ECN 103 Professor Giacomo Bonanno

SECOND MIDTERM EXAM: ANSWERS for VERSION 1

1. It must be that A dominates B in the sense of First-Order Stochastic Dominance. There are only two values of x and y that yield that: $\boxed{x = \frac{17}{36} \text{ and } y = 0}$. Explanation: first of all, in order for B to be a lottery we need the probabilities to add up to 1, which requires $x + y = \frac{17}{36}$ or $y = \frac{17}{36} - x$. Thus we have:

$$\begin{matrix} \\ P_A: \\ cdf_A: \end{matrix} \begin{pmatrix} \$16 & \$18 & \$20 & \$34 & \$36 & \$40 \\ \frac{6}{36} & 0 & \frac{18}{36} & 0 & \frac{3}{36} & \frac{9}{36} \\ \frac{6}{36} & \frac{6}{36} & \frac{24}{36} & \frac{24}{36} & \frac{27}{36} & 1 \end{pmatrix} = A$$

$$\begin{matrix} \\ P_B: \\ cdf_B: \end{matrix} \begin{pmatrix} \$16 & \$18 & \$20 & \$34 & \$36 & \$40 \\ \frac{6}{36} & x & \frac{1}{36} & y = \frac{17}{36} - x & \frac{3}{36} & \frac{9}{36} \\ \frac{6}{36} & \frac{6}{36} + x & \frac{7}{36} + x & \frac{24}{36} & \frac{27}{36} & 1 \end{pmatrix} = B$$

In order for A to dominate B in the sense of First-Order stochastic dominance, it cannot be that $x = 0$, because then $cdf_A(\$20) = \frac{24}{36} > cdf_B(\$20) = \frac{7}{36}$. So it must be that $x > 0$. Then we have that $cdf_A(\$18) < cdf_B(\$18)$, which is fine, but we also need $cdf_A(\$20) = \frac{24}{36} \leq cdf_B(\$20) = \frac{7}{36} + x$ that is, $x \geq \frac{17}{36}$ and this, together with $x + y = \frac{17}{36}$ gives: $x = \frac{17}{36}$ and $y = 0$.

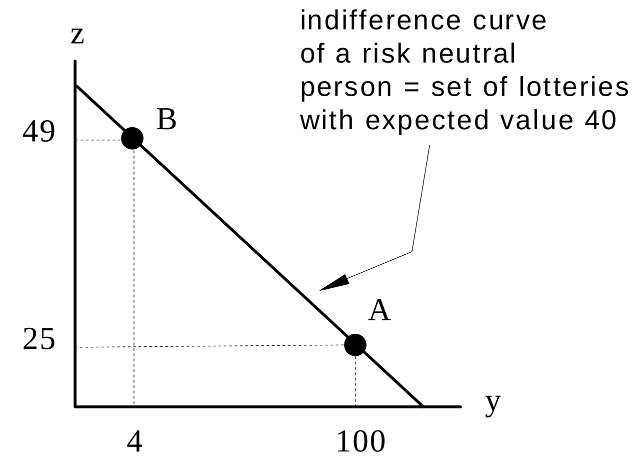
2. First of all, we need the probabilities in lottery A to add up to 1: $\frac{p}{2} + \frac{1}{20} + p + \frac{1}{4} = 1$. Solving this equation we get that $\boxed{p = \frac{7}{15}}$ so that $A = \begin{pmatrix} \$30 & \$36 & \$45 & \$48 \\ \frac{7}{30} & \frac{1}{20} & \frac{7}{15} & \frac{1}{4} \end{pmatrix}$. Two equations need to be satisfied in order for B to be a mean-preserving spread of A: the first ensures that the probabilities in lottery B add up to 1 and the second ensures that the expected value of B is equal to the expected value of A: $\boxed{x + y = \frac{1}{20}}$ and $\boxed{32x + 40y = \frac{1}{20}36}$. [The second equation can also be written as follows, since $E[A] = 41.8$: $\frac{7}{30}30 + 32x + 40y + \frac{7}{15}45 + \frac{1}{4}48 = 41.8$.] [The solution is $x = y = \frac{1}{40}$.]

3. **(a)** Since $\mathbb{E}[A] = \frac{1}{5}100 + \frac{4}{5}25 = 40$ and $\mathbb{E}[B] = \frac{1}{5}4 + \frac{4}{5}49 = 40$, a risk-neutral person would be indifferent between A and B.

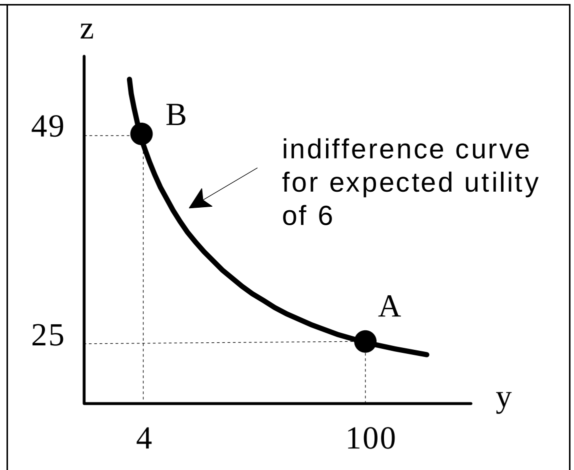
(b) No: we know that John would prefer \$40 for sure to either A or B, but we don't know how he would rank A versus B.

(c) Since $\mathbb{E}[U(A)] = \frac{1}{5}\sqrt{100} + \frac{4}{5}\sqrt{25} = 2 + 4 = 6$ and $\mathbb{E}[U(B)] = \frac{1}{5}\sqrt{4} + \frac{4}{5}\sqrt{49} = \frac{2}{5} + \frac{28}{5} = 6$ Amy is indifferent between A and B.

(d) and **(e)** See the following figures:



(d)



(e)

(f) The slope if the same at every point and equal to $-\frac{p}{1-p} = -\frac{\frac{1}{5}}{\frac{4}{5}} = -\frac{1}{4}$.

(g) $U'(m) = \frac{1}{2\sqrt{m}}$. The slope at A is $-\frac{U'(100)}{U'(25)}\left(\frac{p}{1-p}\right) = -\frac{\frac{1}{2\sqrt{100}}}{\frac{1}{2\sqrt{25}}}\frac{1}{4} = -\frac{1}{8}$ and the slope at B is $-\frac{U'(4)}{U'(49)}\left(\frac{p}{1-p}\right) = -\frac{\frac{1}{2\sqrt{4}}}{\frac{1}{2\sqrt{49}}}\frac{1}{4} = -\frac{7}{8}$.

(h) For a risk-neutral person it is $-\frac{p}{1-p} = -\frac{1}{4}$ and for Amy it is $-\frac{U'(40)}{U'(40)}\left(\frac{p}{1-p}\right) = -\frac{1}{4}$, the same.