firms are as follows (Π_1^B and Π_2^B are given by (2) and $\gamma \in [0, 2]$):

$$\begin{aligned} \Pi_1^\gamma &= \Pi_1^B + (1 - \gamma)\Pi_2^B \\ \Pi_2^\gamma &= (1 - \gamma)\Pi_1^B + \Pi_2^B \end{aligned} \tag{14}$$

Note that (relative to the Bertrand game) the additional term $(1 - \gamma)\Pi_2^B$ in the payoff function of Firm 1 does *not* mean that Firm 1 gets a share $(1 - \gamma)$ of the profits of Firm 2 (indeed $(1 - \gamma)$ can be negative if $\gamma > 1$). The payoff functions (14) have nothing to do with cross ownership: they just express the firms' degree of preference for cooperation.[3] While the *material* payoff of Firm 1 is just its profit Π_1^B, the additional term $(1 - \gamma)\Pi_2^B$ captures a cooperative/competitive component in the utility function of Firm 1: if $\gamma = 0$ then Firm 1's objective is to maximize industry profits (thus a preference for full cooperation), if $\gamma = 1$ then Firm 1's objective it to maximize its own profits (thus no inclination to cooperation) and if $\gamma = 2$ then Firm 1's objective it to maximize the difference

[3]The effects of cross ownership of firms have been studied extensively in the literature: see, for example, Brito et al. (2019), Gilo et al. (2006), Lømo (2024), Reitman (1994), Shelegia and Spiegel (2012).

between its own profits and the competitor's profits (thus a preference for extreme rivalry). Similarly for Firm 2.

Note that, for every $\gamma \in [0, 2)$, in the corresponding game prices are strategic complements; for example, the reaction function of Firm 1 is given by

$$R_1^\gamma = \frac{a + m[b - (1 - \gamma)c]}{2b} + \frac{(2 - \gamma)c}{2b} p_2 \tag{15}$$

which is increasing in p_2, for every $\gamma \in [0, 2)$.

For every $\gamma \in [0, 2]$, the Nash equilibrium of the corresponding game (obtained by solving $\frac{\partial \Pi_i^\gamma}{\partial p_i} = 0$, $i = 1, 2$) is given by

$$p_1^\gamma = p_2^\gamma = \frac{a + (b - c)m + cm\gamma}{2(b - c) + c\gamma} \tag{16}$$

Note that

$$\frac{\partial}{\partial \gamma} p_i^\gamma = -\frac{[a - m(b - c)]c}{(2b - 2c + c\gamma)^2} \quad (i = 1, 2) \tag{17}$$

which is negative (since, by assumption, $b > c > 0$ and $\frac{a}{b-c} > m$); thus, the Nash equilibrium prices are strictly decreasing in γ. We can view γ as a measure of the intensity of competition: higher values of γ reflect more intense competition.

- When $\gamma = 0$, (16) coincides with the monopoly (or collusive) solution (12),
- when $\gamma = 1 - \frac{c}{b}$, (16) coincides with the prices (9) corresponding to the Cournot equilibrium (that is, when $\gamma = 1 - \frac{c}{b}$ the corresponding price-setting game is outcome-equivalent to the Cournot game),
- when $\gamma = 1$, (16) coincides with the Bertrand equilibrium (3),
- values of γ between 0 and $1 - \frac{c}{b}$ correspond to less intense competition than captured by the Cournot outcome and values of γ between $1 - \frac{c}{b}$ and 1 correspond to levels of competition intermediate between the Cournot outcome and the Bertrand outcome,
- when $1 < \gamma \leq 2$, (16) yields even lower prices than the Bertrand equilibrium, that is, more intense competition than captured by the Bertrand game.

