WINTER 2024 - FINAL EXAM: ANSWERS for VERSION 2

1. **(a)** a gets 15 points, b gets 16 and c gets 11. Thus, with sincere voting, outcome b is selected. Consider voter 7 (alternatively, voter 5 or 6). If the others vote sincerely, the votes of 1-6 give 12 points to a, 14 to b and 10 to c. If 7 votes sincerely, the chosen alternative is b. If, instead, he reports the preferences $a \succ c \succ b$ then a gets 15 points, b gets 15 points and c gets 12 points. Thus there is a tie between a and b and, by the given tie-breaking rule, a is chosen. Thus 7 (or 5 or 6) can gain by voting strategically.

(b) With sincere voting, voters 1, 3 and 4 are getting their best alternative (namely b) and thus have nothing to gain from misrepresenting their preferences. Each of voters 5-7 can at most create a tie between a and b, but - given the assumed tie-breaking rule - the chosen alternative would still be b. Now consider Voter 2. If the others vote sincerely, the votes of the others give 13 points to a, 15 to b and 8 to c. If 2 votes sincerely, the chosen alternative is b. If he reports any other ranking, b will get at least 16 points, a at most 16 points and c at most 12. Thus, given the tie-breaking rule, b will always be chosen. Hence nobody can gain by voting strategically.

(c) By the calculations of part (a), the outcome is the social ranking $b \succ a \succ c$.

(d) $\#(a \succ b) = 4, \#(b \succ a) = 3, \#(a \succ c) = 4, \#(c \succ a) = 3, \#(b \succ c) = 6, \#(c \succ b) = 1$. Thus the scores are: $a \succ b \succ c: 4+6+4=14$, $a \succ c \succ b: 4+1+4=9$, $b \succ a \succ c: 3+4+6=13$, $b \succ c \succ a: 6+3+3=12$, $c \succ a \succ b: 3+4+1=8$, $c \succ b \succ a: 1+3+3=7$. Thus the outcome is that the social ranking is $a \succ b \succ c$.

2. The information provided in the question is: $P(U) = \frac{80}{100}$, $P(D) = \frac{20}{100}$, $P(+|U) = \frac{75}{100}$ and $P(-|D) = \frac{60}{100}$ so that $P(+|D) = \frac{40}{100}$.

(a) $P(+) = P(+|U)P(U) + P(+|D)P(D) = \frac{75}{100} \times \frac{80}{100} + \frac{40}{100} \times \frac{20}{100} = \frac{68}{100} = 68\%$.
It follows that $P(-) = 32\%$.

(b) $P(U|+) = \dfrac{P(+|U)P(U)}{P(+)} = \dfrac{\frac{75}{100} \times \frac{80}{100}}{\frac{68}{100}} = \dfrac{15}{17} = 88.24\%$.

(c) $P(D|-) = \dfrac{P(-|D)P(D)}{P(-)} = \dfrac{\frac{60}{100} \times \frac{20}{100}}{\frac{32}{100}} = \dfrac{3}{8} = 37.5\%$.

(d) The table is as follows:

	U	***D***
+	**600**	**80**
-	**200**	**120**

3. **(a.1)** For Voter 1: $B_1(a) = B_1(b) = 3,\ B_1(c) = B_1(d) = 1,\ B_1(e) = 0$

For Voter 2: $B_2(a) = B_2(c) = 1,\ B_2(b) = 0,\ B_2(d) = 3,\ B_2(e) = 4$

Voter 3: $B_3(a) = 3,\ B_3(b) = 0,\ B_3(c) = 4,\ B_3(d) = B_3(e) = 1$

Thus $B(a) = 7,\ B(b) = 3,\ B(c) = 6,\ B(d) = 5,\ B(e) = 5$ so that the social ranking is $a \succ_S c \succ_S d \sim_S e \succ_S b$.

(a.2) The rankings are: $b \succ_1 c \sim_1 d \succ_1 e$, $e \succ_2 d \succ_2 c \succ_2 b$ and $c \succ_3 d \sim_3 e \succ_3 b$. Thus

$B_1(b) = 3,\ B_1(c) = B_1(d) = 1,\ B_1(e) = 0$

$B_2(b) = 0,\ B_2(c) = 1,\ B_2(d) = 2,\ B_2(e) = 3$

$B_3(b) = 0,\ B_3(c) = 3,\ B_3(d) = B_3(e) = 1$

Thus $B(b) = 3,\ B(c) = 5,\ B(d) = 4,\ B(e) = 4$ so that the social ranking is $c \succ_S d \sim_S e \succ_S b$.

(b.1) Additivity: N/A **(b.2) Unanimity: YES** **(b.3) Non-manipulability: N/A**

(b.4) Unrestricted domain (or Freedom of expression): YES **(b.5) Completeness and transitivity (or Rationality) : YES** **(b.6) Non-dictatorship: YES**

(b.7) Order independence: N/A **(b.8) Independence of irrelevant alternatives: NO.**

[The **Independence of Irrelevant** alternatives is not satisfied. This can be established in two ways: (1) by appealing to Arrow's theorem, since the remaining axioms are satisfied, or (2) by noting that when individual preferences are strict, the above rule coincides with the Borda rule and we know that the Borda rule violates IIA.]

4. **(a)** The worst outcome with *T* is w_1 while *NT* gives outcome w_4 for sure. Thus the MaxiMin solution is *NT*.

(b) Set $U(w_1) = 0$ and $U(w_3) = 1$. For every *p*, $H(NT, p) = U(w_4)$.

$H(T, \tfrac{5}{8}) = \tfrac{5}{8}U(w_1) + \tfrac{3}{8}U(w_3) = \tfrac{3}{8}$. Thus $U(w_4) = \tfrac{3}{8}$. The expected utility of $\begin{pmatrix} w_1 & w_3 & w_4 \\ \tfrac{1}{2} & \tfrac{1}{4} & \tfrac{1}{4} \end{pmatrix}$ is $\tfrac{1}{2}\times 0 + \tfrac{1}{4}\times 1 + \tfrac{1}{4}\times\tfrac{3}{8} = \tfrac{11}{32}$. Hence $U(w_2) = \tfrac{11}{32}$. Thus the normalized utility function is $\begin{matrix} w_1 & w_2 & w_3 & w_4 \\ 0 & \tfrac{11}{32} & 1 & \tfrac{3}{8} \end{matrix}$.

(c) The utility table is

	s_1	s_2	s_3
T	1	$\tfrac{11}{32}$	0
NT	$\tfrac{3}{8}$	$\tfrac{3}{8}$	$\tfrac{3}{8}$

so that the regret table is

	s_1	s_2	s_3
T	0	$\tfrac{3}{8} - \tfrac{11}{32} = \tfrac{1}{32}$	$\tfrac{3}{8}$
NT	$\tfrac{5}{8}$	0	0

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Thus the MinMax Regret solution is *T*.

(d) $\mathbb{E}[U(T)] = \tfrac{1}{5}\times 1 + \tfrac{2}{5}\times\tfrac{11}{32} + \tfrac{2}{5}\times 0 = \tfrac{27}{80} = 0.3375$ and $\mathbb{E}[U(NT)] = \tfrac{3}{8} = 0.375$. Thus she will choose *NT*.

(e) $\mathbb{E}[U(T)] = \tfrac{1}{3}\times 1 + \tfrac{1}{3}\times\tfrac{11}{32} + \tfrac{1}{3}\times 0 = \tfrac{43}{96} = 0.448$ and $\mathbb{E}[U(NT)] = \tfrac{2}{5} = 0.4$. Thus she will choose *T*.