Economics 103

PRACTICE EXAM for the FIRST MIDTERM: ANSWERS

1. If Jane does not buy insurance, with probability 0.8 her wealth will be \$10,000 (no theft), while with probability 0.2 it will be \$2000 (the car is stolen). If she buys insurance at premium *h* her wealth will be \$(10,000–*h*) with probability 0.8 (no theft), and \$(10,000– 1,000–*h*) with probability 0.2 (the car is stolen). The maximum premium she is willing to pay is the value of *h* that solves the following equation, where the LHS is Jane's expected utility if she doesn't buy insurance and the RHS is her expected utility if she does:

$$0.8\,[40(10)-(10)^2] + 0.2\,[40(2)-2^2] =$$

$$0.8\left[40\left(10-\frac{h}{1{,}000}\right)-\left(10-\frac{h}{1{,}000}\right)^2\right]+0.2\left[40\left(9-\frac{h}{1{,}000}\right)-\left(9-\frac{h}{1{,}000}\right)^2\right]$$

The solution is h = 1,826.64. Thus Jane is willing to pay up to \$1826.64 for the insurance policy.

2. **(a)** If she doesn't buy insurance then her wealth will be 200,000+120,000 = 320,000 if there is no fire (and this happens with probability 0.99) and 200,000 if there is a fire (and this happens with probability 0.01). Thus her expected wealth is 320,000(0.99) + 200,000(0.01) = 318,800.

(b) If they sign the contract then Carla faces the following lottery:

EVENT	Both houses burn down	Only Carla's house burns down	Only Natasha's house burns down	Neither house burns down
PROBABILITY	(0.01)(0.01) = 0.0001	(0.01)(0.99) = 0.0099	(0.99)(0.01) = 0.0099	(0.99)(0.99)= 0.9801
Carla's wealth	200,000	260,000	260,000	320,000

Thus her expected wealth is:

200,000(0.0001) + 260,000(0.0198) + 320,000(0.9801) = 318,800,
the same as without the contract.

(c) Since her expected wealth is the same with or without the contract, if she is risk-neutral she does not gain by signing the contract (nor does she lose: she is indifferent).

(d) Normalize Carla's utility function so that U(200,000) = 0, U(260,000) = a and U(320,000) = 1 (with $0 < a < 1$). Then her expected utility without the contact is:

$$0.01(0) + 0.99\,(1) = 0.99.$$

(e) Her expected utility with the contract is:

$$0.0001(0)+ 0.0198\,(a) + 0.9801\,(1) = 0.0198\,a + 0.9801.$$

(f.1) From (d) and (e) we deduce that she is better off with the contract if and only if

$$0.0198\,a + 0.9801 > 0.99,$$

that is, if and only if

$$a > 0.5.$$

Thus, since $a = 0.6$, she is better off with the contract.

(f.2) For a risk-averse person the utility of the expected value of a lottery is greater than the expected utility of the lottery. Let us construct a lottery with prizes 200,000 [with probability (1– p)] and 320,000 [with probability p] whose expected value is 260,000:

$$(1 - p)\ 200{,}000 + p\ 320{,}000 = 260{,}000.$$

Solving for p we get: p = 0.5. Now, U(260,000) = 0.6, while denoting by A is the lottery $\begin{pmatrix} 320{,}000 & 200{,}000 \\ 0.5 & 0.5 \end{pmatrix}$, EU(A) = 0.5 U(200,000) + 0.5 U(320,000) = 0.5 (0) + 0.5 (1) = 0.5.

Thus we have that the utility of the expected value of A is greater than the expected utility of A, hence Carla is risk averse.

3. **(a)**

PRIZE	\$2,000	\$4,000	\$8,000	\$16,000	\$32,000	\$64,000	\$0
COIN SEQUENCE	H	TH	TTH	TTTH	TTTTH	TTTTTTH	TTTTTTT
PROBABILITY	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$

(b) The expected prize is:

$$\frac{1}{2}\,2{,}000 + \frac{1}{4}\,4{,}000 + \frac{1}{8}\,8{,}000 + \frac{1}{16}\,16{,}000 + \frac{1}{32}\,32{,}000 + \frac{1}{64}\,64{,}000 = \$6{,}000.$$

Since it costs \$5,000 to play the game, the expected net gain is \$1,000.

(c) If you don't enter the casino, your utility is $\sqrt{15000} = 122.474$

If you enter (by paying \$5,000) and play the game, your expected utility is:

$$\frac{1}{2}\sqrt{12{,}000} + \frac{1}{4}\sqrt{14{,}000} + \frac{1}{8}\sqrt{18{,}000} + \frac{1}{16}\sqrt{26{,}000} + \frac{1}{32}\sqrt{42{,}000} + \frac{1}{64}\sqrt{74{,}000} + \frac{1}{64}\sqrt{10{,}000} = 123.494$$

(note: the last term in the sum is for the case where the outcome is TTTTTT and you are left with the initial \$15,000 minus the price of \$5,000 you paid to play).

Thus you **should** play the game.

4. Suppose Peter does satisfy the axioms of expected utility and let U be his utility-of-money function, normalized so that U(5000) = 1 and U(0) = 0. Let U(1000) = p. Then 0 < p < 1. Now, EU(A) = p, EU(B) = 0.1 (1) + 0.89 (p) + 0.01 (0) = 0.1 + 0.89 p, EU(C) = 0.11 p and EU(D) = 0.1. Then EU(A) > EU(B) if and only if p > 0.1 + 0.89 p, i.e. if and only if $p > \frac{10}{11}$. But if $p > \frac{10}{11}$ then $EU(C) = 0.11p > 0.11\frac{10}{11} = 0.1 = EU(D)$. Thus if Peter satisfies the axioms of expected utility and prefers A to B then he must also prefer C to D. Hence he does **not** satisfy the axioms of expected utility.