ECN 106 : Decision Making Professor Giacomo Bonanno

WINTER 2024 - FINAL EXAM: $\boxed{\textbf{ANSWERS for VERSION 1}}$

1. **(a.1)** For Voter 1: $B_1(a) = B_1(b) = 3,\ B_1(c) = B_1(d) = 1,\ B_1(e) = 0$

For Voter 2: $B_2(a) = B_2(c) = 1,\ B_2(b) = 0,\ B_2(d) = 3,\ B_2(e) = 4$

Voter 3: $B_3(a) = 3,\ B_3(b) = 0,\ B_3(c) = 4,\ B_3(d) = B_3(e) = 1$

Thus $B(a) = 7,\ B(b) = 3,\ B(c) = 6,\ B(d) = 5,\ B(e) = 5$ so that the social ranking is $a \succ_S c \succ_S d \sim_S e \succ_S b$

(a.2) The rankings are: $b \succ_1 c \sim_1 d \succ_1 e$, $e \succ_2 d \succ_2 c \succ_2 b$ and $c \succ_3 d \sim_3 e \succ_3 b$. Thus

$B_1(b) = 3,\ B_1(c) = B_1(d) = 1,\ B_1(e) = 0$

$B_2(b) = 0,\ B_2(c) = 1,\ B_2(d) = 2,\ B_2(e) = 3$

$B_3(b) = 0,\ B_3(c) = 3,\ B_3(d) = B_3(e) = 1$

Thus $B(b) = 3,\ B(c) = 5,\ B(d) = 4,\ B(e) = 4$ so that the social ranking is $c \succ_S d \sim_S e \succ_S b$.

(b.1) Additivity: N/A **(b.2) Unanimity: YES** **(b.3) Non-manipulability: N/A**

(b.4) Anonymity: N/A **(b.5) Completeness and transitivity (or Rationality) : YES**

(b.6) Non-dictatorship: YES **(b.7) Order independence: N/A**

(b.8) Independence of irrelevant alternatives: NO.

[The **Independence of Irrelevant** alternatives is not satisfied. This can be established in two ways: (1) by appealing to Arrow's theorem, since the remaining axioms are satisfied, or (2) by noting that when individual preferences are strict, the above rule coincides with the Borda rule and we know that the Borda rule violates IIA.]

2. **(a)** The worst outcome with *T* is z_3 while *NT* gives outcome z_4 for sure. Thus the MaxiMin solution is *NT*.

(b) Set $U(z_1) = 1$ and $U(z_3) = 0$. For every p, $H(NT, p) = U(z_4)$. $H(T, \frac{3}{5}) = \frac{3}{5}U(z_3) + \frac{2}{5}U(z_1) = \frac{2}{5}$.

Thus $U(z_4) = \frac{2}{5}$. The expected utility of $\begin{pmatrix} z_1 & z_3 & z_4 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$ is $\frac{1}{4}\times 1 + \frac{1}{4}\times 0 + \frac{1}{2}\times\frac{2}{5} = \frac{9}{20}$. Hence

$U(z_2) = \frac{9}{20}$. Thus the normalized utility function is $\begin{matrix} z_1 & z_2 & z_3 & z_4 \\ 1 & \frac{9}{20} & 0 & \frac{2}{5} \end{matrix}$.

(c) The utility table is

	s_1	s_2	s_3
T	1	$\frac{9}{20}$	0
NT	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

so that the regret table is

	s_1	s_2	s_3
T	0	0	$\frac{2}{5}$
NT	$\frac{3}{5}$	$\frac{9}{20} - \frac{2}{5} = \frac{1}{20}$	0

.

Thus the MinMax Regret solution is *T*.

(d) $\mathbb{E}[U(T)] = \frac{1}{3}\times 1 + \frac{1}{3}\times\frac{9}{20} + \frac{1}{3}\times 0 = \frac{29}{60} = 0.4833$ and $\mathbb{E}[U(NT)] = \frac{2}{5} = 0.4$. Thus she will choose *T*.

(e) $\mathbb{E}[U(T)] = \frac{1}{4}\times 1 + \frac{3}{8}\times\frac{9}{20} + \frac{3}{8}\times 0 = \frac{67}{160} = 0.4187$ and $\mathbb{E}[U(NT)] = \frac{2}{5} = 0.4$. Thus she will choose *T*.

3. **(a)** *a* gets 15 points, *b* gets 16 and *c* gets 11. Thus the outcome is the social ranking $b \succ a \succ c$.

(b) $\#(a \succ b) = 4, \#(b \succ a) = 3, \#(a \succ c) = 4, \#(c \succ a) = 3, \#(b \succ c) = 6, \#(c \succ b) = 1$. Thus the scores are: $a \succ b \succ c: 4+6+4 = 14$, $a \succ c \succ b: 4+1+4 = 9$, $b \succ a \succ c: 3+4+6 = 13$, $b \succ c \succ a: 6+3+3 = 12$, $c \succ a \succ b: 3+4+1 = 8$, $c \succ b \succ a: 1+3+3 = 7$. Thus the outcome is that the social ranking is $a \succ b \succ c$

(c) Based on the calculations of part (a), with sincere voting the selected alternative is *b*. Consider voter 1 (alternatively, voter 2 or voter 3). If the others vote sincerely, the votes of 2-7 give 12 points to *a*, 14 to *b* and 10 to *c*. If 1 votes sincerely, the chosen alternative is *b*. If he instead reports the preferences $a \succ c \succ b$ then *a* gets 15 points, *b* gets 15 points and *c* gets 12 points. Thus there is a tie between *a* and *b* and, by the given tie-breaking rule, *a* is chosen. Thus 1 (or 2 or 3) can gain by voting strategically.

(d) With sincere voting, voters 4, 5 and 6 get their best outcome, so they cannot gain by lying. Each of voters 1, 2 and 3 by lying can at most decrease the score of *b* by 1 point and thus create a tie with *a*, but then – given the tie-breaking rule – *b* would still be chosen. This leaves us with Voter 7. The best she can do by lying is to increase the score of *a* by 1 point and thus create a tie with *a*, but then – given the tie-breaking rule – *b* would still be chosen. Thus no individual voter can gain by lying.

4. The information provided in the question is: $P(U) = \frac{4}{5}$, $P(D) = \frac{1}{5}$, $P(\uparrow|U) = \frac{7}{8}$, so that $P(\downarrow|U) = \frac{1}{8}$, and $P(\downarrow|D) = \frac{5}{8}$, so that $P(\uparrow|D) = \frac{3}{8}$.

(a) $P(\downarrow) = P(\downarrow|U)P(U) + P(\downarrow|D)P(D) = \frac{1}{8}\times\frac{4}{5} + \frac{5}{8}\times\frac{1}{5} = \frac{9}{40} = 22.5\%$.

It follows that $P(\uparrow) = \frac{31}{40} = 77.5\%$.

(b) $P(U|\uparrow) = \dfrac{P(\uparrow|U)P(U)}{P(\uparrow)} = \dfrac{\frac{7}{8}\times\frac{4}{5}}{\frac{31}{40}} = \dfrac{28}{31} = 90.32\%$.

(c) $P(U|\downarrow) = \dfrac{P(\downarrow|U)P(U)}{P(\downarrow)} = \dfrac{\frac{1}{8}\times\frac{4}{5}}{\frac{9}{40}} = \dfrac{4}{9} = 44.44\%$.

(d)

	U	D
$\uparrow$	1,400	150
$\downarrow$	200	250