

## Texts in Logic and Games | 3

This volume in the Texts in Logic and Games series grew out of the seventh conference on Logic and the Foundations of the Theory of Games and Decisions (LOFT 7), which took place in Liverpool, in July 2006. The LOFT conferences are interdisciplinary events that bring together researchers from a variety of fields: computer science, economics, game theory, linguistics, logic, multi-agent systems, psychology, philosophy, social choice and statistics. Topics that fall under the LOFT umbrella include epistemic and temporal logic, theories of information processing and belief revision, models of bounded rationality, non-monotonic reasoning, theories of learning and evolution, mental models, etc. The papers collected in this volume reflect the variety of interests and the interdisciplinary composition of the LOFT community.

Bonanno | Van der Hoek | Wooldridge (eds.)

# Logic and the Foundations of Game and Decision Theory (LOFT 7)

EDITED BY  
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THEORY (LOFT 7)



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## Introduction

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This volume in the *Texts in Logic and Games* series was conceived as a ramification of the seventh conference on *Logic and the Foundations of the Theory of Games and Decisions* (LOFT7), which took place in Liverpool, in July 2006.<sup>1</sup>

The LOFT conferences have been a regular biannual event since 1994. The first conference was hosted by the Centre International de Recherches Mathématiques in Marseille (France), the next four took place at the International Centre for Economic Research in Torino (Italy), the sixth conference was hosted by the Graduate School of Management in Leipzig (Germany) and the most recent one took place at the University of Liverpool (United Kingdom).<sup>2</sup>

The LOFT conferences are interdisciplinary events that bring together

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<sup>1</sup> The conference was organized by the editors of this volume with the assistance of a program committee consisting of Thomas Ågotnes, Johan van Benthem, Adam Brandenburger, Hans van Ditmarsch, Jelle Gerbrandy, Wojtek Jamroga, Hannes Leitgeb, Benedikt Löwe, Marc Pauly, Andrés Perea, Gabriella Pigozzi, Wlodek Rabinowicz, Hans Rott, and Krister Segerberg.

<sup>2</sup> Collections of papers from previous LOFT conferences can be found in a special issue of *Theory and Decision* (Vol. 37, 1994, edited by M. Bacharach and P. Mongin), the volume *Epistemic logic and the theory of games and decisions* (edited by M. Bacharach, L.-A. Gérard-Varet, P. Mongin and H. Shin and published by Kluwer Academic, 1997), two special issues of *Mathematical Social Sciences* (Vols. 36 and 38, 1998, edited by G. Bonanno, M. Kaneko and P. Mongin), two special issues of *Bulletin of Economic Research* (Vol. 53, 2001 and Vol. 54, 2002, edited by G. Bonanno and W. van der Hoek), a special issue of *Research in Economics*, (Vol. 57, 2003, edited by G. Bonanno and W. van der Hoek), a special issue of *Knowledge, Rationality and Action* (part of *Synthese*, Vol. 147, 2005, edited by G. Bonanno) and the volume *Proceedings of the 7th conference on Logic and the Foundations of Game and Decision Theory* (edited by G. Bonanno, W. van der Hoek and M. Wooldridge, University of Liverpool, 2006).

researchers from a variety of fields: computer science, economics, game theory, linguistics, logic, multi-agent systems, psychology, philosophy, social choice and statistics. In its original conception, LOFT had as its central theme the application of logic, in particular modal epistemic logic, to foundational issues in the theory of games and individual decision-making. Epistemic considerations have been central to game theory for a long time. The expression “interactive epistemology” has been used in the game-theory literature to refer to the analysis of decision making by agents involved in a strategic interaction, when these agents recognize each other’s intelligence and rationality. What is relatively new is the realization that the tools and methodology that were used in game theory are closely related to those already used in other fields, notably computer science and philosophy. Modal logic turned out to be the common language that made it possible to bring together different professional communities. It became apparent that the insights gained and the methodologies employed in one field could benefit researchers in other fields. Indeed, new and active areas of research have sprung from the interdisciplinary exposure provided by the LOFT conferences.<sup>3</sup>

Over time the scope of the LOFT conference has broadened to encompass a wider range of topics, while maintaining its focus on the general issue of rationality and agency. Topics that have fallen within the LOFT umbrella include epistemic and temporal logic, theories of information processing and belief revision, models of bounded rationality, non-monotonic reasoning, theories of learning and evolution, mental models, etc.

The papers collected in this volume reflect the general interests and interdisciplinary scope of the LOFT conferences.

