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CAN GOOD NEWS LEAD TO A MORE PESSIMISTIC CHOICE OF ACTION? *

ABSTRACT. Adapting a definition introduced by Milgrom (1981) we say that a signal about the environment is good news relative to some initial beliefs if the posterior beliefs dominate the initial beliefs in the sense of first-order stochastic dominance (the assumption being that higher values of the parameter representing the environment mean better environments). We give an example where good news leads to the adoption of a more pessimistic course of action (we say that action a_1 reveals greater pessimism than action a_2, if it gives higher payoff in bad environments and lower payoff in good environments). We then give sufficient conditions for a signal not to induce a more pessimistic choice of action.

Keywords: Bayesian updating, first-order stochastic dominance.

1. INTRODUCTION

This note is concerned with situations where an agent has to choose an action $a \in A$ (where A can be an arbitrary set) in an environment $e \in E$ which he cannot observe. We assume that the parameter e is one-dimensional and that higher values of e represent better environments, in the sense that – for any given action a – the agent's payoff increases with e. Since the environment is not observable, the agent will formulate some *a priori* beliefs about the true state of the environment, which are represented by a density function g. We assume that the agent observes an informative signal s about the environment [1] and updates his beliefs from g to g_s according to Bayes' rule.

Adapting a definition introduced by Milgrom (1981), we say that signal s is *good news relative to the initial beliefs* g if g_s dominates g in the sense of first-order stochastic dominance (bad news relative to g, if g dominates g_s). Milgrom introduced the notions of good news and bad news in order to provide a systematic way to understand a monotonicity property which arose in various areas of the literature. In our case, since we allow for a general (hence possibly multidimensional) set of actions, there is no obvious monotonicity property which can be considered. However, there seems to be a very intuitive 'one-dimensional' way in which any two

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actions may (or may not) be compared. We say that action a_1 *reveals greater pessimism* than action a_2, if it is more geared towards bad environments, that is, if it yields higher payoff than a_2 in bad environments and lower payoff than a_2 in good environments. Since upon receiving good news the agent attaches higher probability than before to good environments (and lower probability to bad environments), intuition suggests that if we observe an agent switch from a less pessimistic to a more pessimistic course of action, the reason must be that he has received bad news. In Section 2 we show that this intuition is wrong, by giving an example where *good* news leads to the choice of a more pessimistic course of action.

It is not difficult to think of situations where two agents, A and B, are both interested in the information contained in signal s, but only agent A can observe the signal (for example, A could be an incumbent monopolist and B a potential entrant and s could be a signal about demand). If agent B tries to infer from A's action whether s was good news or bad news, our example shows that in general he would be wrong to infer that s was bad news from the fact that A switched to a more pessimistic course of action (e.g. a lower price).

The framework we consider can also be useful in analysing situations in which agent A (e.g. a buyer) faces the decision whether or not to disclose some private information to agent B (e.g. a seller), who then takes an action (e.g. sets the price) which affects A's payoff. If the agents are in a strictly competitive situation,[2] intuition suggests that if signal s is bad news for B, then it is in A's interest to reveal it. Again, our example proves this intuition to be wrong.

It is therefore interesting to know if it is possible to tell in advance if a given signal can have the counterintuitive effects explained above. We say that a signal is *universally good news* if it is good news whatever the initial beliefs (similarly for universally bad news).[3] Finally, we call signal *s neutral* if it is uninformative, that is, if the updated beliefs coincide with the *a priori* beliefs. We show that if s is not a neutral signal and is universally good news, then it cannot lead to a more pessimistic choice of action (similarly, if s is non-neutral and universally bad news, then it cannot lead to a less pessimistic choice of action).

2. THE MODEL

Consider an agent who faces an environment $e \in E \subseteq R$ and has available a set of actions A, where A is a general topological space. Let

(1) $$U : A \times E \to R$$

be the agent's payoff function. No continuity assumptions are made about U (indeed, in many economic applications U is not continuous). We assume that a higher value of e means a more favourable environment. Formally, let

(2) $$\hat{a}(e) \in \operatorname*{argmax}_{a} \; U(a,e)$$

and

(3) $$V(e) = U(\hat{a}(e),e).$$

Then our assumption is that $U(a,e)$ is non-decreasing in e for each a and $V(e)$ is strictly increasing in e.