Replacing (16) in the payoff functions (14) we get that the equilibrium payoffs are

$$\Pi_1^{*\gamma} = \Pi_2^{*\gamma} = \frac{[a-(b-c)m]^2(2-\gamma)(b-c+c\gamma)}{(2b-2c+c\gamma)^2} \tag{18}$$

Note that

$$\frac{\partial}{\partial\gamma}\Pi_i^{*\gamma} = -\frac{2[a-(b-c)m]^2}{(2b-2c+c\gamma)^3}\left[(3b-c)c\gamma+2(b-c)^2\right] \quad (i=1,2) \tag{19}$$

which is negative since, by assumption, $b > c > 0$, $\frac{a}{b-c} > m$ and $\gamma \in [0,2]$. Thus, as the intensity of competition (i.e. the parameter γ) increases, equilibrium payoffs decrease.[4]

4 Parameterizing the intensity of competition in quantity-setting games

The approach of Section 3 can be repeated in the context of quantity-setting games. Consider the one-parameter family of quantity-setting games where the payoff functions of the firms are as follows (Π_1^C and Π_2^C are given by (7) and $\beta \in [0,2]$):

$$\begin{aligned} \Pi_1^\beta &= \Pi_1^C + (1-\beta)\Pi_2^C \\ \Pi_2^\beta &= (1-\beta)\Pi_1^C + \Pi_2^C \end{aligned} \tag{20}$$

For every $\beta \in [0,2]$, the Nash equilibrium of the corresponding game (obtained by solving $\frac{\partial \Pi_i^\beta}{\partial q_i} = 0$, $i = 1,2$) is given by

$$q_1^\beta = q_2^\beta = \frac{(b+c)[a-m(b-c)]}{2b+(2-\beta)c} \tag{21}$$

with corresponding prices

$$p_1^\beta = p_2^\beta = \frac{a(b+c-\beta c)+(b+c)(b-c)m}{(b-c)[2b+(2-\beta)c]} \tag{22}$$

[4]The material payoffs, that is, the actual profits, also decrease. At the Nash equilibrium payoffs are $\Pi_i^{*B} = \frac{[a-(b-c)m]^2(b-c+c\gamma)}{(2b-2c+c\gamma)^2}$; thus, $\frac{\partial}{\partial\gamma}\Pi_i^{*B} = -\frac{[a-(b-c)m]^2c^2\gamma}{(2b-2c+c\gamma)^3} < 0$.

Note that

$$\frac{\partial}{\partial\beta}p_i^{\beta} = -\frac{[a-(b-c)m](b+c)c}{(b-c)(2b+2c-\beta c)^2} \quad (i=1,2) \tag{23}$$

which is negative (since, by assumption, $b > c > 0$ and $\frac{a}{b-c} > m$); thus, the prices corresponding to the Nash-equilibrium quantities are strictly decreasing in β. We can view β as a measure of the intensity of competition: higher values of β reflect more intense competition.

- When $\beta = 0$, (21) coincides with the the monopoly quantities (13),
- when $\beta = 1$, (21) coincides with the Cournot equilibrium (8),
- When $\beta = 1 + \frac{c}{b}$, (21) coincides with the quantities corresponding to the Bertrand equilibrium, namely (4) (that is, the quantity-setting game with $\beta = 1 + \frac{c}{b}$ is outcome-equivalent to the Bertrand game),
- values of β between 0 and 1 correspond to less intense competition than captured by the Cournot outcome, values of β between 1 and $1 + \frac{c}{b}$ correspond to levels of competition intermediate between Cournot and Bertrand,
- values of β between $1 + \frac{c}{b}$ and 2 correspond to more intense competition than the Bertrand outcome.

Note that, for every value of $\beta \in [0,2)$, in the corresponding game quantities are strategic substitutes; for example, the reaction function of Firm 1 is given by

$$R_1^{\beta} = \frac{(b+c)[a-m(b-c)]}{2b} - \frac{c(2-\beta)}{2b}q_2 \tag{24}$$

which is decreasing in q_2, for every $\beta \in [0,2)$.

Replacing (21) in the payoff functions (20) we get that the equilibrium payoffs are

$$\Pi_1^{*\beta} = \Pi_2^{*\beta} = \frac{[a-(b-c)m]^2(2-\beta)(b+c)(b+c+-c\beta)}{(b-c)(2b+2c-c\beta)^2} \tag{25}$$

Note that

$$\frac{\partial}{\partial\beta}\Pi_i^{*\beta} = -\frac{2(b+c)[a-(b-c)m]^2}{(b-c)(2b+2c-c\beta)^3}\left[2(b+c)^2-(3b+c)c\beta\right] \quad (i=1,2) \tag{26}$$

which is negative.[5] Thus as the intensity of competition (i.e. the parameter β) increases, equilibrium payoffs decrease.

