GAME THEORY. Third Edition

Copyright © 2015, 2018, 2024 Giacomo Bonanno, all rights reserved

You are free to redistribute this book in pdf format. If you make use of any part of this book you must give appropriate credit to the author. You may not remix, transform, or build upon the material without permission from the author. You may not use the material for commercial purposes.

This book is also available in print in two volumes:

Volume 1: Basic Concepts ISBN-13: 978-1983604638 ISBN-10: 1983604631

Volume 2: Advanced Topics ISBN-13: 978-1985862517 ISBN-10: 1985862514
Kindle Direct Publishing

There is also a companion book that contains an additional set of 180 fully solved exercises for Chapters 2-7 (a similar companion book for Chapters 8-16 is in preparation).

For details see: `http://faculty.econ.ucdavis.edu/faculty/bonanno/GT_Book.html`

Giacomo Bonanno is Distinguished Professor of Economics at the
University of California, Davis
http://faculty.econ.ucdavis.edu/faculty/bonanno/

Contents

1 Introduction 13

I Games with Ordinal Payoffs

2 Ordinal Games in Strategic Form 19

2.1 Game frames and games 19
2.2 Strict and weak dominance 26
2.3 Second-price auction 31
2.4 The pivotal mechanism 34
2.5 Iterated deletion procedures 37
2.5.1 IDSDS 37
2.5.2 IDWDS 39
2.6 Nash equilibrium 41
2.7 Games with infinite strategy sets 46
2.8 Proofs of theorems 48
2.9 Exercises 53
2.9.1 Exercises for Section 2.1: Game frames and games 53
2.9.2 Exercises for Section 2.2: Strict/weak dominance 55
2.9.3 Exercises for Section 2.3: Second price auction 56
2.9.4 Exercises for Section 2.4: The pivotal mechanism 57
2.9.5 Exercises for Section 2.5: Iterated deletion procedures 58
2.9.6 Exercises for Section 2.6: Nash equilibrium 60

2.9.7 Exercises for Section 2.7: Games with infinite strategy sets 60
2.10 Solutions to exercises 62

3 Perfect-information Games 77
3.1 Trees, frames and games 77
3.2 Backward induction 81
3.3 Strategies in perfect-information games 84
3.4 Relationship between backward induction and other solutions 87
3.5 Perfect-information games with two players 91
3.6 Proofs 93
3.7 Exercises 96
3.7.1 Exercises for Section 3.1: Trees, frames and games 96
3.7.2 Exercises for Section 3.2: Backward induction 97
3.7.3 Exercises for Section 3.3: Strategies in perfect-information games 98
3.7.4 Exercises for Section 3.5: Two-player games 100
3.8 Solutions to exercises 103

4 General Dynamic Games 121
4.1 Imperfect Information 121
4.2 Strategies 128
4.3 Subgames 130
4.4 Subgame-perfect equilibrium 133
4.5 Games with chance moves 140
4.6 Exercises 146
4.6.1 Exercises for Section 4.1: Imperfect information 146
4.6.2 Exercises for Section 4.2: Strategies 147
4.6.3 Exercises for Section 4.3: Subgames 147
4.6.4 Exercises for Section 4.4: Subgame-perfect equilibrium 148
4.6.5 Exercises for Section 4.5: Games with chance moves 150
4.7 Solutions to exercises 154

II Games with Cardinal Payoffs

5 Expected Utility Theory 175
5.1 Money lotteries and attitudes to risk 175
5.2 Expected utility: theorems 177
5.3 Expected utility: the axioms 185
5.4 Exercises 193
5.4.1 Exercises for Section 5.1: Money lotteries and attitudes to risk 193
5.4.2 Exercises for Section 5.2: Expected utility theory 194

5.4.3 Exercises for Section 5.3: Expected utility axioms 196
5.5 Solutions to Exercises 198

6 Strategic-form Games 203
6.1 Strategic-form games with cardinal payoffs 203
6.2 Mixed strategies 206
6.3 Computing the mixed-strategy Nash equilibria 211
6.4 Strict dominance and rationalizability 217
6.5 Exercises 221
6.5.1 Exercises for Section 6.1: Strategic-form games with cardinal payoffs . 221
6.5.2 Exercises for Section 6.2: Mixed strategies 222
6.5.3 Exercises for Section 6.3: Computing the mixed-strategy Nash equilibria 222
6.5.4 Exercises for Section 6.4: Strict dominance and rationalizability 224
6.6 Solutions to exercises 227

7 Extensive-form Games 237
7.1 Behavioral strategies in dynamic games 237
7.2 Subgame-perfect equilibrium revisited 245
7.3 Problems with the notion of subgame-perfect equilibrium 249
7.4 Exercises 250
7.4.1 Exercises for section 7.1: Behavioral strategies in dynamic games 250
7.4.2 Exercises for section 7.2: Subgame-perfect equilibrium revisited 252
7.5 Solutions to exercises 257

III Interactive Epistemology

8 Common Knowledge 269
8.1 Individual knowledge 269
8.2 Interactive knowledge 272
8.3 Common knowledge 279
8.4 Belief 284
8.5 Interactive Belief and Common Belief 287
8.6 Exercises 290
8.6.1 Exercises for Section 8.1: Individual knowledge 290
8.6.2 Exercises for Section 8.2: Interactive knowledge 293
8.6.3 Exercises for Section 8.3: Common knowledge 295
8.6.4 Exercises for Section 8.4: Belief 296
8.6.5 Exercises for Section 8.5: Interactive Belief and Common Belief 298
8.7 Solutions to Exercises 301

