- Introduction

- Decision making under certainty

  Preference relations

  Revealed preference

- Decision making under uncertainty

  Acts, states and outcomes

  Dominance

  MaxiMin

  LexiMin

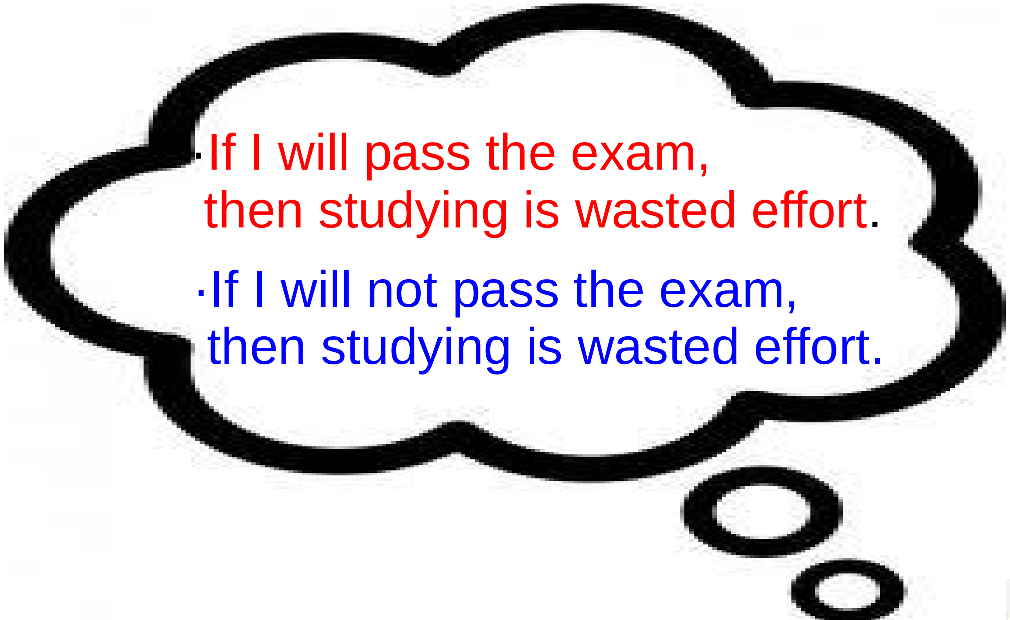
- Money lotteries and attitudes to risk

RATIONAL DECISION MAKING

The psychologist Keith Stanovich defines rationality as the

capacity to make decisions that help you achieve your objectives

1. How to think about choices



Since, whatever will happen, studying is wasted effort,
it is better for me **not** to study.

		possible states	
		Pass	Not pass
possible choices	Study		
	Not study		

possible states

possible choices or acts		Exam is long and difficult	Exam short and easy
	Study		
	Not study		

2. What does ‘Rationality’ mean?

Harold Egbert Camping, president of Family Radio 1958-2011, predicted that the Rapture (the taking up into heaven of God’s elect people) would take place on May 21, 2011 at 6pm.

Some followers of Camping gave up their jobs, sold their homes and spent large sums promoting Camping’s claims.

Did these people act irrationally?

Bob smokes two packets of cigarettes a day. When asked if he would still smoke if he knew that he was going to get lung cancer from smoking, he says “No”. When asked if he is worried about getting lung cancer, he says that he is not and explains that his grandfather was a heavy smoker all his life and died at the age of 98. He also explains that he read an article stating that smoking causes lung cancer only if one has a genetic predisposition to it.

state → act ↓	s_1 : genetically predisposed	s_2 : no genetic predisposition
smoke	get cancer enjoy smoking	no cancer enjoy smoking
not smoke	no cancer no enjoyment	no cancer no enjoyment

3. Framing

I will give you $200: 

and then you will have to choose one of:

OPTION A : I give you an additional $100: 

OPTION B : I toss a coin 

HEADS: I give you an additional $200


TAILS: I give you no additional money

I will give you $400:

and then you will have to choose one of:

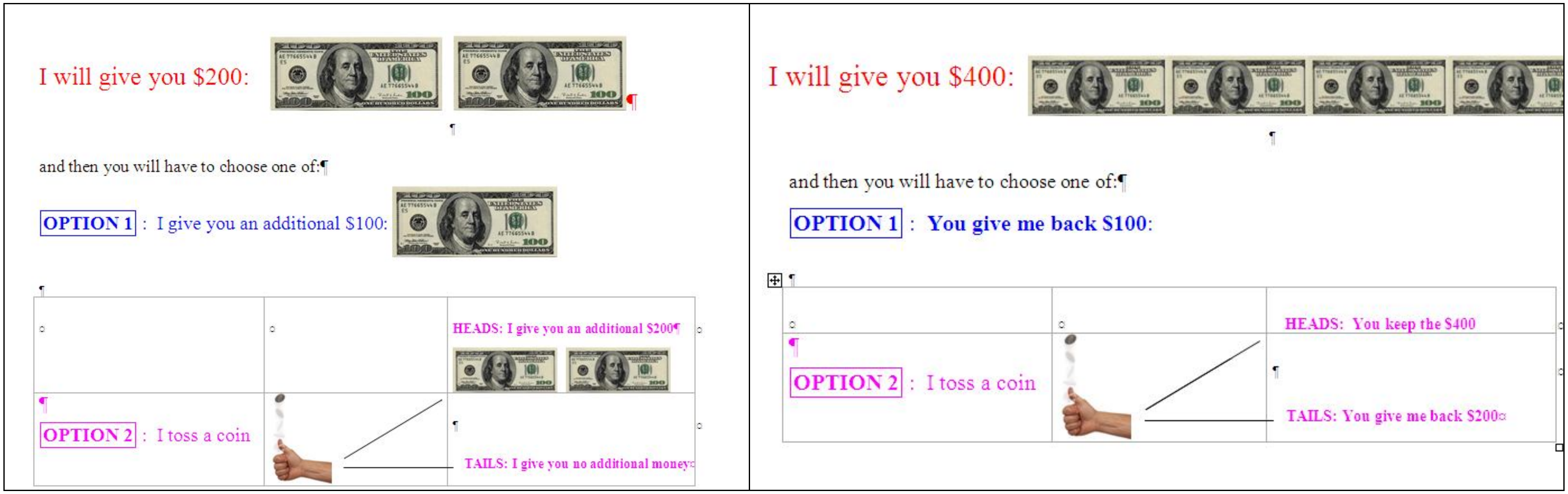
OPTION 1 : **You give me back $100**:

OPTION 2 : I toss a coin

HEADS: You keep the $400

TAILS: You give me back $200

Put the first and third problems side by side:



In both cases:

Option 1 = you end up with $300

Option 2 = you face the uncertain prospect (lottery)

$$\left(\begin{array}{c:c} \text{You end up with \$400} & \text{You end up with \$200} \\ \text{Probability } \frac{1}{2} & \text{Probability } \frac{1}{2} \end{array}\right)$$

Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 60,000 people. Two alternative programs to combat the disease have been proposed.

- If **Program A** is adopted, 20,000 people will be saved.
- If **Program B** is adopted, there is a $\frac{1}{3}$ probability that **all** 60,000 people will be **saved** and a $\frac{2}{3}$ probability that **none** of the 60,000 will be **saved**.

Which of the two programs would you favor?

Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 60,000 people. Two alternative programs to combat the disease have been proposed.

- If **Program C** is adopted, 40,000 people will **die**.

- If **Program D** is adopted, there is a $\frac{1}{3}$ probability that **none** of the 60,000 will **die** and a $\frac{2}{3}$ probability that **all** of the 60,000 people will **die**.

Put the second and fourth problems side by side:

Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 60,000 people. Two alternative programs to combat the disease have been proposed. • If **Program A** is adopted, 20,000 people will be saved. • If **Program B** is adopted, there is a $\frac{1}{3}$ probability that **all** 60,000 people will be **saved** and a $\frac{2}{3}$ probability that **none** of the 60,000 will be **saved**.	Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 60,000 people. Two alternative programs to combat the disease have been proposed. • If **Program C** is adopted, 40,000 people will **die**. • If **Program D** is adopted, there is a $\frac{1}{3}$ probability that **none** of the 60,000 will **die** and a $\frac{2}{3}$ probability that **all** of the 60,000 people will **die**.

