ECN 106 : Decision Making Professor Giacomo Bonanno

WINTER 2024 - THIRD MIDTERM EXAM: ANSWERS for VERSION 2

1. Let M = 90, F = 12, p = 23, b = 40, β = 0.5 and δ = 0.9

(a) It is given by the solution to $\beta\,\delta\,M = \beta\,\delta\,(M - F - p) + \beta\,\delta^2 b$ which is $p = 24$.

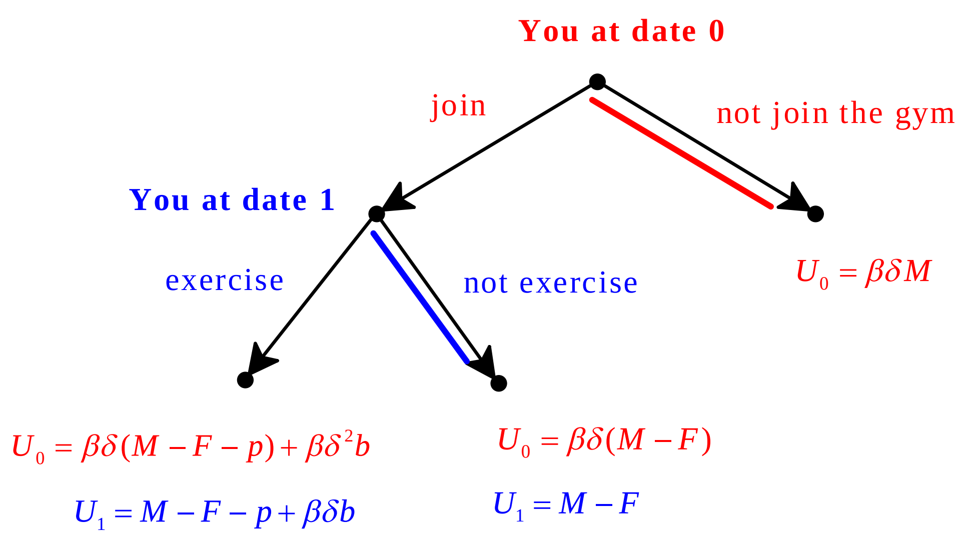
(b) It is given by the solution to $M - F = M - F - p + \beta\,\delta\,b$ which is $p = 18$.

(c) $U_0(A: not\ join) = \beta\,\delta\,M = 40.5$, $U_0(B: join\ and\ no\ exercise) = \beta\,\delta\,(M - F) = 35.1$, $U_0(C: join\ and\ exercise) = \beta\,\delta\,(M - F - p) + \beta\,\delta^2 b = 42.3$. Thus your ranking is $C \succ A \succ B$ and your most preferred plan is to join and exercise.

(d) $U_1(D: no\ exercise) = M - F = 78$, $U_1(E: exercise) = M - F - p + \beta\,\delta\,b = 76$. Thus your ranking is $D \succ E$ and you prefer not to go to the gym.

(e) No, because at date 0 you would plan to join and exercise and then at date 1, when you are a member, you prefer not to go to the gym.

(f) The tree is as follows and the backward-induction solution is shown by double edges. Here M = 90, F = 12, p = 23, b = 40, β = 0.5 and δ = 0.9.



2. First convert the outcomes into utilities:

probability	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
state $\rightarrow$	s_1	s_2	s_3	s_4	s_5
act $\downarrow$					
a	3	4	1	5	1
b	6	2	4	5	8
c	4	5	2	6	2

(a) Since act *a* is strictly dominated by act *c*, we only need to compute the expected utility of *b* and the expected utility of *c*. $EU(b) = \frac{1}{15}6 + \frac{5}{15}2 + \frac{3}{15}4 + \frac{5}{15}5 + \frac{1}{15}8 = \frac{61}{15} = 4.067$ and $EU(c) = \frac{1}{15}4 + \frac{5}{15}5 + \frac{3}{15}2 + \frac{5}{15}6 + \frac{1}{15}2 = \frac{67}{15} =$ 4.467. **Thus she will choose act *c*.**

(b) (b.1) If she received information $\{s_1, s_2\}$ then, using Bayes' rule to update the probabilities, $EU\left(b \mid \{s_1, s_2\}\right) = \frac{1}{6}6 + \frac{5}{6}2 = \frac{16}{6} = 2.667$ and $EU\left(c \mid \{s_1, s_2\}\right) = \frac{1}{6}4 + \frac{5}{6}5 = \frac{29}{6} = 4.833$. Thus **she would choose act *c***. If she received information $\{s_3, s_4, s_5\}$ then, again using Bayes' rule, $EU\left(b \mid \{s_3, s_4, s_5\}\right) = \frac{3}{9}4 + \frac{5}{9}5 + \frac{1}{9}8 = \frac{45}{9} = 5$ and $EU\left(c \mid \{s_3, s_4, s_5\}\right) = \frac{3}{9}2 + \frac{5}{9}6 + \frac{1}{9}2 = \frac{38}{9} = 4.22$. Thus **she would choose act *b***.

(b.2) Her expected utility is $\frac{6}{15}\frac{29}{6} + \frac{9}{15}\frac{45}{9} = \frac{74}{15} = 4.933$

(c) It is $\frac{74}{15} - \frac{67}{15} = \frac{7}{15} = 0.467$.