We don't have to reduce the probability to zero: $L = \begin{pmatrix} \$10 & \$50 & \$110 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$

Take away some of the probability of \$50, say $\frac{3}{10}$ and spread it between a lower amount, say \$15, and a higher amount, say \$90:

$$M = \begin{pmatrix} \$10 & \$15 & \$50 & \$90 & \$110 \\ & & & & \end{pmatrix}$$

For this to be a mean preserving spread we need

$$M = \begin{pmatrix} \$10 & \$15 & \$50 & \$90 & \$110 \\ & & & & \end{pmatrix}$$

Write $L >_{SSD} M$ to mean that ***L* dominates *M*** in the sense of **second-order stochastic dominance.**

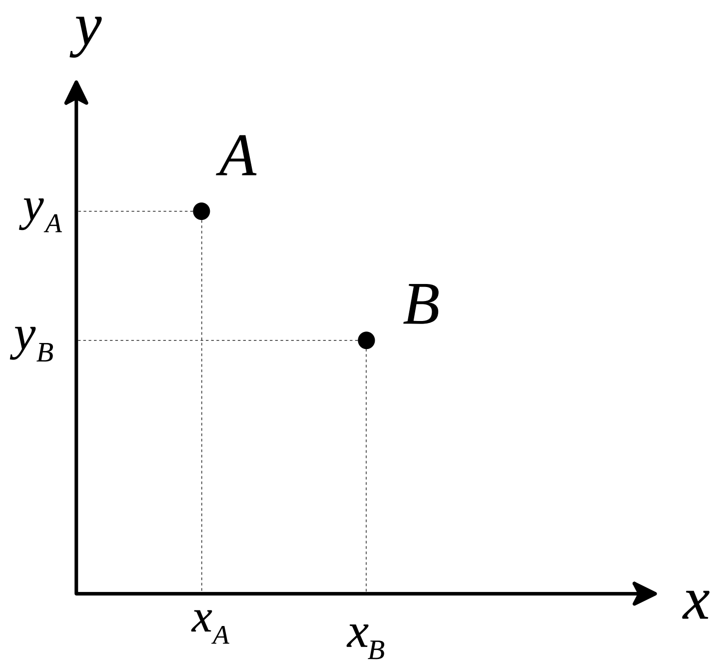
Definition. $L >_{SSD} M$ if M can be obtained from L by a finite sequence of mean-preserving spreads, that is, if there is a sequence of money lotteries $\langle L_1, L_2, \ldots, L_m \rangle$ (with $m \geq 2$) such that:

(1) $L_1 = L,$
(2) $L_m = M$
(3) for every $i = 1, \ldots, m-1,\ L_i \rightarrow_{MPS} L_{i+1}$

Theorem. $L >_{SSD} M$ if and only if $\mathbb{E}[U(L)] > \mathbb{E}[U(M)]$ for every strictly increasing and strictly concave utility function U.

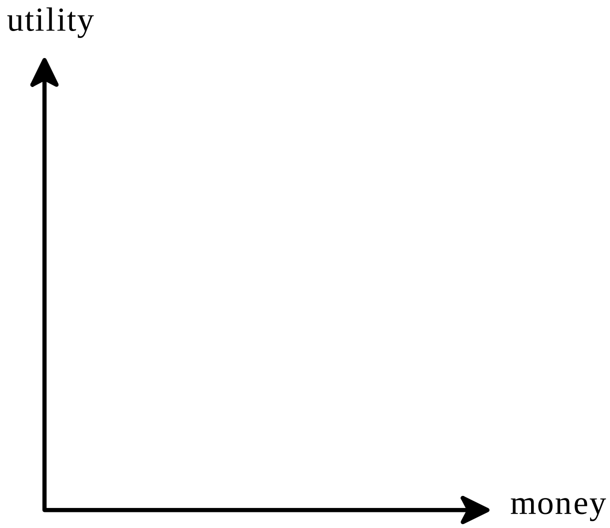
BINARY LOTTERIES

Lotteries of the form $\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$ with p fixed and x and y allowed to vary.