The paper by Alexandru Baltag and Sonja Smets falls within the recent literature that deals with belief revision and update within the Dynamic Epistemic Logic paradigm. The authors develop a notion of doxastic action general enough to cover many examples of multi-agent communication actions encountered in the literature, but also flexible enough to deal with both static and dynamic belief revision. They discuss several epistemic notions: knowledge, belief and conditional belief. For the latter they distinguish between the statement ‘if informed that  $P$ , the agent would believe that  $Q$  *was* the case (before the learning)’ and the statement ‘if informed that  $P$ , the agent would come to believe that  $Q$  *is* the case (in the world after the learning)’. They also study a “safe belief” operator meant to express a weak notion of “defeasible knowledge”: it is belief that is persistent under revision with any true information. Baltag and Smets provide a complete

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<sup>3</sup> community and the community of researchers who are active in another regular, biannual event, namely the conferences on Theoretical Aspects of Rationality and Knowledge (see [www.tark.org](http://www.tark.org)).

axiomatization of the logic of conditional belief, knowledge and safe belief. In the second part of the paper the authors discuss dynamic belief revision in the context of action models.

The paper by Giacomo Bonanno deals with the question of what choices are compatible with rationality of the players and common belief of rationality. He takes a syntactic approach and defines rationality axiomatically. Furthermore, he does not assume von Neumann-Morgenstern payoffs but merely ordinal payoffs, thus aiming for a more general theory of rationality in games. The author considers two axioms. The first says that a player is irrational if she chooses a particular strategy while believing that another strategy of hers is better. He shows that common *belief* of this weak notion of rationality characterizes the iterated deletion of pure strategies that are strictly dominated by another pure strategy. The second axiom says that a player is irrational if she chooses a particular strategy while believing that a different strategy is at least as good and she considers it possible that this alternative strategy is actually better than the chosen one. The author shows that common *knowledge* of this stronger notion of rationality characterizes the iterated deletion procedure introduced by Stalnaker (1994), restricted - once again - to pure strategies.

The paper by Hans van Ditmarsch and Barteld Kooi investigates a dynamic logic describing “epistemic events” that may change both the agents’ information (or beliefs) and what the authors call “the ontic facts” of the world (that is, objective, non-epistemic statements about the world). A sound and complete axiomatization is provided. Some original and interesting semantic results are also proved, in particular the fact that any model change can be simulated by “epistemic events”, and thus any consistent goal can be achieved by performing some such event. The authors illustrate their results in several examples, including card games and logical puzzles.

The paper by Wiebe van der Hoek, Mark Roberts and Michael Wooldridge extends the authors’ previous work on Alternating-time Temporal Logic and its ramifications. They extend it by introducing the notion of a legally possible strategy, that they oppose to a physically possible strategy, and define social belief as truth in all states that are (1) possible for the agent, and (2) are obtained from the initial state by a legally possible strategy. They use this framework to reason about social laws. In a system with social laws, every agent is supposed to refrain from performing certain forbidden actions. Rather than assuming that all agents abide by the law, the authors consider what happens if certain agents act socially, while others do not. In particular, they focus on the agents’ strategic abilities under such mixed conditions.

The paper by Alexander Nittka and Richard Booth deals with the traditional “static” belief revision setting, but with a different twist: rather

than answering the question of how an agent should rationally change his beliefs in the light of new information, they address the question of what one can say about an agent who is observed in a belief change process. That is, the authors study the problem of how to make inferences about an agent's beliefs based on observation of how that agent responded to a sequence of revision inputs over time. They start by reviewing some earlier results for the case where the observation is complete in the sense that (1) the logical content of all formulas appearing in the observation is known, and (2) all revision inputs received by the agent during the observed period are recorded in the observation. They then provide new results for the more general case where information in the observation might be distorted due to noise or because some revision inputs are missing altogether. Their analysis is based on the assumption that the agent employs a specific, but plausible, belief revision framework when incorporating new information.

The paper by R. Ramanujam and Sunil Simon deals with the most important notion of non-cooperative game, namely extensive game. Extensive games provide a richer description of interactive situations than strategic-form games in that they make the order of moves and the information available to a player when it is his turn to move explicit. A strategy for a player in an extensive game associates with every information set of that player a choice at that information set. The authors observe that the game position (or information set) may be only partially known, in terms of properties that the player can test for. Thus - they argue - strategies can be thought of as programs, built up systematically from atomic decisions like *if  $b$  then  $a$*  where  $b$  is a condition checked by the player to hold (at some game position) and  $a$  is a move available to the player at that position. This leads them to propose a logical structure for strategies, where one can reason with assertions of the form “(partial) strategy  $\sigma$  ensures the (intermediate) condition  $\alpha$ ”. They present an axiomatization for the logic and prove its completeness.

The paper by Giacomo Sillari contributes to the very recent and fast growing literature on the notion of (un)awareness. An open problem in this literature has been how to model the state of mind of an individual who realizes that he may be unaware of something, that is, the problem of formalizing the notion of “awareness of unawareness”. Sillari offers a solution to this problem using a new system of first-order epistemic logic with awareness. He also offers a philosophical analysis of awareness structures and proves that a certain fragment of the first-order epistemic language with awareness operators is decidable.

The papers went through a thorough refereeing and editorial process. The editors would like to thank the many referees who provided invaluable help and the authors for their cooperation during the revision stage.