[5] Both the numerator and denominator in the fraction are positive, since $b > c > 0$, $\frac{a}{b-c} > m$ and

4.1 The homogeneous-product case

Can one define, in the context of a homogeneous product, a parameterized family of games that incorporates the collusive outcome, the Cournot outcome and the Bertrand outcome? Given the discontinuity in the demand functions that arise in the case of price competition, it seems that such a parameterized family of games cannot be a one where the games are price-setting games. However, the answer is affirmative for the quantity-setting case.

We consider a homogeneous product duopoly where the inverse demand function is $P = a - bQ$ and both firms have the same constant marginal cost $m \geq 0$ and zero fixed cost.

- In the case of monopoly the profit function can be written as $\Pi^M = q_1[a - b(q_1 + q_2)] - mq_1 + q_2[a - b(q_1 + q_2)] - mq_2$. The symmetric profit-maximizing solution is

$$q_1^M = q_2^M = \frac{a - m}{4b} \tag{27}$$

  with corresponding price $P^M = \frac{a+m}{2}$.

- In the Cournot case the profit functions are

$$\begin{aligned} \Pi_1^C &= q_1[a - b(q_1 + q_2)] - mq_1 \\ \Pi_2^C &= q_2[a - b(q_1 + q_2)] - mq_2 \end{aligned} \tag{28}$$

  The Cournot equilibrium is

$$q_1^C = q_2^C = \frac{a - m}{3b} \tag{29}$$

  with corresponding price $P^C = \frac{a+2m}{3}$.

- In the Bertrand case, the equilibrium has both firms charging a price equal to marginal cost ($p_1^B = p_2^B = m$) with corresponding quantities

$$q_1^B = q_2^B = \frac{a - m}{2b} \tag{30}$$

$\beta \in [0, 2]$. The expression in square brackets is decreasing in β and thus reaches its minimum value in the interval $[0, 2]$ at $\beta = 2$, in which case it becomes $2b^2 + 4bc - 6bc$, which is positive since $b > c$.

Consider now the parameterized family of quantity-setting games analogous to (20) with Π_1^C and Π_2^C given by (28) and, as before, $\beta \in [0, 2]$:

$$\Pi_1^\beta = \Pi_1^C + (1 - \beta)\Pi_2^C$$

$$\Pi_2^\beta = (1 - \beta)\Pi_1^C + \Pi_2^C$$

For every $\beta \in [0, 2]$, the Nash equilibrium of the corresponding game is

$$q_1^\beta = q_2^\beta = \frac{a - m}{(4 - \beta)b} \tag{31}$$

Comparing (31) with (27), (29) and (30), one can see that

- $\beta = 0$ corresponds to the monopoly outcome,
- $\beta = 1$ to the Cournot outcome, and
- $\beta = 2$ to the Bertrand outcome.

In particular, the case $\beta = 2$ provides an alternative interpretation of the Bertrand outcome, in the context of a homogeneous product, as the Nash equilibrium of a quantity-setting game where the objective of each firm is to maximize the difference between its own profits and the profits of the competitor.

5 The incentive to introduce a cost-reducing innovation

In this section we revisit the issue, mentioned in Section 1, of whether more competition implies a stronger or weaker incentive to innovate. We take as starting point the symmetric situation described in Sections 2-4 and focus on Firm 1. We assume that Firm 1 has a chance to reduce its marginal cost from m to $m - \delta$ (with $0 \leq \delta \leq m$) and we investigate – both in the price-setting regime and the quantity-setting regime – whether the payoff gain for Firm 1 from the cost reduction is increasing or decreasing in the intensity of competition (measured by the parameter $\gamma \in [0, 2]$ in the price-setting regime and by the parameter $\beta \in [0, 2]$ in the quantity-setting regime). In order to simplify the expressions, we focus on a particular set of values for the demand and cost parameters, namely[6]

$$a = 20,\ b = 4,\ c = 2 \ \text{ and } \ m = 1 \quad (\text{ thus } 0 \leq \delta \leq 1) \tag{32}$$

First we compare Firm 1's incentive to introduce a cost reduction in the Bertrand game and the Cournot game.