Example. A seller wants to sell a commodity to a buyer whose reservation price is a number $e \in [0,1]$ (thus $E = [0,1]$). Let a be the price set by the seller (thus $A = [0,\infty)$) and let his utility function be linear in money. Then his payoff function is given by

(4) $$U(a,e) = \begin{matrix} a & \text{if } a \leq e \\ 0 & \text{if } a > e \end{matrix}$$

Thus,

(5) $$\hat{a}(e) = e \quad \text{and} \quad V(e) = e.$$

We shall assume that the environment is not observable. Let

(6) $$g : E \to R \quad \left(g(e) \geq 0 \text{ for all } e \text{ and } \int_E g(e) \; \mathrm{d}e = 1 \right)$$

be a probability density function representing the agent's *a priori* beliefs about the environment and let $G : E \to [0,1]$ be the corresponding c.d.f. (thus $G(e) = \int_{x \leqslant e} g(x) \; \mathrm{d}x$).

Let $S \subseteq R^n$ be the set of signals about the environment and let

(7) $$F : S \times E \to R$$

be a function which represents the relationship between environments and signals. Thus for each $e \in E$

$$(8) \qquad F(s,e) \geq 0 \quad \text{for all } s \in S \quad \text{and} \quad \int_S F(s,e)\, \mathrm{d}s = 1.$$

In the absence of signals, the agent will choose that action a^* which maximizes expected utility

$$(9) \qquad EU(a) \equiv \int_E U(a,e)\, g(e)\, \mathrm{d}e.$$

If the agent receives signal $s \in S$ and updates his beliefs according to Bayes' rule, his posterior beliefs will be given by the density function $g_s : E \to R$ defined by

$$(10) \qquad g_s(e) = \frac{F(s,e)\, g(e)}{\int_E F(s,e)\, g(e)\, \mathrm{d}e}$$

Let $G_s : E \to [0,1]$ be the corresponding c.d.f.

DEFINITION 1. We say that signal s is *good news relative to beliefs g* (respectively *bad news relative to* g) if g_s dominates g (resp. g dominates g_s) in the sense of first-order stochastic dominance, that is, if

$$(11) \qquad \text{for all } e \in E,\ G_s(e) \leq G(e) \quad (\text{resp. } G(e) \leq G_s(e)).$$

We say that s is *neutral* if $g_s \equiv g$.

It is easy to see that s is neutral if and only if $F(s,e)$ is constant for all e. It is well-known[4] that g_s dominates g in the sense of first-order stochastic dominance if and only if for every non-decreasing function f

$$(12) \qquad \int_E f(e)\, g_s(e)\, \mathrm{d}e \geq \int_E f(e)\, g(e)\, \mathrm{d}e.$$

Setting $f(e) = U(a,e)$ (for any given a), we obtain an intuitive justification for the expression 'good news' (resp. 'bad news').

When the agent receives signal s, his updated beliefs about the environment will be given by g_s (defined by (10)) and the optimal action will now be that $a^*(s)$ which maximizes

$$(13) \qquad E_s U(a) \equiv \int_E U(a,e)\, g_s(e)\, \mathrm{d}e.$$

DEFINITION 2. We say that *signal s has led to a more pessimistic choice of action* if there exists an environment e_0 such that

$$(14) \qquad 0 < G(e_0) < 1$$

and

$$(15) \qquad \begin{array}{ll} U(a^*(s),e) < U(a^*,e) & \text{for all } e > e_0 \\ U(a^*(s),e) \geq U(a^*,e) & \text{for all } e < e_0 \end{array}$$

(recall that $a^*(s)$ is the optimal action given the updated beliefs and a^* is the optimal action given the *a priori* beliefs). If the inequalities in (15) are reversed we say that signal s has led to a less pessimistic choice of action.[5]

Intuition suggests that if the agent takes a more pessimistic course of action, the reason must be that he has received bad news. Indeed, if the agent has received good news, he will now believe that favourable environments are more likely than before $[1 - G_s(e_0) > 1 - G(e_0)]$ and we expect him to take advantage of this, by switching to an action which gives greater payoff in favourable environments (by a similar argument, we would expect a less pessimistic choice of action to be induced by good news). We now give an example which shows that the above intuition is wrong.

