How to construct or discover a person's von Neumann-Morgenstern utility function

$Z = \{z_1, z_2, z_3, z_4\}$

Question 1: how do you rank the basic outcomes?

		utility
best	z_3	1
	z_1, z_4	
worst	z_2	0

Question 2: what value of *p* would make you indifferent between the following two lotteries?

$$\begin{pmatrix} z_1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} z_3 & z_2 \\ p & 1-p \end{pmatrix}$$

Restating attitudes to risk in terms of utility

$$L = \begin{pmatrix} \$x_1 & \$x_2 & \ldots & \$x_n \\ p_1 & p_2 & \ldots & p_n \end{pmatrix} \text{ money lottery}$$

U utility-of-money function.

$$\mathbb{E}[L] = \underbrace{\qquad\qquad\qquad\qquad}_{\text{expected value of L}} \qquad \mathbb{E}[U(L)] = \underbrace{\qquad\qquad\qquad\qquad}_{\text{expected utility of L}}$$

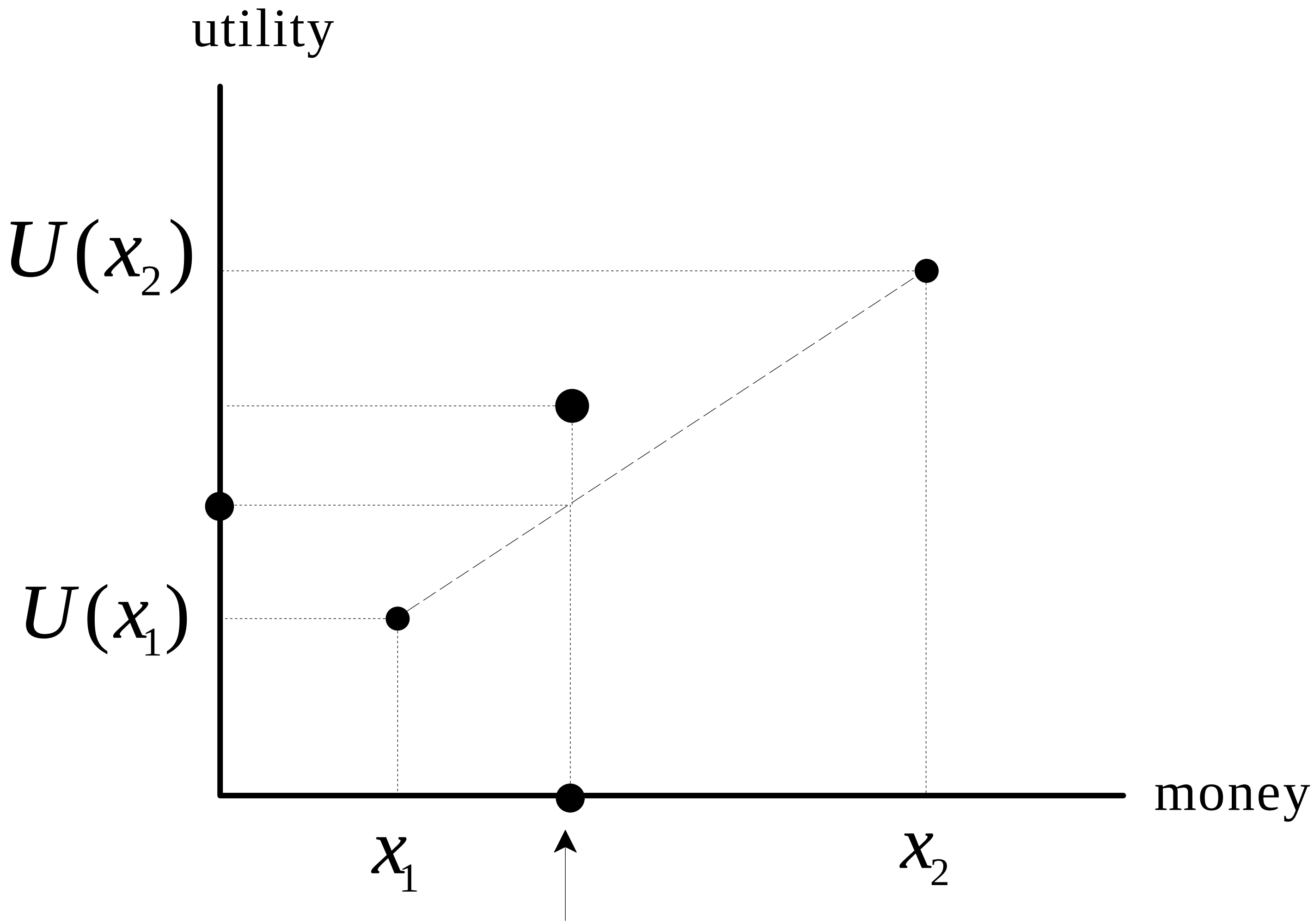
- Risk averse if
- Risk neutral if
- Risk loving if