5.1 The Bertrand game and the Cournot game

In the Bertrand game (Section 2.1) when both firms' marginal cost is $m = 1$ the Nash equilibrium is given by (16), namely (replacing the values (32)) $p_1^B = p_2^B = 4$ with corresponding profits of 36 for Firm 1. When Firm 1's marginal cost is $1-\delta$ (with $0 \leq \delta \leq 1$) and Firm 2's marginal cost is 1, the Nash equilibrium of the corresponding game is given by $p_1 = 4 - \frac{8}{15}\delta$ and $p_2 = 4 - \frac{2}{15}\delta$ with corresponding profits of $36 + \frac{56}{15}\delta + \frac{196}{225}\delta^2$. Thus Firm 1's profit gain from reducing its marginal cost from 1 to $1 - \delta$ is

$$\frac{56}{15}\delta + \frac{196}{225}\delta^2 \tag{33}$$

In the Cournot game (Section 2.2) when both firms' marginal cost is $m = 1$ the Nash equilibrium is given by (8), namely (replacing the values (32)) $q_1^C = q_2^C = \frac{54}{5}$ with corresponding profits of $\frac{972}{25}$ for Firm 1. When Firm 1's marginal cost is $1 - \delta$ (with $0 \leq \delta \leq 1$) and Firm 2's marginal cost is 1, the Nash equilibrium of the corresponding game is given by $q_1 = \frac{54}{5} + \frac{8}{5}\delta$ and $q_2 = \frac{54}{5} - \frac{2}{5}\delta$ with corresponding profits of $\frac{972}{25} + \frac{288}{25}\delta + \frac{64}{75}\delta^2$ for Firm 1. Thus Firm 1's profit gain from reducing its marginal cost from 1 to $1 - \delta$ is

$$\frac{288}{25}\delta + \frac{64}{75}\delta^2 \tag{34}$$

The difference between (34) and (33) is

$$\left(\frac{288}{25}\delta + \frac{64}{75}\delta^2\right) - \left(\frac{56}{15}\delta + \frac{196}{225}\delta^2\right) = \frac{8}{25}\delta - \frac{4}{225}\delta^2$$

which is positive for every $\delta \in (0, 1]$.

Thus *the Cournot model yields a higher incentive for Firm 1 to pursue the cost reduction than the Bertrand model.*

[6]However, extensive simulations confirm the same qualitative conclusions obtained with this set of values.

As remarked in the Introduction, it is not clear whether this result is due to the intensity-of-competition effect (namely to a Schumpeter-like principle that less competition is more favorable to innovation) or to the different strategic properties of the two games. To separate the two effects we turn to the parameterized families of games introduced in Section 3 and 4.

5.2 The parameterized family of price-setting games

When both firms' marginal cost is $m = 1$ the payoff to Firm 1 is given by (18), namely (replacing the values (32))

$$\frac{162(2+\gamma-\gamma^2)}{(2+\gamma)^2} \tag{35}$$

When Firm 1's marginal cost is $1-\delta$ (with $0 \leq \delta \leq 1$) and Firm 2's marginal cost is 1, the Nash equilibrium of the corresponding game is

$$\begin{aligned} p_1^\delta &= \frac{66-(1-\delta)\gamma^2-3\delta(2+\gamma)-5\gamma}{12+4\gamma-\gamma^2} \\ p_2^\delta &= \frac{66-\gamma^2-(2\delta+5)\gamma}{12+4\gamma-\gamma^2} \end{aligned} \tag{36}$$

with corresponding payoff for Firm 1

$$\begin{aligned} &\frac{A}{\left(12+4\gamma-\gamma^2\right)^2} \\ \text{with}\quad & A=\left(4\delta^2-18\delta-162\right)\gamma^4+\left(-4\delta^2+126\delta+2106\right)\gamma^3 \\ &+\left(-44\delta^2-396\delta-7452\right)\gamma^2+\left(96\delta^2+1512\delta+1944\right)\gamma \\ &+144\delta^2+1296\delta+11664 \end{aligned} \tag{37}$$