Example. In the buyer/seller example given above, let the seller's initial beliefs about the buyer's reservation price be given by

$$(16) \qquad g(e) = \begin{array}{ll} 10 & \text{if } 0 \leq e \leq 4/49 \\ 1/5 & \text{if } 4/49 \leq e \leq 1 \end{array}$$

Then[6]

$$(17) \qquad \int_E U(a,e)\, g(e)\, \mathrm{d}e = a[1 - G(a)] = \begin{array}{ll} a - 10a^2 & 0 \leq a \leq 4/49 \\ a/5 - a^2/5 & 4/49 \leq a \leq 1. \end{array}$$

It is easy to check that the unique global maximum of (17) is given by

(18) $a^* = 1/2.$

Now let the agent receive signal s and let

$$(19)\qquad F(s,e) = \begin{cases} 1/100 & 0 \le e \le 4/49 \\ 1/2 & 4/49 \le e \le 2/5 \\ 1/12 & 2/5 \le e \le 1 \end{cases}$$

Then the updated beliefs are given by

$$(20)\qquad g_s(e) = \frac{F(s,e)\,g(e)}{\int_E F(s,e)\,g(e)\,\mathrm{d}e} = \begin{cases} 2 & 0 \le e \le 2/5 \\ 1/3 & 2/5 \le e \le 1 \end{cases}$$

Figure 1a shows the functions g and g_s and Figure 1b the corresponding c.d.f.'s.

It can be seen from Figure 1b that g_s dominates g in the sense of *strict* first-order stochastic dominance and therefore signal s is strictly good news relative to g. We then have

$$(21)\qquad \int_E U(a,e)\,g_s(e)\,\mathrm{d}e = a[1 - G_s(a)] = \begin{cases} a - 2a^2 & 0 \le a \le 2/5 \\ a/3 - a^2/3 & 2/5 \le a \le 1 \end{cases}$$

and the unique global maximum of (21) is given by

(22) $a^*(s) = 1/4.$

Thus signal s, despite being good news, has induced the seller to reduce his price dramatically, that is, to switch to a more pessimistic course of action, according to our definition. In fact, setting $e_0 = 1/2$ we have $0 < G(e_0) = 9/10 < 1$ and

$$(23)\qquad \begin{array}{ll} U(a^*(s),e) = 1/4 < U(a^*,e) = 1/2 & \text{for all } e > e_0 \\ U(a^*(s),e) \ge U(a^*,e) = 0 & \text{for all } e < e_0 \end{array}$$

An intuitive explanation of this phenomenon is as follows. Beliefs g present the seller with a dilemma: he 'knows' that in order to have a good chance to sell the good, he must set a very low price. If he does so, the probability that the commodity will be bought is very high, but his utility will be very small. On the other hand, there is a slight chance that the buyer's reservation price is quite high, and the seller finds the high price/low probability bet more attractive than the low price/high probability bet because the latter involves a very low price indeed (1/20). Upon

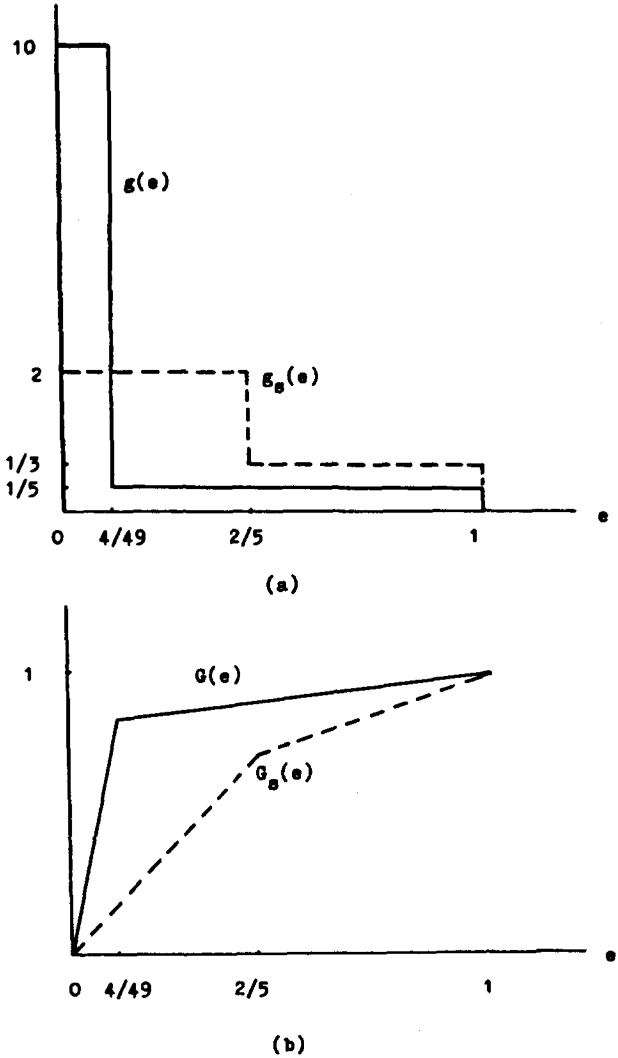

Fig. 1.

receiving the good news conveyed by signal s, the seller's perception of the situation changes: the low price/high probability bet now involves a higher price (1/4) and thus a higher utility and becomes more attractive than the high price/low probability bet (which has remained unchanged: it would still involve setting a price of 1/2).