Example: $L = \begin{pmatrix} \$16 & \$36 & \$64 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$. Then $\mathbb{E}[L]) =$

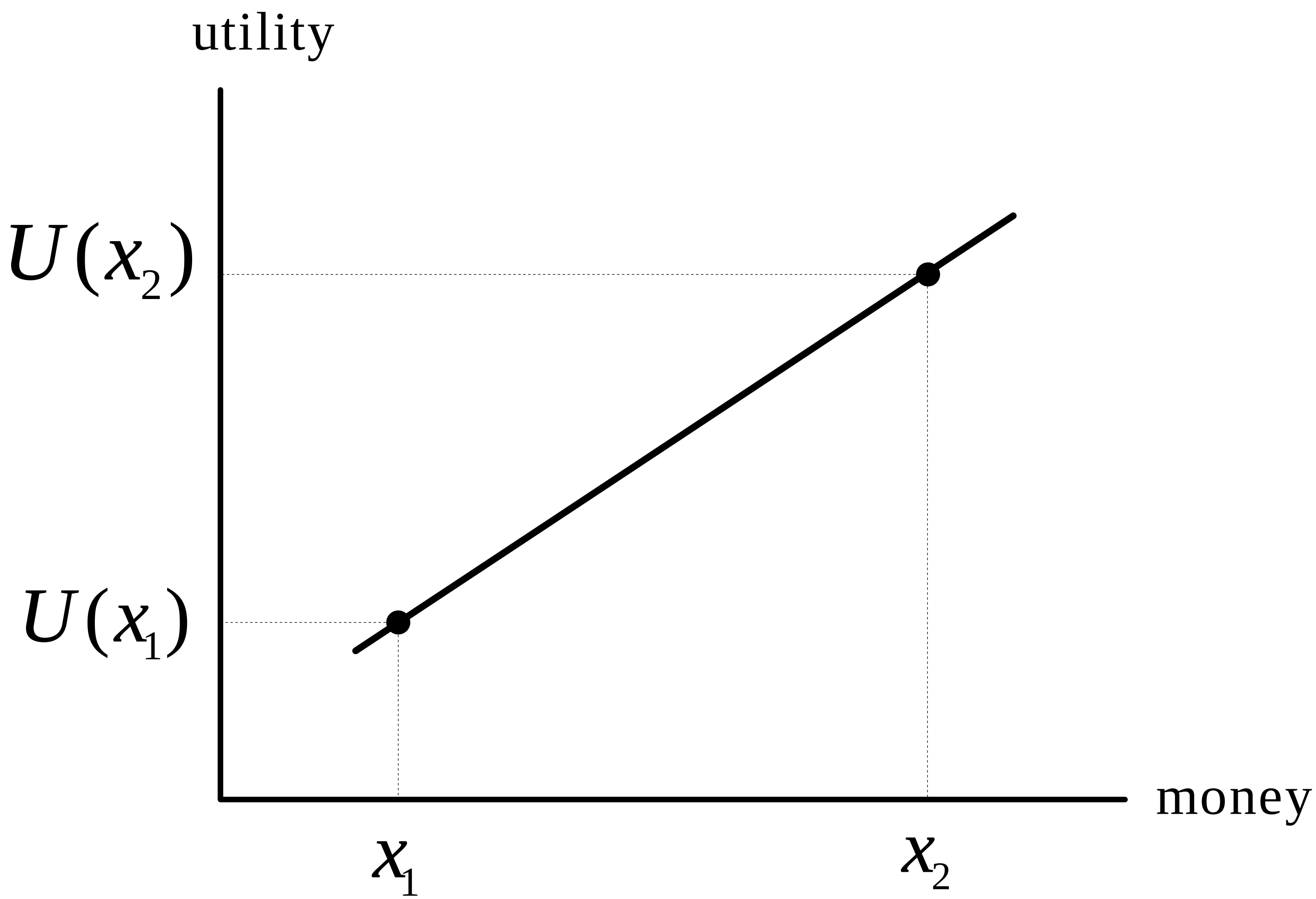