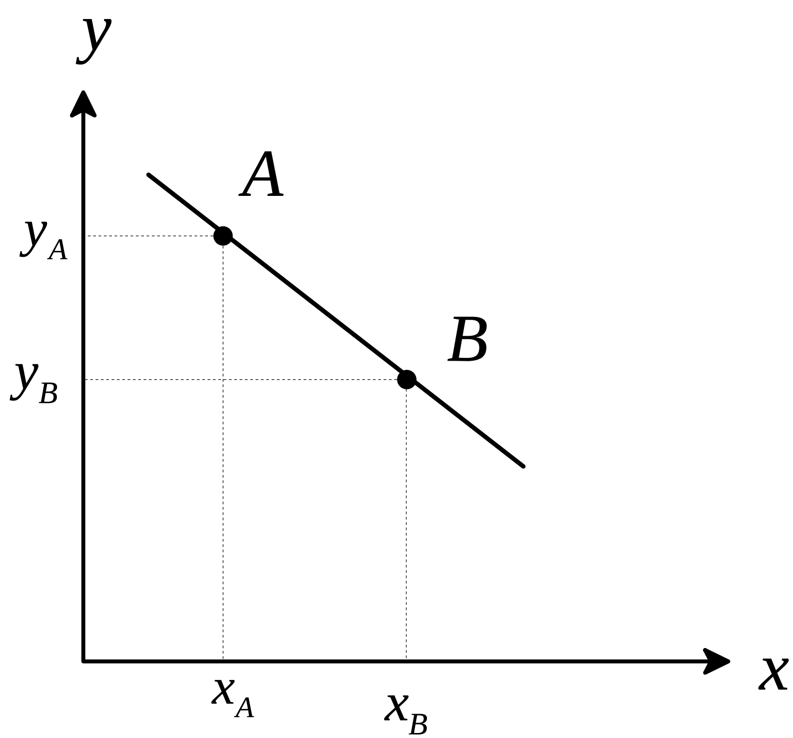


We want to draw indifference curves in this diagram.

Case 1: risk-neutral agent

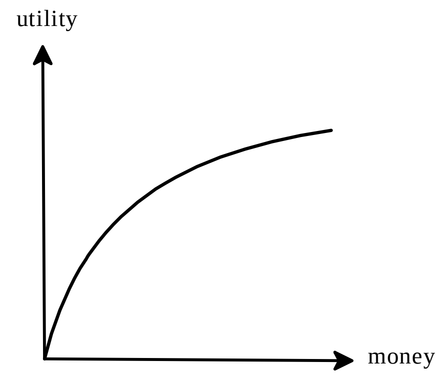


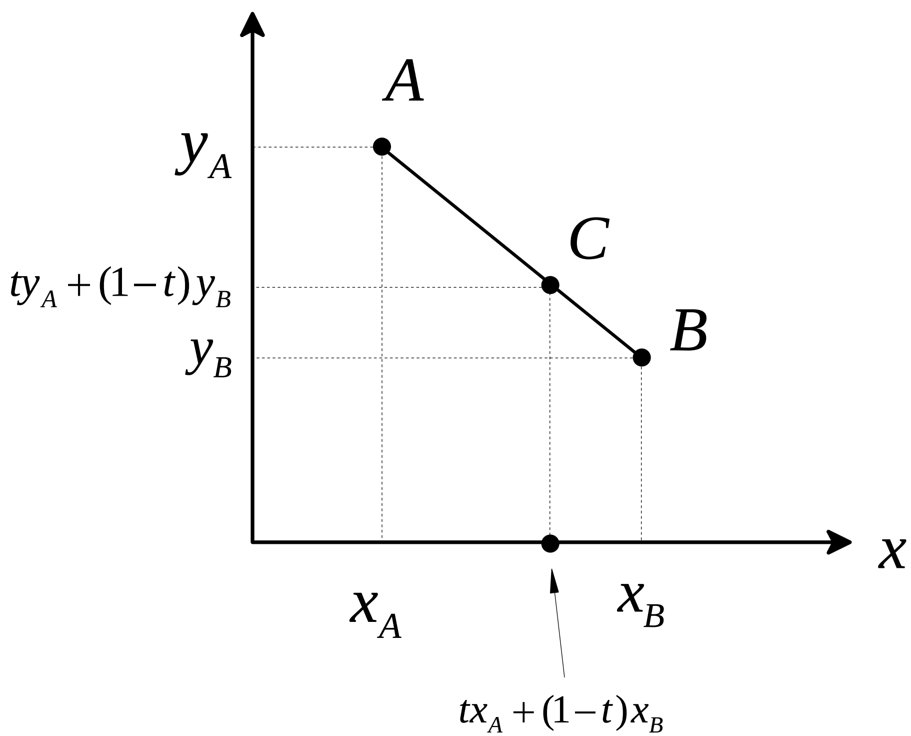
Let A and B be such that $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$:


y
A
y_A
B
y_B
x
x_A
x_B

Case 2: risk-averse agent

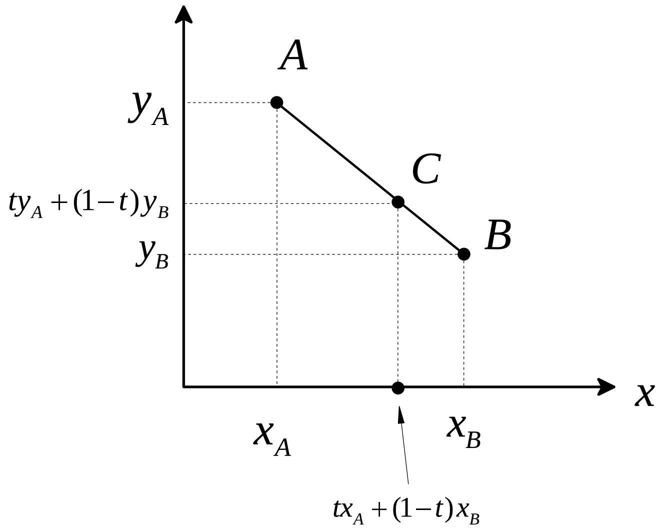
$U(m)$ is strictly concave:





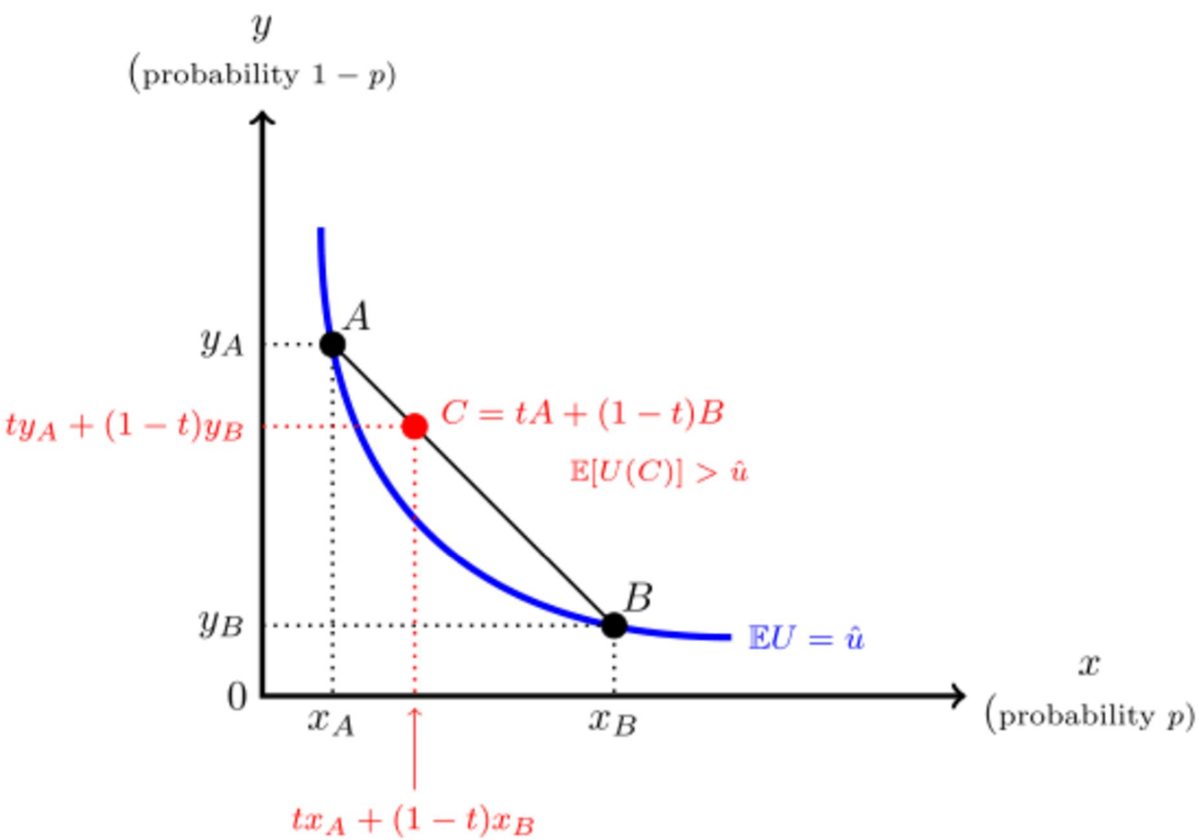
Suppose that $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$

