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On the Interpretation of Taxes in the Pivotal Mechanism

GIACOMO BONANNO *

It is shown that the traditional interpretation of taxes in the pivotal mechanism in terms of the utility loss imposed by the taxed individual on the rest of society is not correct, since is takes into account only the effect that the individual has on the decision concerning the project and disregards the effect that the same individual has on the taxes paid by the other members of society.

1. *Introduction*

The Clarke, or pivotal, mechanism (Clarke, 1971) is now taught in most public economics courses. When applied to the simple case of a fixed-size project, the mechanism asks each individual to state his/her willingness to pay for the project. The decision will then be to carry out the project if and only if the sum of the stated willingness to pay is non-negative. The mechanism also imposes a tax on individuals if and only if they are pivotal. A pivotal individual is one whose vote was decisive: if the individual had not been a voting member of society (or, equivalently, if his/her stated willingness to pay had not been taken into account) the decision concerning the project would have been different. The appealing feature of the pivotal mechanism (shared by all the other members of the class of Groves mechanisms) is that reporting one's own *true* willingness to pay is a dominant strategy for every individual.

The purpose of this note is to argue against not the mechanism itself, but the traditional heuristic *interpretation* of it. It is customary to interpret the mechanism as imposing a *tax* on each individual *equal to the utility loss that the individual's vote imposes on the rest of society*. For example, Tideman and Tullock (1976, pp. 1145) write

"...each individual is offered a chance to change the outcome that would occur without his vote by paying a special charge equal to the net cost to others that results from including his vote in the decision" [1].

* University of California, Department of Economics, Davis, CA 95616 - U.S.A. I have benefited from comments by Aanund Hylland, Louis Makowski and Claudio Mezzetti.

[1] Green and Laffont (1979, pp. 42-43) suggest a similar interpretation.

A careful scrutiny of this interpretation reveals it to be incorrect, since *it takes into account only the effect that an individual has on the decision concerning the project and disregards the effect that the same individual has on the taxes paid by the other members of society*[2]. This point is illustrated in detail in the next section.

The substance of the example given below is that in evaluating the impact that an individual's vote has on the rest of the community one ought to look at the "grand" society and at sub-societies in a "consistent" way. We are referring here to a notion of consistency that has recently been object of study in cooperative game theory (see, for example, Dutta *et al.*, 1987; Greenberg, 1990; Ray, 1989). For instance, Dutta *et al.* (1987, p. 93) observe that "the core and the bargaining set as solution concepts fail to satisfy, at least *a priori*, a natural requirement of consistency", for the following reason. Consider, for example, the notion of the core. Suppose that "blocking" is the criterion to be used in order to decide whether or not any given imputation is sensible or acceptable. Let x be an imputation and suppose that coalition (or sub-society) S_0 can block it, that is, can ensure for its members a payoff vector x_0 which is higher than x. Then at first thought it seems that one ought to rule out x as sensible or acceptable. However, *the deviating coalition S_0 is a potential society of its own and should therefore be analyzed with the same criteria that are used to analyze the grand coalition.* In particular, one ought to check that there is no subcoalition S_1 of S_0 that can ensure itself a payoff vector x_1 which is higher than x_0. If there is such a subcoalition, then x_0 is not a consistent or credible objection to x by S_0.

In the example of section 2 we apply the same line of reasoning. Suppose that the pivotal mechanism is the decision rule that is to be used. Then in evaluating an individual's impact on the rest of the community we need to compare the outcome of applying the pivotal mechanism to the "grand" society (that includes the individual under consideration) with the outcome that would obtain if one were to remove the individual from society *while still applying the pivotal mechanism to the resulting sub-society.*

2. *An example*

Consider the simplest possible case of a costless fixed-size project. There are n individuals each with an additively separable utility function

$$u_i(x_i, d) = \begin{cases} x_i & \text{if } d = No \\ x_i + v_i & \text{if } d = Yes \end{cases}$$

[2] A somewhat related, but conceptually different, observation was made by Tideman (1983, p. 18). Tideman notes that if voters are not purely selfish but rather are motivated by empathy and/or moral regard, then they will take into account also how their vote will affect the taxes paid by others. In this note we do not depart from the standard assumption of selfishness and therefore do not question the conclusion that truth-telling is a dominant strategy. Our remark concerns the consistency of the *interpretation* of the pivotal tax.

where x_i is individual i's consumption of the private good, v_i is a constant (i's "willingness to pay" for the project) and d is the decision whether or not to carry out the project. The pivotal mechanism asks each individual to state a $w_i \in \Re$ (where $\Re$ denotes the set of real numbers) which is interpreted as his/her willingness to pay for the project and the resulting allocation is given by:

$$(1) \qquad d = Yes \quad if \quad \sum_{j=1}^{n} w_j \geq 0 \quad and \quad d = No \quad otherwise$$

$$(2) \qquad t_i = \begin{cases} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j + \min \left\{ 0, - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j \right\} & if \quad \sum_{j=1}^{n} w_j \geq 0 \\ \min \left\{ 0, - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j \right\} & otherwise \end{cases}$$

where t_i is the transfer *to* individual i, which is never positive, hence it is a tax. Thus individual i is taxed if and only if the decision concerning the project would have been different had he/she not been a voting member of the community, that is, if and only if the sign of $\sum_{j=1}^{n} w_j$ is different from the sign of $\sum_{\substack{j=1 \\ j \neq i}}^{n} w_j$.

