PRACTICE SECOND MIDTERM: ANSWERS

1. **(a)** Jennifer's expected utility if she bets \$2000 is:

$$\frac{3}{4}\left\{200-\left[12-\frac{6000+2000}{1000}\right]^2\right\}+\frac{1}{4}\left\{200-\left[12-\frac{6000-2000}{1000}\right]^2\right\}=172.$$

(b) If Jennifer does not bet her utility is:

$$\left\{200-\left[12-\frac{6000}{1000}\right]^2\right\}=164$$

(e) Let \$$x$ be the bet. Then Jennifer is choosing between not betting, which corresponds to the lottery $\begin{pmatrix}\$6,000\\1\end{pmatrix}$, and betting \$$x$, which corresponds to the lottery $L_x=\begin{pmatrix}\$(6,000+x) & \$(6,000-x)\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$. Now, the expected value of L_x is $\frac{1}{2}(6,000+x)+\frac{1}{2}(6,000-x)=6,000$. Thus she will be indifferent between not betting and betting if she is risk neutral and she will prefer not betting if she is risk averse. To check what her attitude to risk is, try a bet, say x = \$4,000. Then the choice is between $\begin{pmatrix}\$6,000\\1\end{pmatrix}$ and $L_{4,000}=\begin{pmatrix}\$10,000 & \$2,000\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$. The expected utility of the first is $200-\left(12-\frac{6,000}{1,000}\right)^2=164$. The expected utility of the second is $\frac{1}{2}\left\{200-\left[12-\frac{10000}{1000}\right]^2\right\}+\frac{1}{2}\left\{200-\left[12-\frac{2000}{1000}\right]^2\right\}=148$. Thus she prefers not betting to betting \$4,000, that is, she is risk averse. Hence she will prefer not betting to any bet.

The proof can also be given for a general bet $0 \le y \le 6,000$. If she does not bet then her utility is U(6,000) = 164. If she bets \$$y$ then her expected utility is

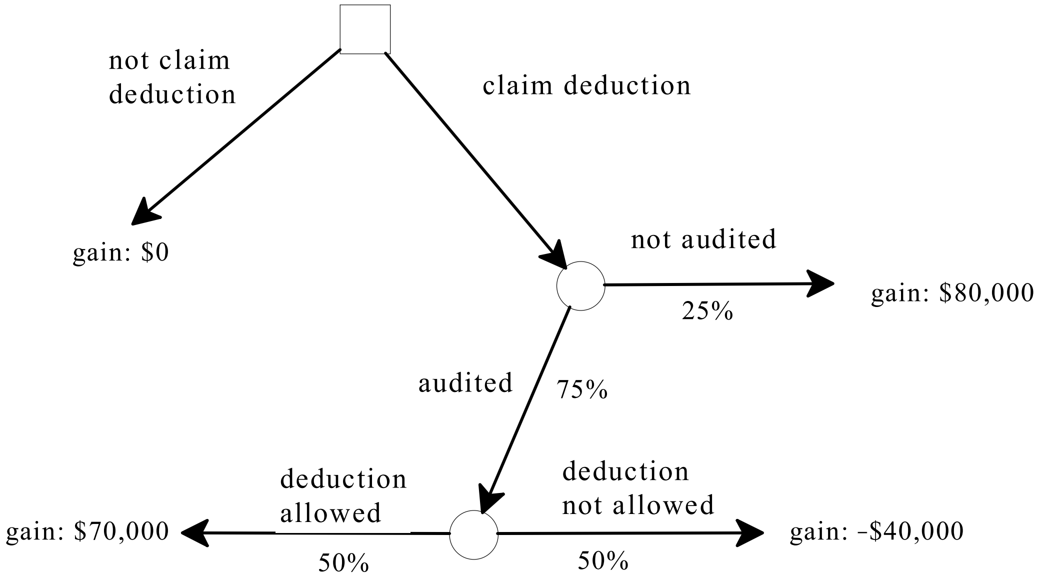
$$\tfrac{1}{2}U(6,000+y)+\tfrac{1}{2}U(6,000-y)=\tfrac{1}{2}\left[U(6,000+y)+U(6,000-y)\right]=164-\frac{y^2}{1,000,000}$$

which is less than 164 if $y > 0$.