Paradoxically, if s is a piece of information which is in the hands of the buyer, it is in his interest to disclose it, even though it induces the seller to believe that higher reservation prices are now more likely.

It is easy to construct an example along the same lines where signal s is bad news relative to the initial beliefs and yet it leads to the choice of a less pessimistic course of action. In the buyer/seller example it would therefore be unwise for the buyer to reveal the information contained in signal s, because it would induce the seller to switch to a much higher price (even though in the seller's mind the probability of obtaining that price would be very small indeed).

We now want to determine conditions under which good news does *not* lead to a more pessimistic choice of action.

DEFINITION 3. We say that signal s is *universally good news* (resp. *universally bad news*) if for every initial beliefs g, g_s dominates g (resp. g dominates g_s) in the sense of first-order stochastic dominance.

LEMMA. *Signal s is universally good news (resp. universally bad news) if and only if $F(s,e)$ is a non-decreasing (resp. non-increasing) function of e.*

The proof of the above lemma is along the lines of Proposition 1 in Milgrom (1981) and is given in the Appendix.

We can now prove the following proposition.

PROPOSITION. *If s is nonneutral and universally good news (resp. nonneutral and universally bad news), then – whatever the initial beliefs (provided they are nondegenerate) – it cannot lead to a more pessimistic choice of action (resp. to a less pessimistic choice of action).*[7]

Proof. We shall prove the proposition for the case of good news. The case of bad news is proved similarly. Let $g: E \rightarrow R$ be an arbitrary (nondegenerate) density function and let

$$(24) \qquad a^* \in \operatorname*{argmax}_{a} \int_E U(a,e)\, g(e)\, de.$$

Let $F(s,.):E \to R$ be a non-negative, non-constant, non-decreasing function (which, by the above lemma, is equivalent to s being universally good news and nonneutral) and let $g_s:E \to R$ be defined by (10). Let

$$a^*(s) \in \operatorname*{argmax}_a \int_E U(a,e)\, g_s(e)\; de. \tag{25}$$

Suppose there exists an $e_0 \in E$ such that $0 < G(e_0) < 1$ and

$$\begin{aligned} U(a^*,e) &> U(a^*(s),e) \quad \text{for all } e > e_0 \\ U(a^*,e) &\leq U(a^*(s),e) \quad \text{for all } e < e_0. \end{aligned} \tag{26}$$

By definition of $a^*(s)$

$$\int_E U(a^*(s),e)\, g_s(e)\; de \geq \int_E U(a^*,e)\, g_s(e)\; de \tag{27}$$

which is equivalent to

$$\begin{aligned} &\int_{e<e_0} [U(a^*(s),e) - U(a^*,e)]\, g_s(e)\; de \geq \\ &\geq \int_{e>e_0} [U(a^*,e) - U(a^*(s),e)]\, g_s(e)\; de. \end{aligned} \tag{28}$$

By (26) the expressions in square brackets are non-negative and this, together with the fact that $F(s,e)$ is non-decreasing in e, implies that

$$\text{RHS of (28)} \geq \frac{F(s,e_0)}{\Omega} \int_{e>e_0} [U(a^*,e) - U(a^*(s),e)]\, g(e)\; de \tag{29}$$

and

$$\text{LHS of (28)} \leq \frac{F(s,e_0)}{\Omega} \int_{e<e_0} [U(a^*(s),e) - U(a^*,e)]\, g(e)\; de \tag{30}$$

where

$$\Omega = \int_E F(s,e)\, g(e)\; de \tag{31}$$

and at least one of the inequalities in (29) and (30) must be strict, because, by assumption, $F(s,e)$ is a non-constant function of e. Thus

$$\int_{e<e_0} [U(a^*(s),e) - U(a^*,e)]\, g(e)\, \mathrm{d}e > \tag{32}$$
$$> \int_{e>e_0} [U(a^*,e) - U(a^*(s),e)]\, g(e)\, \mathrm{d}e$$

which is equivalent to

$$\int_E U(a^*(s),e)\, g(e)\, \mathrm{d}e > \int_E U(a^*,e)\, g(e)\, \mathrm{d}e \tag{33}$$

which contradicts the definition of a^*. ■

The proposition is not true if s is a neutral signal.[8]