- $U(\$x) = \sqrt{x}$. Then

- $U(\$x) = \dfrac{x}{2}$. Then

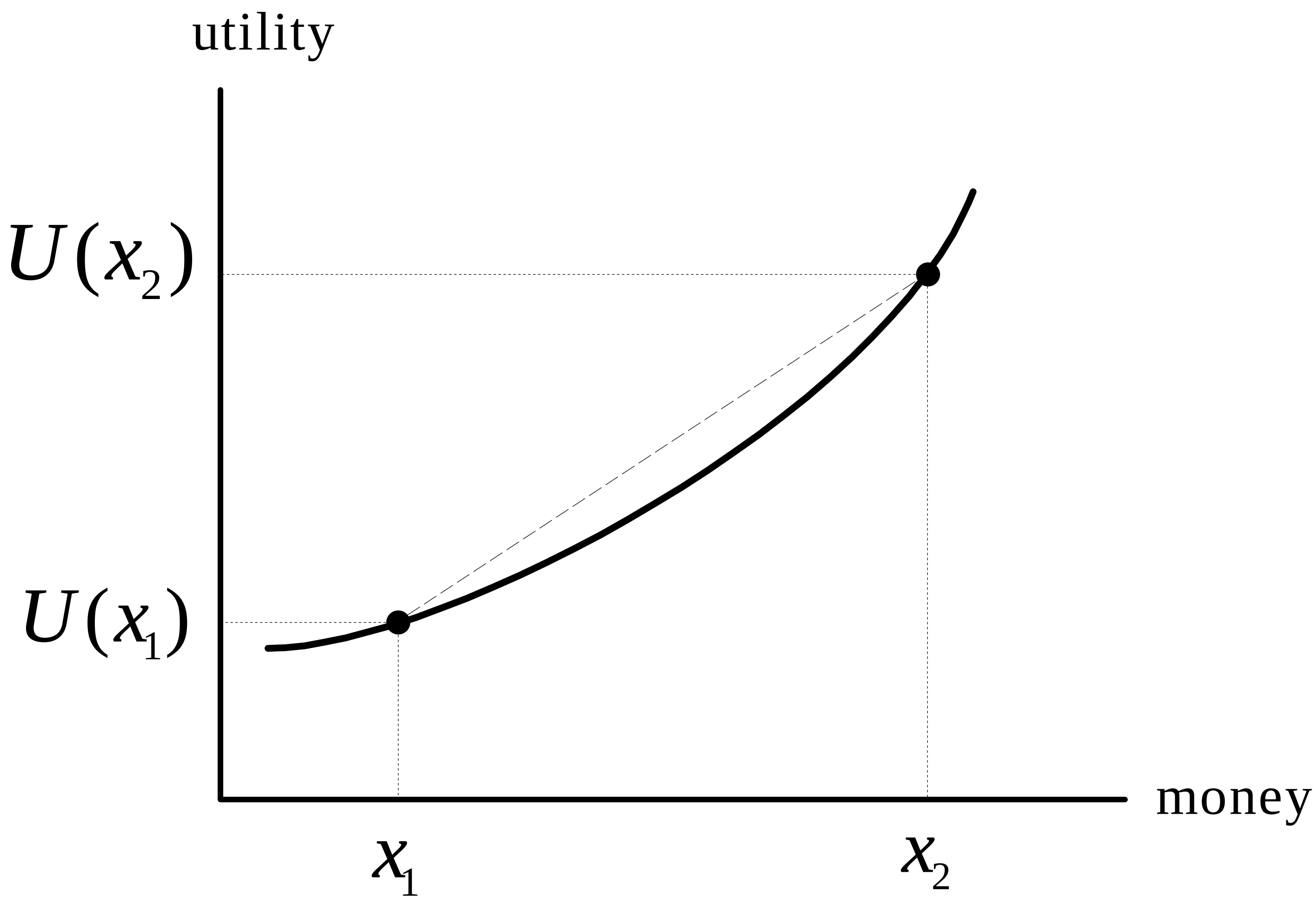
- $U(\$x) = x^2$. Then


utility
$U(x_2)$