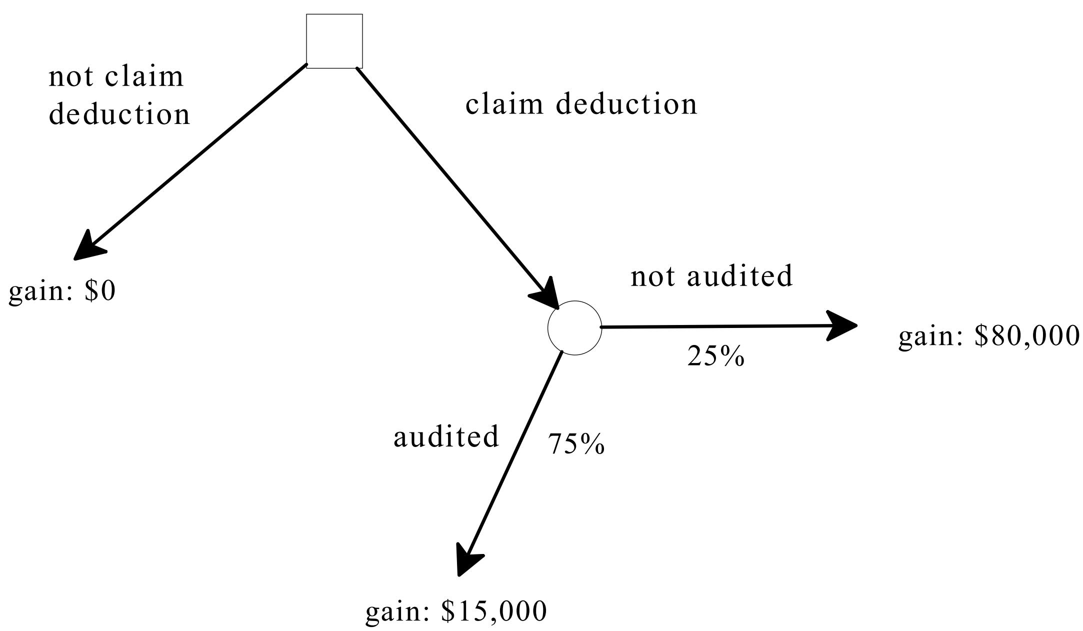
2. Suppose Peter does satisfy the axioms of expected utility and let U be his utility-of-money function, normalized so that U(5000) = 1 and U(0) = 0. Let U(1000) = p. Then 0 < p < 1. Now, EU(A) = p, EU(B) = 0.1 (1) + 0.89 (p) + 0.01 (0) = 0.1 + 0.89 p, EU(C) = 0.11 p and EU(D) = 0.1. Then EU(A) > EU(B) if and only if p > 0.1 + 0.89 p,

i.e. if and only if $p > \frac{10}{11}$. But if $p > \frac{10}{11}$ then EU(C)= $0.11p > 0.11\,\frac{10}{11} = 0.1$ = EU(D). Thus if Peter satisfies the axioms of expected utility and prefers A to B then he must also prefer C to D. Hence he does **not** satisfy the axioms of expected utility.

3. (a) The decision tree is as follows:



(a) The bottom Chance node corresponds to the lottery $\begin{pmatrix} \$70,000 & -\$40,000 \\ 50\% & 50\% \end{pmatrix}$, which has an expected value of $70,000\frac{1}{2} - 40,000\frac{1}{2} = \$15,000$. Thus we can simplify the decision tree as follows



Now the remaining chance node corresponds to the lottery $\begin{pmatrix} \$15,000 & \$80,000 \\ 75\% & 25\% \end{pmatrix}$, which has an expected value of $15,000\frac{3}{4} + 80,000\frac{1}{4} = \$31,250$. Hence you should advise your client to claim the deduction.