
 HOMEWORK # 5 ANSWERS

- (a) It is a Social Choice Function: its output is a single alternative not a ranking of the four alternatives.
- (b) (b.1) There are 24 ways of ordering four alternatives, thus 24 possible rankings that voter 3 could report; hence 24 tables. (b.2) 24 rows and 24 columns.
- (c) (c.1) Yes: if everybody ranks the same alternative at the top then that alternative gets the highest score (namely, 12) and is thus chosen (every other alternative gets less than 12 points).
 (c.2) Yes: if one person ranks x as his top alternative then x is not necessarily chosen (e.g. in the case where the others rank a different alternative y as their top alternative and rank x as their worst alternative: in that case x gets 6 points, while y gets at least 9 points).
- (d) Now each voter has 6 possible rankings that he can report and thus we have 6 tables, each with 6 rows and 6 columns, for a total of $6 \times 6 \times 6 = 216$ cells. For each cell there are three choices (a , b or c). Hence there are 3^{216} possible ways to the appropriate tables: a huge number! ($3^{216} = 1,143.38 \times 10^{100}$).
- (e) (e.1) a gets 7 points, b gets 5 points and c gets 6 points; thus $f(abc, acb, cba) = a$.
 (e.2) a gets 6 points, b gets 6 points and c gets 6 points; thus, according to the tie-breaking rule, $f(bac, acb, cba) = a$.
 (e.3) a gets 4 points, b gets 8 points and c gets 6 points; thus, $f(bac, bca, cba) = b$.
- (f) Use case (e.2) above: the chosen alternative is a , which is 3's worst (assuming that cba is her true ranking). If voter 3 instead reported bca then a would get 6 points, b would get 7 points and c would get 5 points, so that b would be chosen (that is, $f(bac, acb, bca) = b$); since according to her true ranking cba she prefers b to a , she is better off lying (by reporting the false ranking bca instead of the true ranking cba).