Department of Economics, University of California, Davis

Ecn 106 – Decision Making – Professor Giacomo Bonanno

HOMEWORK # 5 (for due date see web page)

There are four alternatives: *a, b, c, d* and three voters: 1, 2 and 3. Consider the following voting procedure: (1) each voter is asked to submit a strict ranking of the alternatives (strict means that indifference is not allowed), (2) the Borda rule is then used to calculate the score of each alternative and (3) the alternative with the largest score is chosen; if two or more alternatives have the largest score, then the one that comes first in alphabetical order is chosen (for example, if *a* and *c* get the largest score then *a* is chosen).

(a) Is this a Social Preference Function in the sense of Arrow or a Social Choice Function?

(b) We can represent this voting procedure using tables as follows: each table has as many rows as the number of rankings that Voter 1 can submit (each row is labeled with one such ranking) and as many columns as the number of rankings that Voter 2 can submit (each column is labeled with one such ranking); furthermore we have as many tables as the number of rankings that Voter 3 can submit (each table is labeled with one such ranking).
(b.1) If you represent this voting procedure using tables, as explained above, how many tables would you need?
(b.2) How many rows and how many columns would each table have?

(c) **(c.1)** Does this voting procedure satisfy Unanimity? [Explain your answer]
(c.2) Does this voting procedure satisfy Non-Dictatorship? [Explain your answer]

For the following questions let us simplify the problem by having only three alternatives: *a, b* and *c* (and still three voters).

(d) How many ways are there to fill in all the cells of all the tables? [The tables are constructed as explained above.]

(e) Write a strict ordering as follows: (*xyz*) and interpret it to mean that *x* is better than *y* and *y* is better that *z* (where *x*, *y* and *z* are place-holders for the three possible alternatives *a, b* and *c*) Given three orderings $(x_1y_1z_1),(x_2y_2z_2)$ and $(x_3y_3z_3)$ (where the first one is interpreted as the reported ranking of voter 1, the second as the reported ranking of voter 2 and the third as the reported ranking of voter 3), denote by $f\left(x_1y_1z_1, x_2y_2z_2, x_3y_3z_3\right)$ the alternative that is chosen (according to the rules given above). Calculate (and show your calculations) the following:
(e.1) $f\left(abc, acb, cba\right)$, **(e.2)** $f\left(bac, acb, cba\right)$, **(e.3)** $f\left(bac, bca, cba\right)$.

(f) Show that this voting procedure is manipulable (or not strategy-proof).