$U(x_1)$
money
x_1
x_2

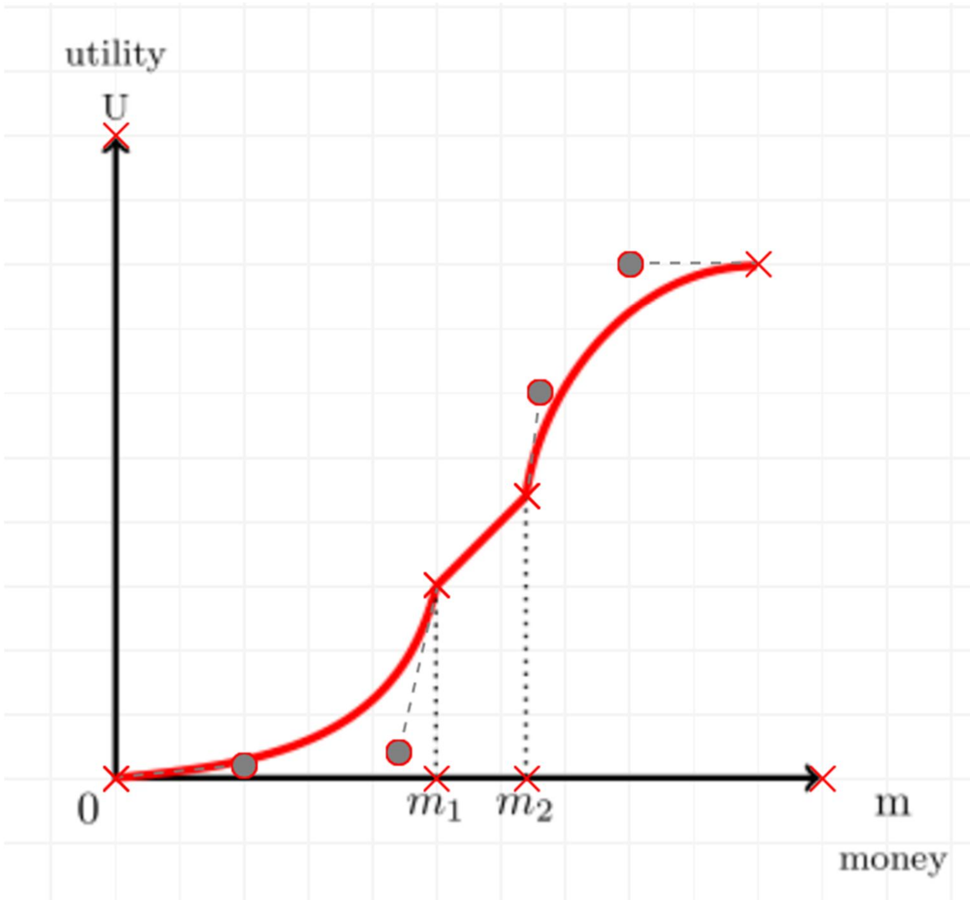
Risk neutrality: straight line



Risk loving: strictly convex



Mixture of attitudes:



A function $U(x)$ is strictly concave if and only if

Examples:

$$U(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$U'(x) =$$

$$U''(x) =$$

$$U(x) = \ln(x)$$

$$U'(x) =$$

$$U''(x) =$$

A function $U(x)$ is strictly convex if and only if

Examples:

$$U(x) = x^2 \qquad U(x) = e^x$$

$$U'(x) = \qquad U'(x) =$$

$$U''(x) = \qquad U''(x) =$$

A function $U(x)$ is has a straight-line graph if and only if

Measuring risk aversion

How to identify risk aversion: $U''(x) < 0$

Can there be more or less risk aversion?

Even the same utility function, **the degree of risk aversion of an individual varies with her level of wealth.**

$U(x) = \sqrt{x}$. Initial wealth: W_0.

What is the **risk premium** associated with this lottery? I**t depends on** W_0**.**

Suppose that $W_0 = 50$

Suppose that Suppose that $W_0 = 1,000$

Thus she is less risk averse when her wealth is $1,000 than when her wealth is $50.

We compared two related lotteries **given some fixed preferences (i.e. a fixed utility function).**

Now **fix a lottery *L* and consider different preferences (that is, different utility functions)**.

Take the risk premium of the lottery as a measure of the intensity of risk aversion.

Initial wealth: 50. Wealth lottery: $L = \begin{pmatrix} 0 & 100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\mathbb{E}[L] = 50$

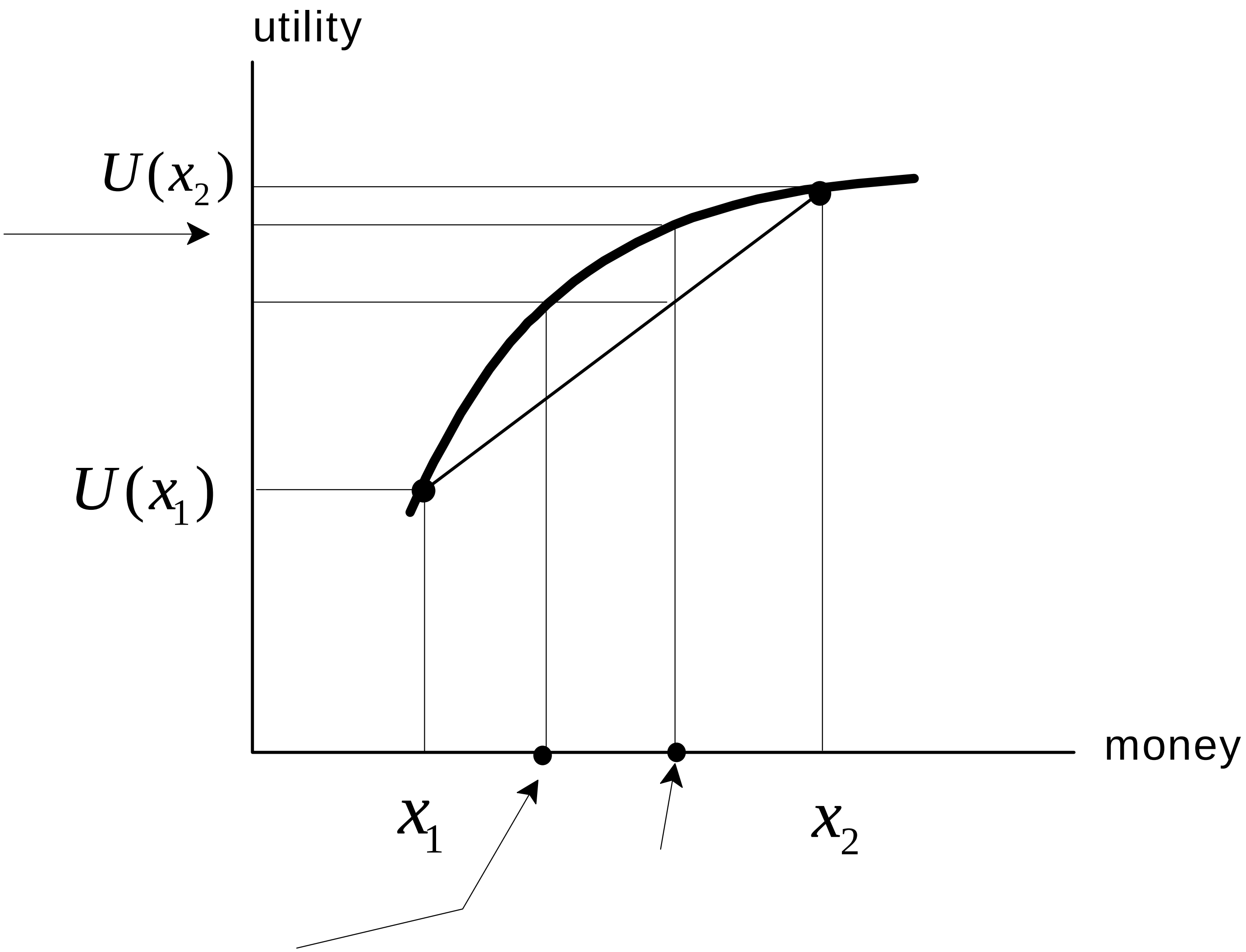
- $\boxed{U(x) = \sqrt{x}}$ then, as we saw before, the risk premium is the solution to