9 Adding Beliefs to Knowledge 315

9.1 Sets and probability: brief review 315

9.1.1 Sets 315

9.1.2 Probability 317

9.2 Probabilistic beliefs 318

9.3 Conditional probability and Bayes' rule 320

9.3.1 Conditional probability 320

9.4 Changing beliefs in response to information 323

9.4.1 Belief updating 324

9.4.2 Belief revision 325

9.5 Harsanyi consistency of beliefs or like-mindedness 330

9.6 Agreeing to disagree 335

9.7 Proof of the Agreement Theorem 342

9.8 Exercises 343

9.8.1 Exercises for Section 9.1: Sets and probability 343

9.8.2 Exercises for Section 9.2: Probabilistic beliefs 344

9.8.3 Exercises for Section 9.3: Conditional probability and Bayes' rule 345

9.8.4 Exercises for Section 9.4: Changing beliefs in response to new information 348

9.8.5 Exercises for Section 9.5: Harsanyi consistency of beliefs or like-mindedness 351

9.8.6 Exercises for Section 9.6: Agreeing to disagree 352

9.9 Solutions to Exercises 353

10 Rationality 369

10.1 Strategic-form games with ordinal payoffs 370

10.1.1 Models of ordinal games 370

10.1.2 Common belief of weak rationality 373

10.2 Strategic-form games with cardinal payoffs 376

10.2.1 Models of cardinal games 376

10.2.2 Common knowledge of Bayesian rationality 380

10.3 Beyond common knowledge of rationality 382

10.4 Common knowledge of rationality in extensive-form games 382

10.5 Proofs of Theorems 384

10.6 Exercises 388

10.6.1 Exercises for Section 10.1.1: Models of ordinal games 388

10.6.2 Exercises for Section 10.1.2: Common belief of weak rationality 390

10.6.3 Exercises for Section 10.2.1: Models of cardinal games 392

10.6.4 Exercises for Section 10.2.2: Common knowledge of Bayesian rationality 394

10.6.5 Exercises for Section10.4: Common knowledge of rationality in extensive-form games 395

10.7 Solutions to Exercises 395

IV Refinements of Subgame Perfection

11 Weak Sequential Equilibrium 405

11.1 Assessments and sequential rationality 405

11.2 Bayesian updating at reached information sets 412

11.3 A first attempt: Weak sequential equilibrium 414

11.4 Exercises 424

11.4.1 Exercises for Section 11.1: Assessments and sequential rationality 424

11.4.2 Exercises for Section 11.2: Bayesian updating at reached information sets 427

11.4.3 Exercises for Section 11.3: Weak sequential equilibrium 429

11.5 Solutions to Exercises 432

12 Sequential Equilibrium 443

12.1 Consistent assessments 443

12.2 Sequential equilibrium 449

12.3 Is 'consistency' a good notion? 451

12.4 Exercises 453

12.4.1 Exercises for Section 12.1: Consistent assessments 453

12.4.2 Exercises for Section 12.2: Sequential equilibrium 454

12.5 Solutions to Exercises 458

13 Perfect Bayesian Equilibrium 469

13.1 Belief revision and AGM consistency 469

13.2 Bayesian consistency 473

13.3 Perfect Bayesian equilibrium 476

13.4 Adding independence 481

13.4.1 Weak independence 481

13.4.2 Strong independence 484

13.5 Characterization of SE in terms of PBE 486

13.6 History-based definition of extensive-form game 489

13.7 Proofs 492

13.8 Exercises 494

13.8.1 Exercises for Section 13.1: Belief revision and AGM consistency 494

13.8.2 Exercises for Section 13.2: Bayesian consistency 495

13.8.3 Exercises for Section 13.3: Perfect Bayesian equilibrium 497

13.8.4 Exercises for Section 13.4: Adding independence 499

13.8.5 Exercises for Section 13.5: Characterization of sequential equilibrium in terms of PBE 500

13.9 Solutions to Exercises 504

V Incomplete Information

14 Static Games 523
14.1 Interactive situations with incomplete information 523
14.2 One-sided incomplete information 525
14.3 Two-sided incomplete information 532
14.4 Multi-sided incomplete information 536
14.5 Exercises 540
14.5.1 Exercises for Section 14.2: One-sided incomplete information. 540
14.5.2 Exercises for Section14.3: Two-sided incomplete information. 542
14.5.3 Exercises for Section14.4: Multi-sided incomplete information. 544
14.6 Solutions to Exercises 548

15 Dynamic Games 561
15.1 One-sided incomplete information 561
15.2 Multi-sided incomplete information 576
15.3 Exercises 583
15.3.1 Exercises for Section 15.1: One-sided incomplete information 583
15.3.2 Exercises for Section 15.2: Multi-sided incomplete information 587
15.4 Solutions to Exercises 590

16 The Type-Space Approach 607
16.1 Types of players 607
16.2 Types that know their own payoffs 608
16.3 The general case 612
16.4 Exercises 615
16.4.1 Exercises for Section 16.2: Types that know their own payoffs 615
16.4.2 Exercises for Section 16.3: The general case 617
16.5 Solutions to Exercises 619

Index 631