The payoff gain for Firm 1 from reducing its marginal cost from 1 to $1-\delta$ is given by the difference between (37) and (35), namely

$$\begin{aligned} \Delta\Pi_1^{*\gamma,\delta} &= \frac{4\delta B}{\left(12+4\gamma-\gamma^2\right)^2} \\ \text{with}\qquad & B=(\delta-\tfrac{9}{2})\gamma^4+(\tfrac{63}{2}-\delta)\gamma^3+(-11\delta-99)\gamma^2 \\ &+(24\delta+378)\gamma+36\delta+324 \end{aligned} \tag{38}$$

We are interested in how the gain from innovating, given by (38), varies with $\gamma \in [0, 2]$. Taking the derivative of (38) with respect to γ we get

$$\begin{aligned} \frac{\partial}{\partial \gamma} \Delta\Pi_1^{*\gamma,\delta} &= \frac{2\delta C}{(12 + 4\gamma - \gamma^2)^3} \\ \text{with} \qquad C &= \left(14\gamma^4 + 44\gamma^3 + 72\gamma^2 - 432\gamma\right)\delta \\ &\quad - 9\gamma^4 - 576\gamma^3 + 4536\gamma^2 - 5184\gamma + 3888 \end{aligned} \tag{39}$$

It is shown in the Appendix that $\frac{\partial}{\partial \gamma} \Delta\Pi_1^{*\gamma,\delta} > 0$; thus, the payoff *gain from innovation is an increasing function of the intensity of competition* (measured by the parameter γ).

5.3 The parameterized family of quantity-setting games

When both firms' marginal cost is $m = 1$ the payoff to Firm 1 is given by (25), namely (replacing the values (32))

$$\frac{486(6 - 5\beta + \beta^2)}{(6 - \beta)^2} \tag{40}$$

When Firm 1's marginal cost is $1 - \delta$ (with $0 \leq \delta \leq 1$) and Firm 2's marginal cost is 1 the Nash equilibrium of the corresponding game is

$$\begin{aligned} q_1^\delta &= \frac{108 + 24\delta + 54\beta}{12 + 4\beta - \beta^2} \\ q_2^\delta &= \frac{108 - 12\delta + 6\delta\beta + 54\beta}{12 + 4\beta - \beta^2} \end{aligned} \tag{41}$$

with corresponding payoff for Firm 1

$$\begin{aligned} &\frac{D}{(12 + 4\beta - \beta^2)^2} \\ \text{with} \quad &D = \left(12\beta^3 - 60\beta^2 + 96\beta + 144\right)\delta^2 + \\ &\left(54\beta^4 - 162\beta^3 - 108\beta^2 + 1512\beta + 1296\right)\delta \\ &+ 486\beta^4 - 486\beta^3 - 4860\beta^2 + 1944\beta + 11664 \end{aligned} \tag{42}$$

The payoff gain for Firm 1 from reducing its marginal cost from 1 to $1-\delta$ is given by the difference between (42) and (40), namely

$$\Delta\Pi_1^{*\beta,\delta} = \frac{12\delta\left(\delta+\frac{9}{2}\beta+9\right)(12+8\beta-5\beta^2+\beta^3)}{(12+4\beta-\beta^2)^2} \tag{43}$$

We are interested in how the gain from innovating, given by (43), varies with $\beta \in [0,2]$. Taking the derivative of (43) with respect to β we get

$$\begin{aligned} \frac{d}{d\beta}\Delta\Pi_1^{*\beta,\delta} &= \frac{12\delta\, E}{(12+4\beta-\beta^2)^3} \\ \text{with} \qquad E &= \delta\left(-104\beta+60\beta^2-6\beta^3+\beta^4\right) \\ &\quad + \left(648-288\beta-108\beta^2+144\beta^3+\tfrac{45}{2}\beta^4\right) \end{aligned} \tag{44}$$

It is shown in the Appendix that $\frac{\partial}{\partial\beta}\Delta\Pi_1^{*\beta,\delta} > 0$; thus, the payoff *gain from innovation is an increasing function of the intensity of competition* (measured by the parameter β).

Hence the conclusion that the gain from innovation (and thus incentive to innovate) increases with the intensity of competition does not depend on whether the firms' decision variables are strategic complements or substitutes.

The analysis of this section confirms Arrow's (Arrow (1962)) intuition that more intense competition creates a more favorable environment for investment in a cost-reducing innovation.

