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|-------------|----------------|---------------|---------------|---------------|---------------|
| probability | $\frac{1}{12}$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{6}$ |
| state → | s_1 | s_2 | s_3 | s_4 | s_5 |
| act ↓ | | | | | |

1. First convert the outcomes into utilities:

| | | | | | |
|-----|---|---|---|---|---|
| a | 3 | 4 | 1 | 8 | 6 |
| b | 6 | 2 | 4 | 5 | 8 |
| c | 2 | 3 | 0 | 7 | 5 |

(a) Since act c is strictly dominated by act a , we only need to compute the expected utility of a and the expected utility of b . $EU(a) = \frac{1}{12}3 + \frac{2}{12}4 + \frac{4}{12}1 + \frac{3}{12}8 + \frac{2}{12}6 = \frac{51}{12} = 4.25$ and

$$EU(b) = \frac{1}{12}6 + \frac{2}{12}2 + \frac{4}{12}4 + \frac{3}{12}5 + \frac{2}{12}8 = \frac{57}{12} = 4.75. \text{ Thus she will choose act } b.$$

(b) (b.1) If she received information $\{s_1, s_2\}$ then, using Bayes' rule to update the probabilities,

$$EU(a|\{s_1, s_2\}) = \frac{1}{3}3 + \frac{2}{3}4 = \frac{11}{3} = 3.67 \text{ and } EU(b|\{s_1, s_2\}) = \frac{1}{3}6 + \frac{2}{3}2 = \frac{10}{3} = 3.33. \text{ Thus she}$$

would choose act a . If she received information $\{s_3, s_4, s_5\}$ then, again using Bayes' rule,

$$EU(a|\{s_3, s_4, s_5\}) = \frac{4}{9}1 + \frac{3}{9}8 + \frac{2}{9}6 = \frac{40}{9} = 4.44 \text{ and}$$

$$EU(b|\{s_3, s_4, s_5\}) = \frac{4}{9}4 + \frac{3}{9}5 + \frac{2}{9}8 = \frac{47}{9} = 5.22. \text{ Thus she would choose act } b.$$

(b.2) Her expected utility is $\frac{3}{12} \frac{11}{3} + \frac{9}{12} \frac{47}{9} = \frac{58}{12} = 4.83$

(c) It is $\frac{58}{12} - \frac{57}{12} = \frac{1}{12} = 0.083$

2. (a) It is given by the solution to $(0.6)(0.9)100 = (0.6)(0.9)(100 - 10 - p) + (0.6)(0.9)^2 40$ which is $p = 26$.

(b) It is given by the solution to $100 - 10 = 100 - 10 - p + (0.6)(0.9)40$ which is $p = 21.6$

(c) $U_0(A: \text{not join}) = (0.6)(0.9)100 = 54$, $U_0(B: \text{join and no exercise}) = (0.6)(0.9)(100 - 10) = 48.6$, $U_0(C: \text{join and exercise}) = (0.6)(0.9)(100 - 10 - 23) + (0.6)(0.9)^2 40 = 55.62$. Thus your ranking is $C \succ A \succ B$ and your most preferred plan is to join and exercise.

(d) $U_1(D: \text{no exercise}) = M - F = 90$, $U_1(E: \text{exercise}) = M - F - p + \beta \delta b = 88.6$. Thus your ranking is $D \succ E$ and you prefer not to go to the gym.

(e) No, because at date 0 you would plan to join and exercise and then at date 1, when you are a member, you prefer not to go to the gym.

(f) The tree is as follows and the backward-induction solution is shown by double edges. Here $M = 100$, $F = 10$, $p = 23$, $b = 40$, $\beta = 0.6$ and $\delta = 0.9$.