  $\sqrt{50 - R} = \underbrace{5}_{=\mathbb{E}[U(L)]}$ which is $\boxed{R = \$25}$

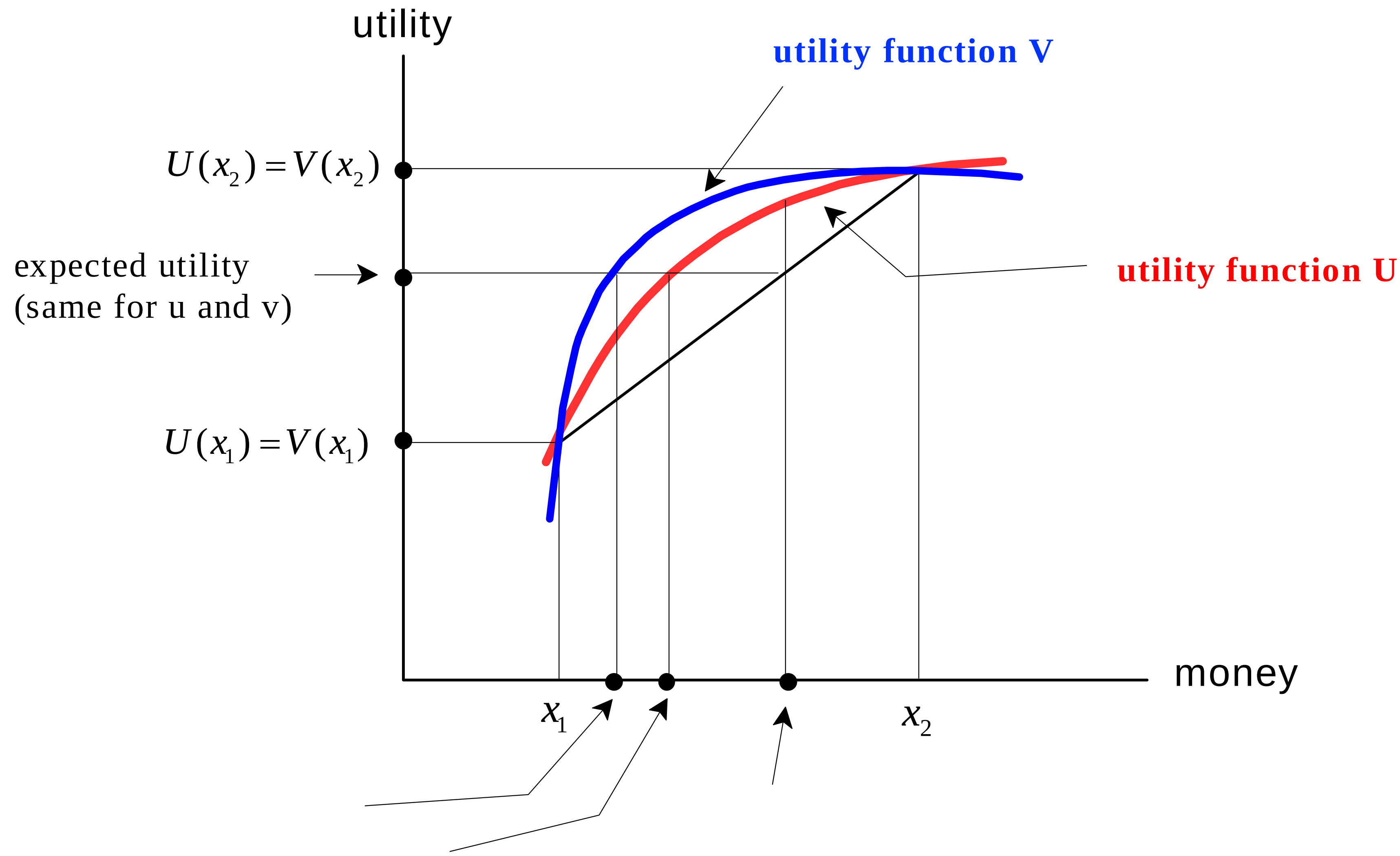
- If her utility function is $\boxed{U(x) = \ln(x+1)}$

