University of California, Davis -- Department of Economics

ECON 106 : DECISION MAKING Professor Giacomo Bonanno

WINTER 2024 - FINAL EXAM **Version 1**

Answer all questions. **If you don't explain (= show your work for) your answers you will get no credit.**

NAME:________________________________ University ID:___________________

CIRCLE THE NAME OF YOUR TA: **Kalyani Chaudhuri** or **Joaquin Paleo**

If you don't know the name of your TA, then circle your Section:

A01, Tuesday 5-6 **A02, Tuesday 6-7** **A03, Tuesday 7-8** **A04, Tuesday 8-9**

- **By writing your name on this exam you certify that you have not violated the University's Code of Academic Contact** (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).
- **If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.**
- **If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.**

1. [20 points] Consider the following social preference function. Let W be the set of alternatives. Each voter $i \in \{1,2,...,n\}$ submits a complete and transitive "at least as good" relation R_i on W (thus indifference is allowed). Then the following procedure is applied. Fix an individual i and construct for every alternative $w \in W$ a number $B_i(w)$ as follows: $B_i(w)$ is the number of alternatives that individual i considers **worse than** w (according to the reported relation R_i). For example, if there are four alternatives: $W = \{w_1, w_2, w_3, w_4\}$ and individual 2 reports the following relation: $w_3 \succ_2 w_1 \sim_2 w_4 \succ_2 w_2$, then $B_2(w_1) = B_2(w_4) = 1$, $B_2(w_2) = 0$, $B_2(w_3) = 3$. Then, for every alternative $w \in W$, the number $B(w)$ is constructed by taking the sum of the numbers for each voter, that is, $B(w) = B_1(w) + B_2(w) + ... + B_n(w)$. Finally, for any two alternatives $w, w' \in W$, w is declared to be at least as good for society as w' if and only if $B(w) \geq B(w')$.

(a) **(a.1)** [6 points] Apply the above social preference function to the following situation: $n = 3$, $W = \{a,b,c,d,e\}$, R_1 is given by $a \sim_1 b \succ_1 c \sim_1 d \succ_1 e$, R_2 is given by $e \succ_2 d \succ_2 c \sim_2 a \succ_2 b$ and R_3 is given by $c \succ_3 a \succ_3 d \sim_3 e \succ_3 b$. What is the social preference relation?

(a.2) [6 points] Suppose now that alternative a becomes unavailable so that the set of alternatives becomes $\{b,c,d,e\}$. Apply again the above social preference function to determine the social ranking of $\{b,c,d,e\}$.

(b) For each of the following axioms write one of the following:

- **N/A** if it is not one of Arrow's axioms.
- **YES** if it is one of Arrow's axioms and the procedure described above satisfies it.
- **NO** if it is one of Arrow's axioms and the procedure described above does **not** satisfy it.

(b.1) [1 point] **Additivity.**

(b.2) [1 point] **Unanimity.**

(b.3) [1 point] **Non-manipulability.**

(b.4) [1 point] **Anonymity.**

(b.5) [1 point] **Completeness and transitivity (or Rationality).**

(b.6) [1 point] **Non-dictatorship.**

(b.7) [1 point] **Order independence.**

(b.8) [1 point] **Independence of irrelevant alternatives.**

2. [26 points] Ann has a crush on Brad. She is thinking whether she should tell him (call this choice *T*) or not tell him (call this choice *NT*). She thinks that there are three possibilities concerning Brad:

- s_1 : he is also interested in her,
- s_2 : he is not interested in her but he is a kind person,
- s_3 : he is not interested in her and he is unkind.

Thus, Ann thinks that, if she confesses her feelings to Brad, there are three possible outcomes:

- z_1 : Brad welcomes Ann's romantic confession.
- z_2 : Brad rejects Ann, but in a kind and gentle way.
- z_3 : Brad rejects Ann in a hurtful way.

Another possible outcome, namely the *status quo,* is the one that arises if Ann chooses *NT* (in which case, of course, she does not learn anything about Brad's type). Call this outcome z_4.

Ann's ranking of the outcomes is $z_1 \succ z_2 \succ z_4 \succ z_3$. Ann has vNM preferences over lotteries involving these outcomes.

