PRACTICE FOR FINAL EXAM

Note: Question 1 is longer than a typical exam question, but it covers several concepts so it is useful practice to try it.

1. Jonathan is facing a dilemma. Tomorrow is the last day of the Quarter and he has his last final exam in the morning. He has been invited to a party, where Kate is going to be. Jonathan has a crush on Kate. This would be the last chance to see Kate before the next academic year. Jonathan does not know if Kate is interested in him. He has three choices.

- The first choice is to skip the party and study for the final. If he does so, then he expects to get an A if the exam is easy but only a C if the exam is difficult.
- His second choice is to go to the party and approach Kate. Of course, this implies that he will not study for the final, with the consequence that he will get a C if the exam is easy and an F if the exam is difficult. Furthermore, he doesn't know if Kate is interested in him. If he approaches her and she is welcoming then he will have a great time, while if she rejects him then he will feel awful. He is not really interested in the party itself: all he cares about is Kate.
- His third choice is to go to the party and "play cool", that is, not approach Kate. The point of this is to avoid rejection. His hope is that Kate will approach him, which she will do if she is interested in him and is not shy, but she won't do if she is shy (whether or not she is interested in him).

(a) Write down what the possible outcomes (or consequences) are. Note that in Jonathan's mind there is no difference between (1) approaching Kate and finding her welcoming and (2) playing cool and being approached by her, since in both cases he ends up having a good time with her. On the other hand, being rejected (after approaching her) is definitely *not* the same as not being approached by her!

(b) Represent Jonathan's decision problem in terms of states, outcomes and acts.

What should Jonathan do? Of course it depends on how Jonathan feels about the possible outcomes. The best outcome for him is to have a good time with Kate at the party (either because he approaches her and she is welcoming or because he plays cool and she approaches him) and pass the exam. The second best is to get an A in the exam (this is a very important class for him). The third best is to have a good time with Kate at the party and fail the exam. The fourth outcome is skipping the party and passing the exam; Jonathan is indifferent between this outcome and the outcome where he is "cool" at the party, not approached by Kate and passes the exam. The next outcome in the ranking is to be "cool" at the party, not be approached by Kate and fail the exam. The worst outcome is approach Kate at the party and be rejected: this would be so devastating that he doesn't even care whether or not he passes the exam (that is, if rejected, he is indifferent between passing and not passing).

(c) Represent Jonathan's ranking by means of an ordinal utility function.

(d) Re-write the representation under (b) replacing outcomes with utilities. Is there one act that dominates another act? If so, is it weak dominance or strict dominance?

Let E be the event that the exam is easy (so that its complement $\neg E$ is the event that the exam is difficult). Let S be the event that Kate is shy (so that $\neg S$ is the event that she is not shy). Finally, let A be the event that Kate is attracted to (or interested in) Jonathan (so that $\neg A$ is the event that she is not).

(e) Write down the events $E, \neg E, S, \neg S, A$ and $\neg A$.

Jonathan knows that these events are pair-wise independent, that is, whether or not the exam is easy is independent of whether or not Kate is shy, which in turn is independent of whether or not Kate is attracted to him, etc.

(f) Let P be the probability distribution over the set of states. Express, in terms of P, the pairwise independence of the above events. **Note: two events A and B are independent if the probability of the intersection of A and B is equal to the product P(A)P(B).**

Now we want to construct P using available information. After looking at all the past exams on the web page for the class, Jonathan noticed that 40% of the past exams were easy and 60% were difficult. He also found a web page where you input the character traits of an individual and it tells you the proportion of individuals with those characteristics that are shy. In the case of people who are like Kate the verdict is that 80% are shy and 20% are not. Finally there is the matter of whether or not Kate is interested in Jonathan. Unfortunately Google cannot be of help on this! So Jonathan turns to his friend James, who has a reputation of knowing everything about women; James tells him that he thinks that there is a 50% chance that Kate is indeed attracted to Jonathan.

(g) Using the above information and the pair-wise independence of events A, E, S and their complements calculate the following probabilities: (1) that the exam is easy and Kate is shy, (2) that the exam is easy and Kate is attracted to Jonathan, (3) that Kate is attracted to Jonathan and is shy.