The two problems are the same. In both cases,

if Program A/C is adopted, 20,000 people are saved and 40,000 die;

Program B/D corresponds to the following lottery $\left(\begin{array}{c:c} \text{all 60,000 are saved} & \text{nobody is saved} \\ \text{= nobody dies} & \text{= all 60,000 die} \\ \text{Probability } \frac{1}{3} & \text{Probability } \frac{2}{3} \end{array}\right)$.

4. How to process information

- In the US, **1% of women of age 40 have breast cancer**.
- If a woman **has** breast cancer, the probability that she tests **positive** on a screening mammogram is **90%**.
- If she **does not have** breast cancer, the probability that she tests **negative** on a screening mammogram is **90%**.

That is, mammograms have a **90% accuracy**.

Susan is a 40-year old woman who tested **positive** *on a mammogram.*

What are the chances that she actually has breast cancer?

You are at the CVS store in West Covell Blvd about to buy a watch that costs $14. You bump into a friend who says that the same watch costs only $7 at Target. Are you willing to drive 15 minutes across town to get the better deal?

You are about to buy a washing machine that costs $420. A friend tells you that a different store, which is a 15-minute drive from here, sells the same washing machine for $413. Are you willing to drive 15 minutes across town to get the better deal?

Recommended viewing:

1. Dan Ariely, Are we in control of our own decisions?, on Ted.com:

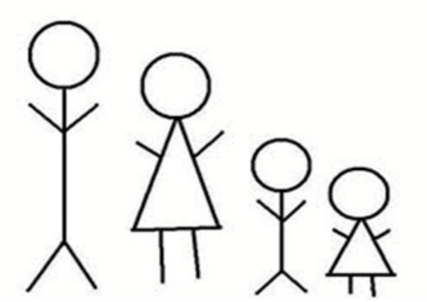
http://www.ted.com/talks/view/lang/en//id/548

2. Dan Gilbert, Why we make bad decisions, on Ted.com:

http://www.ted.com/talks/lang/en/dan_gilbert_researches_happiness.html

Decisions under certainty

A binary relation R on a set Z is a set of ordered pairs (x,y) with both x and y elements of Z.



Alex Beth Carl Dana

Z = {A, B, C, D}

R is the "taller than" relation: (x,y) means that x is taller than y

REVEALED PREFERENCE

Z = {a, b, c, d, e, f}

Observations:

{a, c, f} $\longrightarrow$ c (1)

{a, d, b} $\longrightarrow$ a (2)

{d, e, f} $\longrightarrow$ f (3)

{a, f} $\longrightarrow$ f (4)

Assume also that his ranking is a strict ranking (no indifference)
Is there a complete and transitive strict ranking of Z that rationalizes the observations?

$Z = \{a, b, c, d, e, f\}$

Observations:

$$\{a, c, f\} \longrightarrow c \qquad (1)$$

$$\{a, d, b\} \longrightarrow a \qquad (2)$$

$$\{d, e, f\} \longrightarrow f \qquad (3)$$

$$\{a, f\} \longrightarrow f \qquad (4)$$

Decisions under **uncertainty**

Act a ***weakly dominates*** act b if, for every state s, $a(s) \succsim b(s)$ and, furthermore, there is at least one state $\hat{s}$ such that $a(\hat{s}) \succ b(\hat{s})$.
Using utility, $U\left(a(s)\right) \geq U\left(b(s)\right)$ for every state s and there is at least one state $\hat{s}$ such that $U\left(a(\hat{s})\right) > U\left(b(\hat{s})\right)$.

state →	s_1	s_2	s_3
act ↓			
a_1	1	3	1
a_2	0	2	1
a_3	1	3	3

- a_1 weakly dominates a_2
- a_3 weakly dominates a_1
- a_3 strictly (and thus also weakly) dominates a_2.

a and b are ***equivalent***, if, for every state s, $a(s) \sim b(s)$ or, in terms of utility, $U\left(a(s)\right) = U\left(b(s)\right)$.