(a) [4 points] Find the MaxiMin solution for Ann's decision problem, without attempting to construct her utility function.

(b) [10 points] Let $H(x, p)$ be the Hurwicz index of act *x* when the degree of **pessimism** is *p*. Given the following information, calculate Ann's normalized vNM utility function: (1) $H(T, \frac{3}{5}) = H(NT, \frac{3}{5})$, (2) Ann is indifferent between the following two lotteries: $\begin{pmatrix} z_2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_3 & z_4 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$

(c) [6 points] Using the vNM utility function of part (b), construct the regret table and find the MinMax Regret solution.

(d) [3 points] If Ann believes that all three possibilities s_1, s_2 and s_3 are equally likely, and she makes her decision based on expected utility, what will she do?

(e) [3 points] If Ann believes that the probability of s_1 is 25% and the other two possibilities are equally likely, and she makes her decision based on expected utility, what will she do?

3. [34 points] There are three alternatives: *a*, *b* and *c* and seven voters, with the following preferences:

Voters 1, 2 and 3:	$a \succ b \succ c$
Voters 4 and 5:	$b \succ c \succ a$
Voter 6:	$b \succ a \succ c$
Voter 7:	$c \succ a \succ b$

(a) [6 points] Suppose that the Borda count is used as a social **preference** function and that there is sincere voting. What is the outcome?

Voters 1, 2 and 3:	$a \succ b \succ c$
Voters 4 and 5:	$b \succ c \succ a$
Voter 6:	$b \succ a \succ c$
Voter 7:	$c \succ a \succ b$

(b) [12 points] Suppose that the Kemeny-Young method is used as a social **preference** function and that there is sincere voting. What is the outcome? [Consider only strict rankings.]

Voters 1, 2 and 3:	$a \succ b \succ c$
Voters 4 and 5:	$b \succ c \succ a$
Voter 6:	$b \succ a \succ c$
Voter 7:	$c \succ a \succ b$

(c) [8 points] Suppose now that the Borda count is used as a social **choice** function, with the rule that if two or more alternatives get the largest number of points, then the one that comes **first** in alphabetical order is chosen. Given the above preferences, if everybody else votes sincerely, can any one individual gain by voting strategically? If No, explain why not, if Yes explain who and how.

(d) [8 points] Continue to assume that the Borda count is used as a social **choice** function, but now the rule is that if two or more alternatives get the largest number of points, then the one that comes **last** in alphabetical order is chosen. Given the above preferences, if everybody else votes sincerely, can any one individual gain by voting strategically?

4. [20 points] You are thinking about investing your savings in the stock market. This is a good idea if the Dow Jones index will go up (*U*) and a bad idea if it will go down (*D*). Assume throughout that the index cannot remain constant: it will either go up or down. At the moment you attach probability 80% to *U* and 20% to *D*. You can consult an expert who will either tell you that she is confident that the index will go up ($\uparrow$) or that she is confident that the index will go down ($\downarrow$). You looked at her past performance and saw that of all the cases where the index actually went **up** (*U*), her prediction was correct (that is, it was $\uparrow$) 7 times out of 8 (that is, 87.5% of the time) and of all the cases where the index actually went **down** (*D*), her prediction was correct (that is, it was $\downarrow$) 5 times out of 8 (that is, 62.5% of the time).

(a) [4 points] What is the probability that, if you consult her, she will tell you $\downarrow$ (that is, that she is confident that the index will go down)?

(b) [4 points] Suppose that you have consulted her and she tells you $\uparrow$ (that is, that she is confident that the index will go **up**). What is the probability that she is correct, that is, that the index will actually go **up** (*U*)?

(c) [4 points] Suppose that you have consulted her and she tells you $\downarrow$ (that is, that she is confident that the index will go **down**). What is the probability that she is wrong, that is, that the index will actually go **up** (U)?

(d) [8 points] Suppose that the number of past cases you looked at was 2,000. Draw a table that classifies these 2,000 cases in terms of U, D, $\uparrow$ and $\downarrow$ (two rows, labeled $\uparrow$ and $\downarrow$, and two columns, labeled U and D).