Thus the utility function ln(x+1) embodies more risk aversion then the function $\sqrt{x}$ relative to lottery $\begin{pmatrix} 0 & 100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. But perhaps there is another lottery relative to which the function $\sqrt{x}$ displays more (or the same) risk aversion than the utility function ln(x +1)?

Graphical representation of the risk premium:



A more concave utility function is associated with a larger risk premium for the same lottery:



(2) Check that the risk premium is a meaningful measure, that is, that it is invariant to an allowed transformation of the utility function.

$$L = \begin{pmatrix} \$x_1 & \$x_2 & \ldots & \$x_n \\ p_1 & p_2 & \ldots & p_n \end{pmatrix}, \quad \mathbb{E}[L] = p_1 x_1 + p_2 x_2 + \ldots + p_n x_n$$

Utility function $U(\$x)$. $\mathbb{E}[U(L)] =$

R_{UL} solution to

Now let $V(x) = aU(x) + b$ with $a > 0$

R_{VL} solution to

$V\left(\mathbb{E}[L] - R\right) = \mathbb{E}[V(L)]$ if and only if $U\left(\mathbb{E}[L] - R\right) = \mathbb{E}[U(L)]$. Hence $R_{VL} = R_{UL}$

Definition. Utility function U embodies more risk aversion that utility function V if

$R_{UL} > R_{VL}$ for every non-degenerate money lottery L.

Short of trying every possible lottery, is there a way to determine if U embodies more risk aversion than V?

Arrow-Pratt measure of risk aversion:

First, let us verify that it is a meaningful measure, that is, that it is invariant to an allowed transformation of the utility function

Let $V(x) = aU(x) + b$ for every $x \geq 0$ with $a > 0$. $V'(x) =$ and $V''(x) =$

Examples.

$$\boxed{U(x)=\sqrt{x}} = x^{\frac{1}{2}}$$

$$\boxed{U(x)=\ln(x)}$$

Note that both display decreasing risk aversion as *x* increases

Theorem. Let U(x) and V(x) be two strictly concave functions. Then the following conditions are equivalent:

1. $R_{VL} > R_{UL}$ for every non-degenerate wealth lottery *L*

2. $A_V(x) > A_U(x)$ for every $x > 0$.