3. CONCLUSION

The purpose of this short note was to show that if we observe that an agent – upon receipt of a signal about the environment – switches to an action which is more geared towards bad environments, then in general we cannot infer that the signal was bad news for the agent (in the sense that the agent's updated beliefs – in the light of the information contained in the signal – attach higher probability to bad environments and lower probability to good environments, as compared to the initial beliefs). Indeed we gave an example where *good news* can lead to a more pessimistic choice of action. We also showed that a sufficient condition for a signal not to induce counterintuitive changes in behaviour is that it be universally good news (or universally bad news), that is, that it be good news (resp. bad news) whatever the initial beliefs.

In Bonanno (1986) the relevance of these results is shown in the context of negotiations between a firm and a union over a requested pay rise by the latter. It is shown that if the firm pursues a policy which aims to make the union increasingly pessimistic about the size of the surplus to be divided, then that policy may eventually backfire and induce intransigence on the part of the union. It is therefore important for the firm to be able to know in advance if disclosing a certain signal, which is known to be bad news for the union, can be counterproductive.

APPENDIX 1

Proof of lemma. Necessity. Let $e_1, e_2 \in E$ with $e_1 < e_2$. Let $g(e_1) = g(e_2) = 1/2$. Then it must be true that

$$\text{(A1)} \qquad g_s(e_2) = 1 - g_s(e_1) \geq g_s(e_1).$$

By Bayes' rule,

$$\text{(A2)} \qquad g_s(e_1) = \frac{F(s,e_1)}{F(s,e_1) + F(s,e_2)} \quad \text{and} \quad g(e_2) = \frac{F(s,e_2)}{F(s,e_1) + F(s,e_2)}.$$

Thus for (A1) to be satisfied, we must have

$$\text{(A3)} \qquad F(s,e_2) \geq F(s,e_1)$$

Sufficiency. Fix any e^* such that $0 < G(e^*) < 1$ and let $e_1 \leq e^*$. Then for every $e_2 \geq e^*$ (A3) is satisfied and

$$\text{(A4)} \qquad \int_{e_2 \geq e^*} F(s,e_2)\, g(e_2)\, de_2 \geq$$

$$\geq F(s,e_1) \int_{e_2 \geq e^*} g(e_2)\, de_2 = F(s,e_1)[1 - G(e^*)].$$

Dividing both sides by $\int_E F(s,e)\, g(e)\, de$ (A4) becomes

$$\text{(A5)} \qquad 1 - G_s(e^*) \geq [1 - G(e^*)] \frac{F(s,e_1)}{\int_E F(s,e)\, g(e)\, de}$$

from which we obtain

$$\text{(A6)} \qquad \int_{e_1 \leq e^*} [1 - G_s(e^*)]\, g(e_1)\, de_1 \geq$$

$$\geq \int_{e_1 \leq e^*} [1 - G(e^*)] \frac{F(s,e_1)}{\int_E F(s,e)\, g(e)\, de} g(e_1)\, de_1$$

which is equivalent to

$$\text{(A7)} \qquad [1 - G_s(e^*)]\, G(e^*) \geq [1 - G(e^*)]\, G_s(e^*)$$

which implies $G_s(e^*) \leq G(e^*)$. ■

APPENDIX 2

In this appendix we discuss in detail the example of Note 7. Let $V(w)$ be the utility of wealth function, which is increasing, concave and displays increasing absolute risk aversion (i.e. $-V''(w)/V'(w)$ is increasing in w). Let x be the initial wealth, π_0 a given positive number and Z a random variable (whose c.d.f. is denoted by F) with zero mean (and such that $\text{Prob}\{z<\pi_0\}>0$). Let $A=\{a_1,a_2\}$, $E=\{e_1,e_2\}$ with $e_2>e_1$, and for every $i=1,2$

$$\text{(A8)} \qquad U(a_1,e_i)=V(x+e_i-\pi_0)$$

$$\text{(A9)} \qquad U(a_2,e_i)=\int V(x+e_i+z)\,\mathrm{d}F(z).$$

[Note that for every given $a\in A$, $U(a,e)$ is increasing in e].