Act a is ***weakly dominant*** if, for every other act b, either a weakly dominates b or a and b are equivalent.

In the above example, …

Another example:

state →	s_1	s_2	s_3	s_4
act ↓				
a_1	1	3	3	2
a_2	0	2	1	2
a_3	1	3	3	2

You are bidding against a computer for an item that you **value at $30**. The allowed bids are $10, $20, $30, $40 and $50. The computer will pick one of these bids randomly. Let x be the bid generated by the computer. If your bid is greater than or equal to x then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than x then you get nothing and pay nothing.

computer's bid → / your bid ↓	$10	$20	$30	$40	$50
$10					
$20					
$30					
$40					
$50					

Now same as above, but if you win the object and **pay your own bid**.

computer's bid → / your bid ↓	$10	$20	$30	$40	$50
$10					
$20					
$30					
$40					
$50					

state → act ↓	s_1	s_2	s_3
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9
a_4	z_{10}	z_{11}	z_{12}
a_5	z_{13}	z_{14}	z_{15}

	Utility
best	z_4, z_{10}
	z_7, z_{15}
	z_1
	z_2, z_8
	z_5, z_6, z_9, z_{14}
	z_3, z_{11}
worst	z_{12}

state → act ↓	s_1	s_2	s_3
a_1			
a_2			
a_3			
a_4			
a_5			

state → act ↓	s_1	s_2	s_3
a_1	4	3	1
a_2	6	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	2	5

Dominance:

So we can simplify

state → act ↓	s_1	s_2	s_3
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

What then?

First a different example:

state →	s_1	s_2	s_3
act ↓			
a_1	4	3	1
a_2	3	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	3	4

One criterion that can be used is the **MaxiMin** criterion.

Now back to the previous problem:

state →	s_1	s_2	s_3
act ↓			
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

MaxiMin =

A refinement is the **LexiMin**

state →	s_1	s_2	s_3
act ↓			
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

Here the LexiMin picks

One more example:

state →	s_1	s_2	s_3	s_4
act ↓				
a_1	2	3	1	5
a_2	6	2	2	3
a_3	5	3	2	4
a_4	6	1	0	7
a_5	3	2	5	1

MaxiMin =

LexiMin =

Special case: outcomes are sums of money

state $\rightarrow$ / act $\downarrow$	s_1	s_2	s_3	s_4
a_1	\$12	\$30	\$0	\$18
a_2	\$36	\$6	\$24	\$12
a_3	\$6	\$42	\$12	\$0

Suppose that we are able to assign probabilities to the states:

state $\rightarrow$	s_1	s_2	s_3	s_4
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$

a_1 is the lottery

a_2 is the lottery

a_3 is the lottery

The expected values are:

Definition of attitude to risk ….

Given a money lottery *L*, imagine giving the individual a choice between *L* and the expected value of *L* for sure, that is, the choice

$$\text{between } \begin{pmatrix} \mathbb{E}[L] \\ 1 \end{pmatrix} \text{ and } L \quad \text{or, written more simply, between } \mathbb{E}[L] \text{ and } L$$

If she says that

- $\mathbb{E}[L] \succ L$ we say that she is **risk** **relative to *L***
- $\mathbb{E}[L] \sim L$ we say that she is **risk** **relative to *L***
- $L \succ \mathbb{E}[L]$ we say that she is **risk** **relative to *L***

So in the above example, if we assume that the agent is risk neutral relative to every lottery and her preferences are transitive, then, since

$$\mathbb{E}[a_1] = 10.5$$
$$\mathbb{E}[a_2] = 24$$
$$\mathbb{E}[a_3] = 14$$

Can we infer risk attitudes from choices?

Let $L = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Then $\mathbb{E}[L] =$

Suppose Ann's preferences are transitive, she prefers more money to less and she says that she prefers \$49 to *L*.

Suppose Bob's preferences are transitive, he prefers more money to less and he says that he prefers \$51 to *L*.