

★ Manipulability of Borda SCF
Manipulability of Kemeny-Young SCF

★ Psychology of decision making

★ Decoys

★ Framing

★ Loss aversion

★ REVIEW

MANIPULABILITY of the BORDA count

Four alternatives: a, b, c and d

Three voters

	1	2	3	score
best				
worst				

a:
b:
c:
d:

	1 changes to:				
	1	2	3	score	
best					
worst					

a:
b:
c:
d:

MANIPULABILITY of the KEMENY-YOUNG method

The Kemeny-Young procedure is a social **preference** function. However, just like the Borda rule, it can be converted to a social **choice** function by picking the top-ranked alternative in the selected ranking.

Consider the following tie-breaking rule: if two or more rankings are selected by the Kemeny-Young procedure, then pick the one whose top alternative comes first in alphabetical order.

	voter 1	voter 2	voter 3
best	<i>A</i>	<i>C</i>	<i>B</i>
	<i>B</i>	<i>A</i>	<i>C</i>
worst	<i>C</i>	<i>B</i>	<i>A</i>

Ranking	Kemeny-Young score
$A \succ B \succ C$	$\#(A \succ B) + \#(A \succ C) + \#(B \succ C) =$
$A \succ C \succ B$	$\#(A \succ C) + \#(A \succ B) + \#(C \succ B) =$
$B \succ A \succ C$	$\#(B \succ A) + \#(B \succ C) + \#(A \succ C) =$
$B \succ C \succ A$	$\#(B \succ C) + \#(B \succ A) + \#(C \succ A) =$
$C \succ A \succ B$	$\#(C \succ A) + \#(C \succ B) + \#(A \succ B) =$
$C \succ B \succ A$	$\#(C \succ B) + \#(C \succ A) + \#(B \succ A) =$

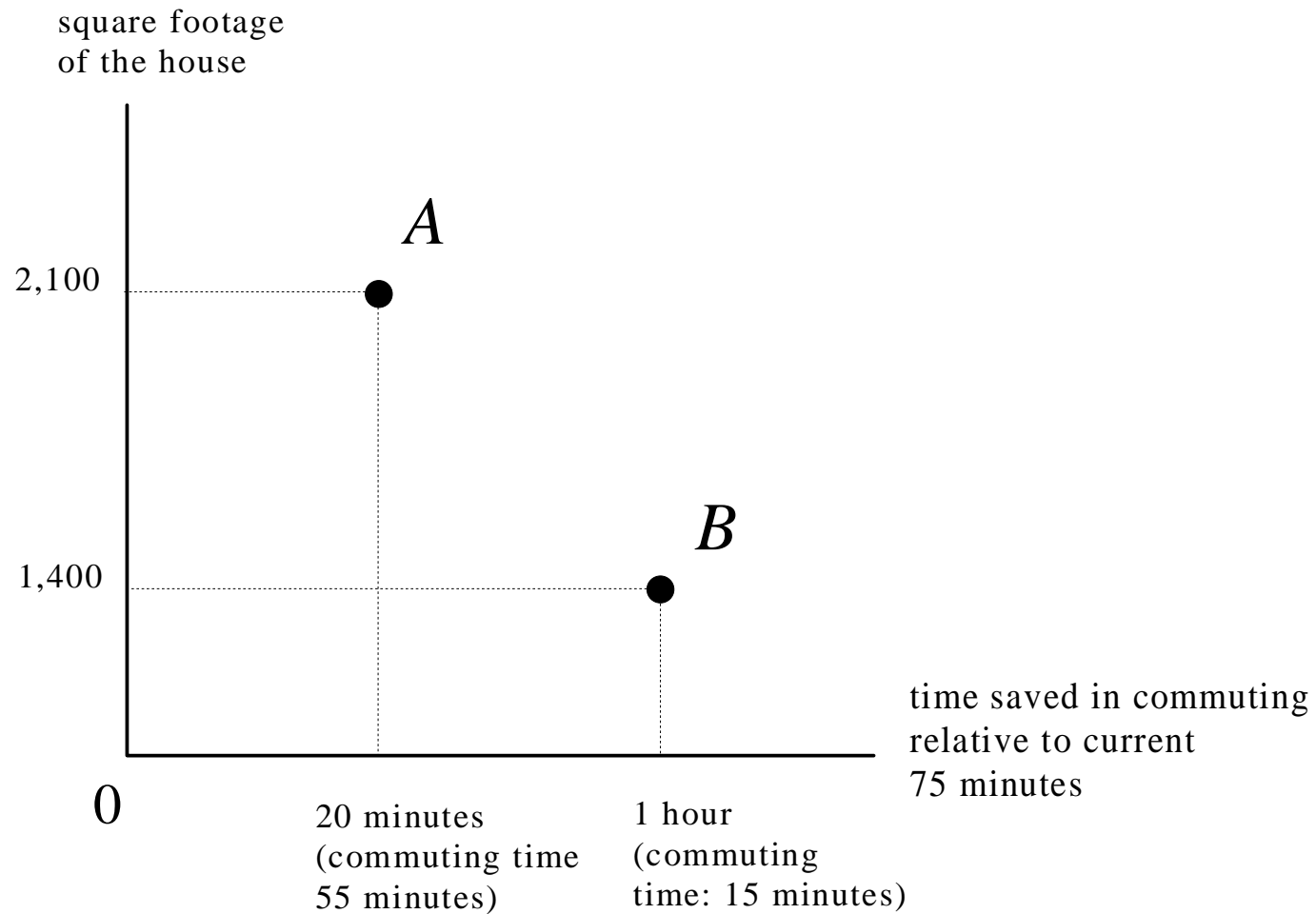
If Voter 3 (for whom A is the worst alternative) lies and reports $C \succ B \succ A$ instead of the true $B \succ C \succ A$