Want to show that
$\mathbb{E}[U(C)] > \mathbb{E}[U(A)](= \mathbb{E}[U(B)])$

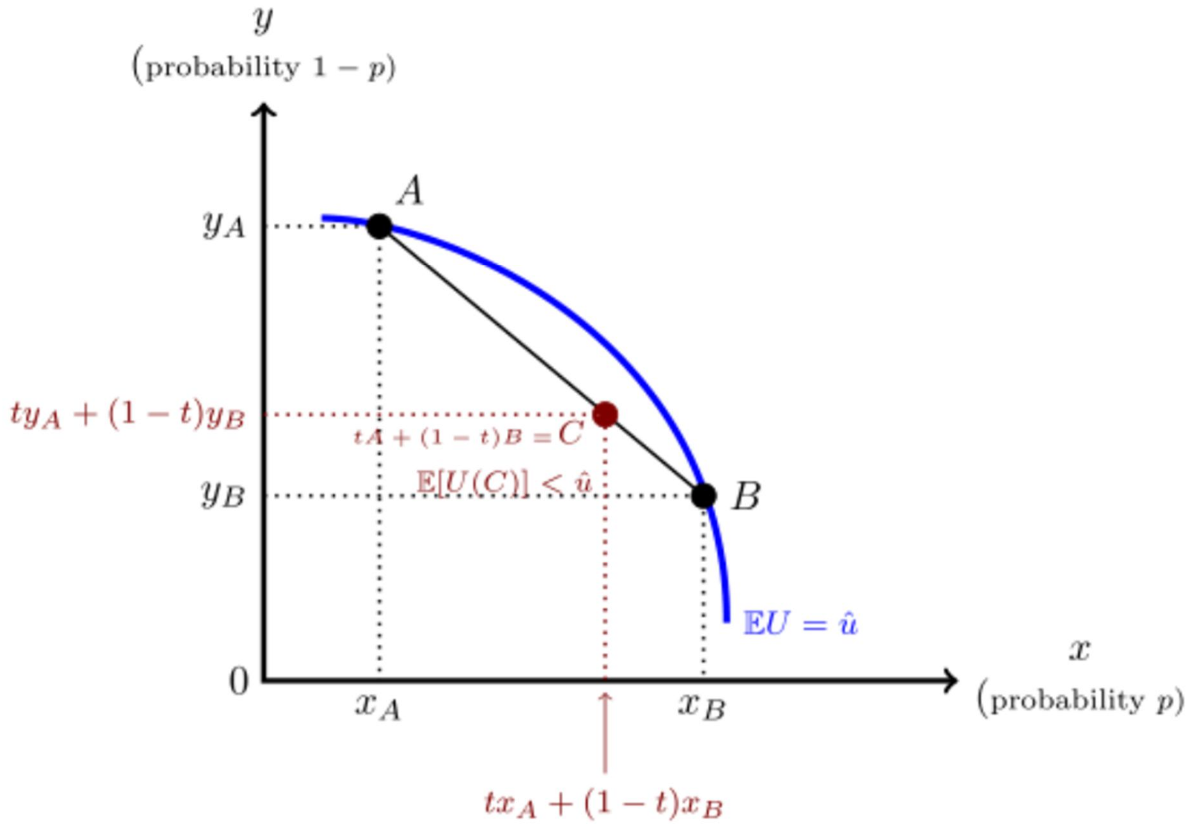


$\mathbb{E}[U(C)] =$

The indifference curve must lie below the straight-line segment joining A and B.