Suppose now that independence is even stronger than said above: not only are events A, E, S and their complements pair-wise independent, but the also the following events are independent of S: $A \cap E$, $A \cap \neg E$, $\neg A \cap E$ and $\neg A \cap \neg E$.

(h) Calculate all the values of the probability distribution P.

(i) Represent Jonathan's choices as lotteries.

Assume that Jonathan satisfies the axioms of expected utility. Let z_4 be the outcome where Jonathan goes to the party, has a good time with Kate and gets an F in the exam, z_3 the outcome where Jonathan goes to the party, has a good time with Kate and gets a C in the exam and z_5 the outcome where Jonathan goes to the party, is rejected by Kate and gets a C in the exam. Jonathan says that he is indifferent between the following two lotteries: $\begin{pmatrix} z_4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} z_3 & z_5 \\ 0.6 & 0.4 \end{pmatrix}$; furthermore, he is indifferent between not going to the party and going and approaching.

(j) Using this information, determine, as much as possible, the values of Jonathan's von Neumann-Morgenstern utility function.

(k) If you wanted to help Jonathan determine all the values of his von Neumann-Morgenstern utility function, how many and what questions would you need to ask him?

(l) Suppose that the answers to those questions are: 0.8 and 0.05. What choice should Jonathan make?

2. (A) Carla's intertemporal preferences satisfy the discounted utility hypothesis. For any sum of money $*m*, her instantaneous utility at any date *t* is $u_t(m) = m$. A time period represents a year. Carla's discount **rate** is $\rho = \frac{1}{9}$

(a) Today she is given a choice between receiving $100 in six years' time or $200 in eight years' time. What does she choose?

(b) After six years she is given the choice to either confirm the choice she made six years ago or change her choice (that is, she is asked whether she prefers $100 right away or $200 after two years). What does she choose?

(c) Are Carla's preferences time consistent?

(B) Carla's intertemporal preferences are represented by a utility function with hyperbolic discounting. For any sum of money $*m*, her instantaneous utility at any date *t* is $u_t(m) = m$. A time period represents a year. Carla's parameters are $\beta = 0.6$ and $\delta = 0.9$.

(d) Today she is given a choice between receiving $100 in six years' time or $200 in eight years' time. What does she choose?

(e) After six years she is given the choice to either confirm the choice she made six years ago or change her choice (that is, she is asked whether she prefers $100 right away or $200 after two years). What does she choose?

(f) Are Carla's preferences time consistent?

3. There are four candidates for one position: *a*, *b*, *c* and *x*. There are seven voters, with the following preferences (most preferred at the top and least preferred at the bottom).

1	**2**	**3**	**4**	**5**	**6**	**7**
x	*a*	*b*	*x*	*a*	*b*	*x*
c	*x*	*a*	*c*	*x*	*a*	*c*
b	*c*	*x*	*b*	*c*	*x*	*b*
a	*b*	*c*	*a*	*b*	*c*	*a*

Suppose that the Borda rule is used as a social preference function and that there is sincere (or honest) voting.

(a) (What is the social ranking? Who wins the election? If voting takes place over the four candidates and then, **after the election**, candidate *x* is disqualified, who is the chosen candidate according to the social ranking?

(b) Suppose that, **just before the vote**, candidate *x* drops out (e.g. because he is disqualified). What is the new social ranking, given that voting (using the Borda count) is only over candidates *a, b* and *c*? Who wins the election?

4. **(a)** The Gibbard-Satterthwaite theorem says that if the range of a Social Choice Function (SCF) contains at least 3 alternatives (i.e. if there are at least 3 alternatives among the ones that are actually chosen by the SCF) and the SCF satisfies Freedom of Expression, Unanimity and Non-manipulability (also called strategy-proofness), then it is a dictatorship. The theorem is false if the range of the SCF contains **two** elements. In this case can you think of a voting procedure (or SCF) which satisfies Freedom of Expression, Unanimity, Non-dictatorship and Non-manipulability?

(b) Consider the following voting procedure. There are three individuals and three alternatives, *a, b* and *c*. Each individual reports a strict ranking of the alternatives. If at least two individuals rank the same alternative at the top, then that alternative is chosen. Otherwise (that is, if there is complete disagreement) the alternative ranked highest by individual 1 is chosen. Represent this voting procedure by means of tables and explain whether it satisfies Freedom of Expression, Unanimity, Non-dictatorship and Non-manipulability.