	voter 1	voter 2	voter 3
best	A	C	C
	B	A	B
worst	C	B	A

Ranking	Kemeny-Young score
$A \succ B \succ C$	$\#(A \succ B) + \#(A \succ C) + \#(B \succ C) =$
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$B \succ A \succ C$	$\#(B \succ A) + \#(B \succ C) + \#(A \succ C) =$
$B \succ C \succ A$	$\#(B \succ C) + \#(B \succ A) + \#(C \succ A) =$
$C \succ A \succ B$	$\#(C \succ A) + \#(C \succ B) + \#(A \succ B) =$
$C \succ B \succ A$	$\#(C \succ B) + \#(C \succ A) + \#(B \succ A) =$

The Psychology of Decision Making

1. Manipulation of Choices Through Decoys



GROUP 1. Choose one of the two:



Handsome A



Handsome B

GROUP 2. Choose one of the three:



Handsome A



“uglified” version of A



Handsome B

GROUP 3. Choose one of the three:



Handsome A



“uglified” version of B



Handsome B

2. Framing Effects: Gains versus Losses

I will give you \$200:



and then you will have to choose one of:

OPTION 1 : I give you an additional \$100:



OPTION 2 : I toss a coin



HEADS: I give you an additional \$200



TAILS: I give you no additional money

I will give you \$400:



and then you will have to choose one of:

OPTION 1 : You give me back \$100:


OPTION 2 : I toss a coin



HEADS: You keep the \$400


TAILS: You give me back \$200



I will give you \$200:




and then you will have to choose one of:

OPTION 1 : I give you an additional \$100:




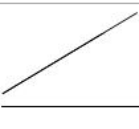
<p>OPTION 2 : I toss a coin</p> 		<p>HEADS: I give you an additional \$200</p> <p>TAILS: I give you no additional money</p>
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I will give you \$400:



and then you will have to choose one of:

OPTION 1 : You give me back \$100:

<p>OPTION 2 : I toss a coin</p> 		<p>HEADS: You keep the \$400</p> <p>TAILS: You give me back \$200</p>
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In both cases:

Option 1 = you end up with \$300

Option 2 = you face the uncertain prospect (lottery)

$$\left(\begin{array}{c|c} \text{You end up with \$400} & \text{You end up with \$200} \\ \hline \text{Probability } \frac{1}{2} & \text{Probability } \frac{1}{2} \end{array} \right)$$

Non-monetary example of effect of FRAMING in terms of GAINS vs LOSSES

You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table.
Out of every 100 patients who chose surgery 90 survived the operation, 68 were alive after 1 year and 34 were alive after 5 years.
- Radiation. Out of every 100 patients who chose radiation 100 survived the treatment, 77 were alive after 1 year and 22 were alive after 5 years.