y
(probability $1-p$)
y_A
A
$ty_A + (1-t)y_B$
$C = tA + (1-t)B$
$\mathbb{E}[U(C)] > \hat{u}$
B
y_B
$\mathbb{E}U = \hat{u}$
x
(probability p)
0
x_A
x_B
$tx_A + (1-t)x_B$

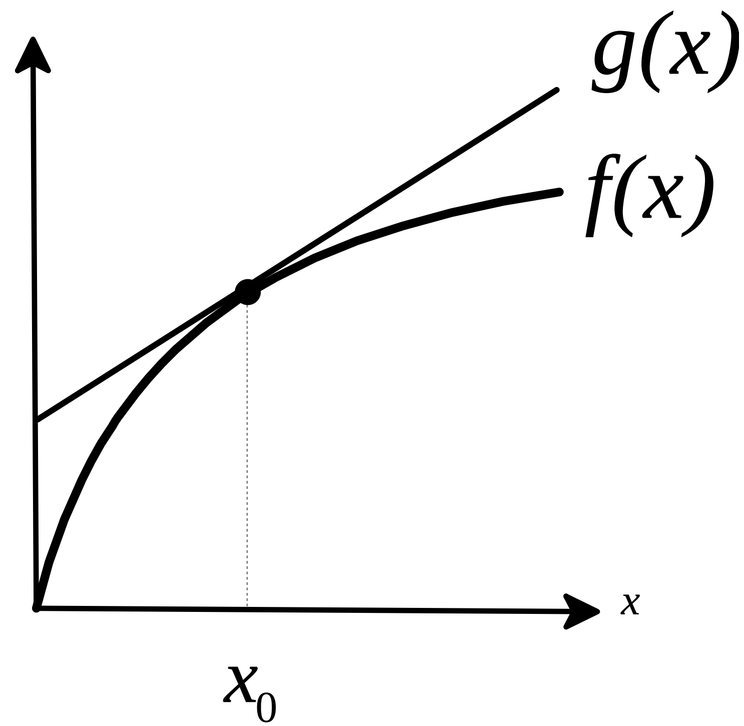
Case 2: risk-loving agent



Slope of an indifference curve

Preliminaries on the meaning of the derivative.

$f(x) = \sqrt{x} + \frac{x^2}{3}$. Then $f'(x) =$ The derivative is used to construct a linear function to approximate the function $f(x)$ at a point x_0:



$f(x)=\sqrt{x}+\frac{x^2}{3}$. Then $f'(x)=\frac{1}{2\sqrt{x}}+\frac{2x}{3}$. Let $x_0=9$.

$f(9)=$ and $f'(9)=$

so that $g(x)=$ Let's see how well g approximates f

Take x = 9.1. Then $f(9.1)=$ $g(9.1)=$

Take x = 12. Then $f(12)=$ $g(12)=$

END OF PRELIMINARIES

Slope of indifference curve

Let A and B be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$,

(*)

- Since x_B is close to x_A, $U(x_B) \simeq$
- Since y_B is close to y_A, $U(y_B) \simeq$