6 Conclusion

The Cournot model typically yields higher prices than the Bertrand model and thus can be thought of as capturing a regime of less intense competition than the Bertrand model. This fact has been used in the literature to study the issue of whether less intense competition yields a lower or higher incentive to introduce a cost-reducing innovation. Unfortunately, the two models have different strategic properties, making it difficult to disentangle the intensity-of-competition effect from the strategic effect. To separate the two effects, we constructed a one-parameter family of price-setting games, where the parameter γ unambiguously captures the intensity of competition, while maintaining the strategic properties of the Bertrand game. Similarly, we constructed a one-parameter family of quantity-setting games, where the parameter β unambiguously captures the intensity of competition, while maintaining the strategic properties of the Cournot game. Then we used these two parameterized families of games to show that, as first suggested by Arrow (1962), the incentive to innovate *increases* with the intensity of competition.

A Appendix

First we show that $\frac{d}{d\gamma}\Delta\Pi_1^{*\gamma,\delta} > 0$. Recall that (see (39))

$$\frac{\partial}{\partial\gamma}\Delta\Pi_1^{*\gamma,\delta} = \frac{2\delta C}{(12+4\gamma-\gamma^2)^3}$$

$$\text{with} \qquad C = \underbrace{\left(14\gamma^4+44\gamma^3+72\gamma^2-432\gamma\right)}_{f(\gamma)}\delta$$

$$\underbrace{-9\gamma^4-576\gamma^3+4536\gamma^2-5184\gamma+3888}_{g(\gamma)}$$

Since $\gamma \in [0,2]$, $(12+4\gamma-\gamma^2) > 0$. Thus, since $\delta > 0$, $\frac{2\delta}{\left(12+4\gamma-\gamma^2\right)^3} > 0$. Hence $\frac{\partial}{\partial\gamma}\Delta\Pi_1^{*\gamma,\delta} > 0$ if and only if $C > 0$. The function

$$f(\gamma) = \left(14\gamma^4+44\gamma^3+72\gamma^2-432\gamma\right)$$

is strictly convex in the interval $\gamma \in [0,2]$, achieves its minimum value at $\gamma = 1.16$ and $f(1.16) = -310.208$. The function

$$g(\gamma) = -9\gamma^4-576\gamma^3+4536\gamma^2-5184\gamma+3888$$

is strictly convex in the interval $\gamma \in [0,2]$, achieves its minimum value at $\gamma = 0.654$ and $g(0.645) = 2275$. Thus, since $\delta \in (0,1]$, $C \geq -310.208 + 2275 > 0$. □

Next we show that $\frac{d}{d\beta}\Delta\Pi_1^{*\beta,\delta} > 0$. Recall that (see (44))

$$\frac{\partial}{\partial\beta}\Delta\Pi_1^{*\beta,\delta} = \frac{12\delta\, E}{(12+4\beta-\beta^2)^3}$$

$$\text{with} \qquad E = \delta\underbrace{\left(-104\beta + 60\beta^2 - 6\beta^3 + \beta^4\right)}_{F(\beta)} + \underbrace{\left(648 - 288\beta - 108\beta^2 + 144\beta^3 + \frac{45}{2}\beta^4\right)}_{G(\beta)}$$

Since $\beta \in [0,2]$, $(12+4\beta-\beta^2)^3 > 0$. Thus, since $\delta > 0$, $\frac{12\delta}{(2+\beta)^3(6-\beta)^3} > 0$. Hence $\frac{\partial}{\partial\beta}\Delta\Pi_1^{*\beta,\delta} > 0$ if and only if $E > 0$. The function

$$F(\beta) = -104\beta + 60\beta^2 - 6\beta^3 + \beta^4$$

is strictly convex in the interval $\beta \in [0,2]$, achieves its minimum value at $\beta = 0.979$ and $F(0.979) = -49.021$. The function

$$G(\beta) = 648 - 288\beta - 108\beta^2 + 144\beta^3 + \frac{45}{2}\beta^4$$

is strictly convex in the interval $\beta \in [0,2]$, achieves its minimum value at $\beta = 0.98$ and $G(0.98) = 418.322$. Thus, since $\delta \in (0,1]$, $E \geq -49.021 + 418.322 > 0$. □

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