Since truth-telling is a dominant strategy, each individual will announce his true willingness to pay v_i. Hence if individual i is pivotal, he will have to pay a tax equal to $\left| \sum_{\substack{j=1 \\ j \neq i}}^{n} v_j \right|$ and this is normally interpreted as the utility loss or externality that individual i imposes on the rest of the community [3]. For example, if $\sum_{j=1}^{n} v_j < 0$ and $\sum_{\substack{j=1 \\ j \neq i}}^{n} v_j > 0$, then individual i imposes an externality on the rest of society by depriving them of a project that has a net positive benefit to them of $\sum_{\substack{j=1 \\ j \neq i}}^{n} v_j$. And the tax individual i would pay under the pivotal mechanism is exactly equal to $\sum_{\substack{j=1 \\ j \neq i}}^{n} v_j$.

We want to show that the interpretation according to which the pivotal

[3] Makowski and Ostroy (1987) have suggested a different interpretation of the pivotal mechanism, based on the notion of an individual's marginal product to society.

tax reflects the utility loss imposed by the taxed individual on the rest of the community is not correct. The reason is as follows. If the pivotal mechanism is the decision rule that has been agreed upon, then it will be used both in the "grand" society, which includes individual i, and in the hypothetical sub-society obtained by eliminating individual i. Thus if we want to compute the externality that, say, individual 1 imposes on the rest of society, we need to compute:

(i) the total utility of individuals *2 to n* when the pivotal mechanism is applied to the society consisting of individuals *1 to n*,

(ii) the total utility of individuals *2 to n* when the pivotal mechanism is applied to the sub-society consisting of individuals *2 to n* only

(iii) the difference between (i) and (ii), all it E_1.

If $E_1 < 0$, individual 1 is imposing a negative externality on the rest of society and this fact could be the basis for a tax on him/her equal to E_1. The following example shows that, in general, E_1 is *not* equal to $(v_2 + v_3 + \ldots + v_n)$.

Let $n = 4$, $v_1 = 2$, $v_2 = -8$, $v_3 = 5$, $v_4 = 10$. If the pivotal mechanism is used, only individual 4 is pivotal. The others don't pay any taxes, while individual 4 pays a tax equal to $-(v_1 + v_2 + v_3) = 1$. Does this tax represent the externality that individual 4 imposes on individuals 1 to 3 by being a member of this society? The answer is negative. If individual 4 were not a member of society and the pivotal mechanism were applied to the sub-society consisting of individuals 1, 2 and 3, the project would not be carried out. Individuals 1 and 3 would not be pivotal, while individual 2 would be. She would have to pay a tax equal to $(v_1 + v_3) = 7$. When individual 4 is added to this sub-society, the project is carried out. Individual 2 will have a utility loss of 8 (because of the project) but will no longer have to pay a tax of 7; hence she will experience a net utility loss of 1. On the other hand, individuals 1 and 3 will experience an increase in utility equal $2 + 5 = 7$. So the net externality that individual 4 imposes on individuals 1 to 3 is a *positive* one: $7 - 1 = 6$. Hence if transfers are to reflect externalities, individual 4 should receive a subsidy of 6 rather than have to pay a tax of 1!

In view of the above example, one can ask if there exists a mechanism in the class of Groves mechanisms that has the advantages of the pivotal mechanism (namely that it never generates a budget deficit) and, at the same time, it never requires an individual to pay a tax unless that individual is imposing a negative externality on the rest of society, where the externality is computed in a consistent way, as explained above. In other words, such mechanism would satisfy the following properties:

(a) $t_i(w) \leqslant 0$ for all w, and

(b) if $t_i(w) < 0$ then $E_i < 0$, where E_i is the externality (imposed by individual i) which is computed as explained above, i.e. in a consistent way.

The above example can be used to show that there is no such mechanism. The general class of Groves mechanisms is obtained by replacing

$$\min \left\{ 0, - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j \right\}$$

in (2) above with an arbitary function $h_i(w_{-i})$ where w_{-i} represents the vector of announcements by all individuals except individual i. Condition (a) is then equivalent to

$$h_i(w_{-i}) \leqslant \min \left\{ 0, - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_j \right\} ,$$

and this inequality implies that in the above example individual 4 will have to pay a tax of at least 1, while we know that he imposes a *positive*, rather than a negative, externality on the rest of society. Thus condition (b) is violated.

3. *Conclusion*

We argued against the traditional interpretation of the pivotal mechanism according to which individuals are taxed in an amount equal to the utility loss that they impose on the rest of society. We showed that this interpretation takes into account only the effect of an individual's vote on the "physical" decision concerning the project and fails to take into account the effect that the same individual has on the taxes paid by the other members of society. The point made in this note, however, ought to be qualified as follows. The traditional interpretation is correct if a new agent is added to the community, one who *has no interest at all in the project and who receives the total amount of the tax* (and, in order not to distort incentives, we also need to add the requirement that the members of society do not care about the welfare of this new agent). Call this new agent individual 0. Then the traditional view is correct, in the sense that the tax paid by an individual under the pivotal mechanism is equal to the externality the individual imposes on the rest of society, *if the latter is interpreted in a broader sense so as to include also individual 0.*

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