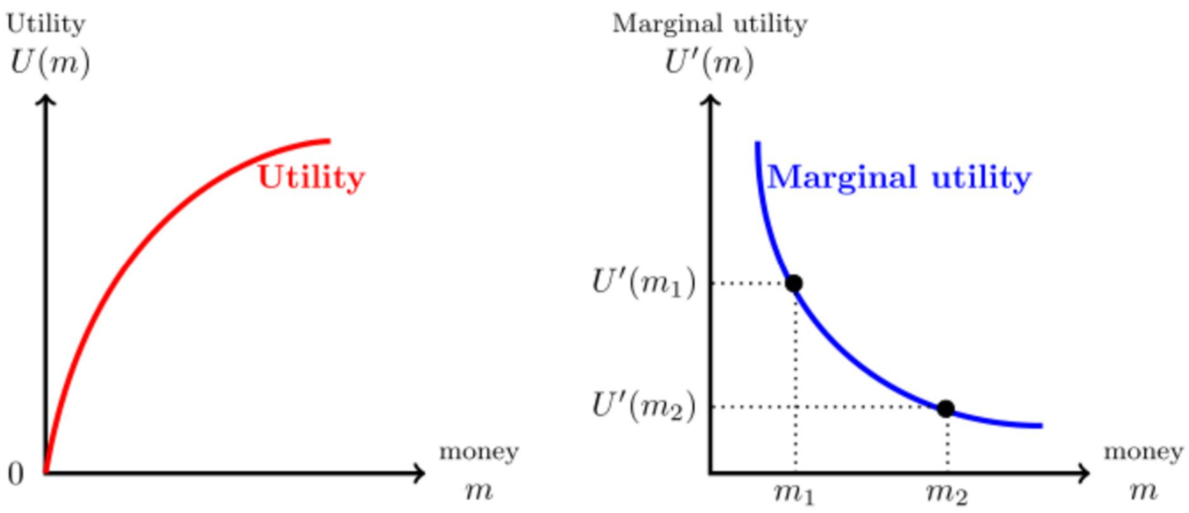
Thus the RHS of (*) can be written as

So (*) becomes

that is, which can be written as

Comparing the slope at a point with the ratio $\frac{p}{1-p}$

Look at the case of risk aversion but the other cases are similar.



- at a point **above** the 45° line, where $x < y$,

- at a point **on** the 45° line, where $x = y$,

- at a point **below** the 45° line, where $x > y$,

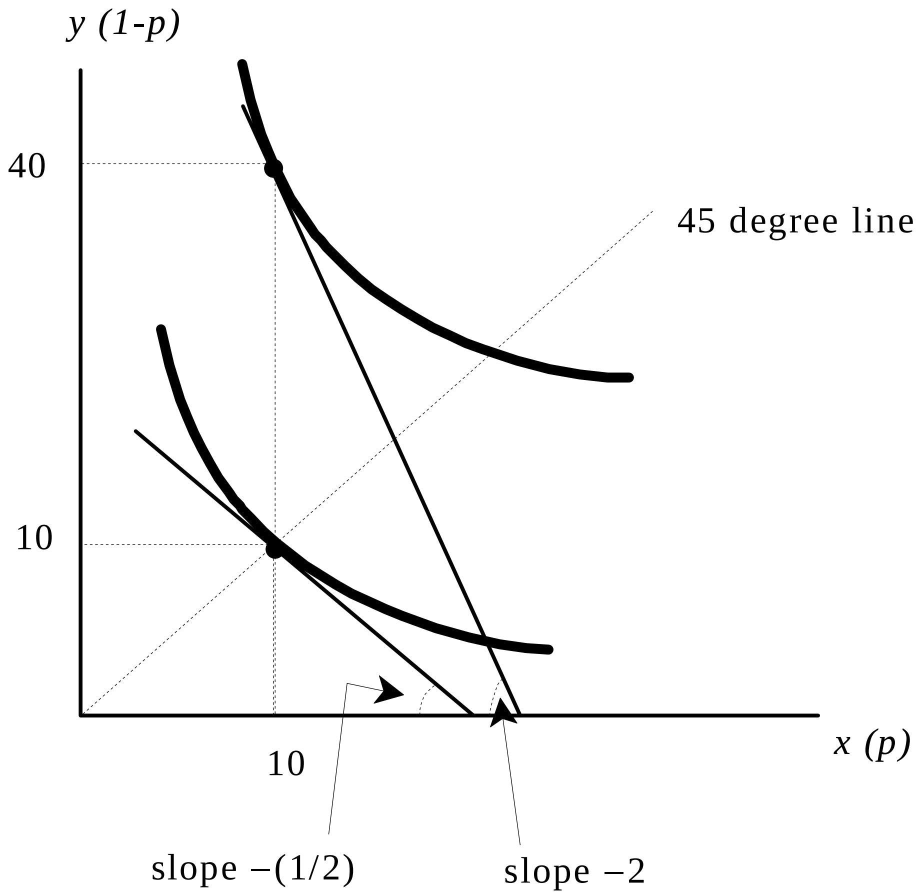
Example. $U(m) = \ln(m)$, $p = \frac{1}{3}$. What is the slope of the indifference curve at points A = (10,40) and B = (10,10)?

The expected utility of lottery $A = \begin{pmatrix} 10 & 40 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

The slope of the indifference curve at point A is equal to .

The expected utility of lottery $B = \begin{pmatrix} 10 & 10 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

The slope of the indifference curve at point B is equal to .



$A = (10, 40)$

$B = (10,10)$

$p = \frac{1}{3}$

Thus $-\frac{p}{1-p} = -\frac{1}{2}$