Let $\pi_i\,(i=1,2)$ be the (unique) solution to

$$\text{(A10)} \qquad V(x+e_i-\pi_i)=\int V(x+e_i+z)\,\mathrm{d}F(z)$$

(that is, π_i is the risk premium when the initial wealth is $x+e_i$). Then, because of increasing absolute risk aversion (and $e_2>e_1$)

$$\text{(A11)} \qquad \pi_1<\pi_2$$

[see Pratt (1964)]. Assume that π_0 satisfies

$$\text{(A12)} \qquad \pi_1<\pi_0<\pi_2.$$

Then, since V is an increasing function, using (A8)–(A10) and (A12) we obtain

$$\text{(A13)} \qquad U(a_1,e_1)<U(a_2,e_1)$$

and

$$\text{(A14)} \qquad U(a_1,e_2)>U(a_2,e_2)$$

Given *any* probability distribution (prior beliefs) over $E=\{e_1,e_2\}$, the signal which reveals that the true value of e is e_2 represents good news. It is clearly possible that, before the signal was received, the agent chose a_2 (if enough weight was attached to e_1: cf. (A13)) and after the signal is received, the agent switches to a_1. One could argue (cf. Note 7) that action a_1 is 'more pessimistic' than action a_2. However, according to our definition, in order for a_1 to be more pessimistic than a_2 it is necessary that a_1 performs better than a_2 in the bad environment e_1 and worse than a_2 in the good environment e_2, that is,

(A15) $\quad U(a_1,e_2) < U(a_2,e_2)$

and

(A16) $\quad U(a_1,e_1) \geq U(a_2,e_1)$

But (A15) and (A16) contradict (A13) and (A14).

Thus our definition of 'more pessimistic action' has a different intuitive content to the expression 'more risk averse'. According to the terminology we have chosen, a price reduction represents a more pessimistic choice for the seller (cf. the example given in Section 2), while the switch from an uncertain prospect with expected value $(x+e)$ to the certain prospect $(x+e-\pi_0)$ is not necessarily more pessimistic.

NOTES

* A first version of this paper was written when the author was Heyworth Research Fellow at Nuffield College, Oxford, and presented at the Second Annual Congress of the European Economic Association (Copenhagen, August 1987). The author is grateful to an anonymous referee for helpful comments.

[1] As Milgrom (1981, p. 381) points out, a signal can be anything from a financial or geological report to a television news show, and '... the form that a signal takes is theoretically irrelevant to its ability to convey information'.

[2] For an example (involving negotiations between a firm and a union) see Bonanno (1986).

[3] Thus our definition of 'universally good (bad) news' corresponds to Milgrom's definition of 'good (bad) news'.

[4] See, for example, Lippman and McCall (1982, p. 216).

[5] See Note 7 and Appendix 2 for a further discussion of the intuitive content of the expression 'more pessimistic action' as defined above.

[6] It is worth noting that even though the payoff function U and the density functions g and g_s are not continuous, the expected utility functions (17) and (21) are continuous; furthermore, they are differentiable everywhere, with the exception of the points $a=4/49$ for (17) and $a=2/5$ for (21), which are kinked local minima.

[7] A referee suggested the following example and wondered how it would fit into the framework of this paper:

'Say I have initial wealth x, increasing absolute risk aversion, and there are two actions: accept (in addition to x) a given additive random variable $(e+Z)$ or accept (in addition to x) the degenerate random variable $(e-\pi)$ (Z has zero mean and involves possible losses greater than π). Now, say the signal that I learn is the value of e. Clearly, the latter action is more pessimistic. But we know from standard utility theory that with increasing absolute risk aversion, the knowledge that I have a higher e will make me *more inclined* to take this latter action. In other words, *unambiguously favourable* news about the environment leads to a more pessimistic choice of action!'

This example is examined in detail in Appendix 2, where we show that it is not at variance with our results, because, according to our definition, accepting $(e-\pi)$ is not a more pessimistic choice: 'more pessimistic' does not mean 'more risk averse'.

[8] If the optimal solution is not unique, then the seller could switch to a more pessimistic action upon receipt of a neutral signal (e.g. in the example $g(e)=2$ for $e \leq 1/3$, and $g(e)=0.5$ for $e > 1/3$, in which case the expected utility function (9) has two global maxima, $a^*=1/4$ and $a^*=1/2$). These cases, however, are not generic.

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