

WINTER 2024 - THIRD MIDTERM EXAM Version 2

Answer all questions. **If you don't explain (= show your work for) your answers you will get no credit.**

NAME: _____ **University ID:** _____

CIRCLE THE NAME OF YOUR TA: Kalyani Chaudhuri or Joaquin Paleo

If you don't know the name of your TA, then circle your Section:

A01, Tuesday 5-6

A02, Tuesday 6-7

A03, Tuesday 7-8

A04, Tuesday 8-9

- **By writing your name on this exam you certify that you have not violated the University's Code of Academic Contact** (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- **If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.**

- **If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.**

1. [50 points] Today is January 1st and your New Year's resolution is to exercise more during the year. This is why you came to the gym to find out about pricing. To make things simple, suppose that there are only three dates. Today (date 0) you have to decide whether or not to join the gym by signing a membership agreement. If you don't sign there will **not** be another chance to join later. If you do sign then the membership fee of \$12 will be deducted from your paycheck of \$90 **tomorrow (date 1)**. The membership fee is **non-refundable**, so it will be deducted from your paycheck, whether or not you actually make use of the gym. If you sign the contract today (date 0), then tomorrow (date 1) you will have to decide whether to actually go to exercise or to stay at home; if you do go to the gym then you have to pay \$ p for using it (this is a charge **in addition** to the \$12 membership fee that you paid). Exercising, in itself, neither increases nor decreases your utility. If you don't go to exercise, you experience a benefit of 0; if you do go to exercise then you will experience a health benefit the day after tomorrow (**date 2**) and you consider that benefit to be **equivalent to receiving \$40**. For every date $t \in \{1, 2\}$, your instantaneous utility from having \$ m to spend at date t is $u_t(\$m) = m$. Whatever money you are left with at date 1, you spend at date 1 and the benefit from exercise experienced at date 2 is the same as having \$40 to spend at date 2. You are a hyperbolic discounter with parameters $\beta = 0.5$ and $\delta = 0.9$.

(a) [5 points] For what value of p would you be indifferent **at date 0** between (1) not signing the contract and (2) signing the contract and **committing** yourself to exercising at date 1?

(b) [5 points] For what value of p would you be indifferent **at date 1** between going to the gym and not going to the gym? [This assumes that at date 0 you chose to become a member.]

(c) [10 points] Suppose that $p = 20$. Today (**date 0**), how do you rank the following plans? (A) Do not sign the contract, (B) Sign the contract and do not go to exercise tomorrow, (C) Sign the contract and go to the gym to exercise tomorrow [You need to compute the utility of each option in order to get credit.]

(d) [10 points] Continue to suppose that $p = 20$. Suppose also that you sign the contract and become a member of the gym. Now it is **date 1**. How do you rank the two options you have: (D) Do not exercise, (E) Go to the gym to exercise? [Again, you need to compute the utility of each option in order to get credit.]

(e) [5 points] Are your preferences time consistent? [Explain.]

- (f) [15 points] Continue to suppose that $p = 20$. Draw a decision tree to show your options at dates 0 and 1. Assuming that you cannot commit at date 0 to go to exercise at date 1, use backward induction to determine your optimal decision at date 0.

2. [50 points] Consider the following decision problem, where the outcomes are levels of **total** wealth of an agent whose vNM utility-of money function is $U(\$m) = \sqrt{m}$ and the probabilities are the agent's initial beliefs.

probability	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
state \rightarrow	s_1	s_2	s_3	s_4	s_5
act \downarrow					
a	\$9	\$16	\$1	\$25	\$1
b	\$36	\$4	\$16	\$25	\$64
c	\$16	\$25	\$4	\$36	\$4

(a) [15 points] What act will the agent choose?

(b) Suppose that the agent is offered the option of consulting, **for free**, an expert who will be able to provide the agent with some reliable information concerning the true state. The information provided by the expert is only partial and is represented by the partition $\{\{s_1, s_2\}, \{s_3, s_4, s_5\}\}$ (this means that, for example, if the true state is s_4 , then the expert will be able to inform the agent that the true state is either s_3 or s_4 or s_5).

probability	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
state \rightarrow	s_1	s_2	s_3	s_4	s_5
act \downarrow					
a	\$9	\$16	\$1	\$25	\$1
b	\$36	\$4	\$16	\$25	\$64
c	\$16	\$25	\$4	\$36	\$4

(b.1) [25 points] For every possible item of information, find the optimal decision for the agent who has received such information.

(b.2) [5 points] Calculate, **from the initial point of view** (that is, before the information is obtained), the expected utility of taking advantage of the option of consulting the expert.

(c) [5 points] Calculate, **from the initial point of view** (that is, before the information is obtained), the increase in expected utility that the agent obtains by agreeing to consult the expert.