About 80% of experimental subjects chose surgery

You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table.
Out of every 100 patients who chose surgery 10 died during the operation, 32 died after 1 year and 66 died within 5 years.
- Radiation. Out of every 100 patients who chose radiation none died during the treatment, 23 after 1 year and 78 died within 5 years.

About 50% of experimental subjects chose surgery.

Loss Aversion:

We are happy when we gain something, but

Twice unhappy when we lose it

The Pain of Paying

fMRI studies show that the pain centers of the brain light up when one has to part with one's cash.



The Pain of Paying

FMRI studies show that the pain centers of the brain light up when one has to part with one's cash.



Less pain with credit.



Functional magnetic resonance imaging (fMRI) is a procedure that measures brain activity by detecting associated changes in blood flow.

People tend to be risk-averse towards gains, but risk-loving towards losses.

Can such an attitude be compatible with expected utility?

Choice between $A: \begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B: \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Suppose that **she prefers** the sure gain: she prefers **A**. Then she displays **risk-aversion towards gains** (the expected value of these two options is the same).

Choice between $C: \begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D: \begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Suppose that **she prefers** the risky prospect: she prefers **D**. **Then she is risk-loving towards losses** (the expected value of these two options is the same).

Is there a von Neumann-Morgenstern utility function that is consistent with these choices?

Suppose that her initial wealth is \$100.

outcome	U	$A = \begin{pmatrix} +\$50 \\ 1 \end{pmatrix} \succ B = \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	
\$200			
\$150			
\$100			
\$50			
\$0			

Hence it is possible for an expected-utility maximizing individual to display risk aversion towards a gain and risk love towards a symmetric loss.

However, this cannot happen at every wealth level.

Beginning wealth: \$200. Choice between $A:\begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B:\begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Beginning wealth: \$200. Choice between $C:\begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D:\begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Can she prefer A to B and also D to C? Let’s see.

		Since she prefers D to C, she prefers	
outcome	U		
\$200	1		
\$150	a		
\$100	b		
\$50	c		
\$0	0		

Thus people who are consistently (that is, at every initial level of wealth) risk-averse towards gains and risk-loving towards losses cannot satisfy the axioms of expected utility. If those axioms capture the notion of rationality, then those people are irrational.

People tend to be risk-averse towards gains, but risk-loving towards losses.

Can such an attitude be compatible with expected utility?

Choice between $A: \begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B: \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Suppose that **she prefers** the sure gain: she prefers **A**. Then she displays **risk-aversion towards gains** (the expected value of these two options is the same).

Choice between $C: \begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D: \begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Suppose that **she prefers** the risky prospect: she prefers **D**. **Then she is risk-loving towards losses** (the expected value of these two options is the same).

Is there a von Neumann-Morgenstern utility function that is consistent with these choices?

Suppose that her initial wealth is \$100.

outcome	U	$A = \begin{pmatrix} +\$50 \\ 1 \end{pmatrix} \succ B = \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	
\$200			
\$150			
\$100			
\$50			
\$0			

Hence it is possible for an expected-utility maximizing individual to display risk aversion towards a gain and risk love towards a symmetric loss.

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Thus people who are consistently (that is, at every initial level of wealth) risk-averse towards gains and risk-loving towards losses cannot satisfy the axioms of expected utility. If those axioms capture the notion of rationality, then those people are irrational.

Review

1. Choice under certainty. Completeness and transitivity.
Ordinal utility function.

2. Choice under **uncertainty**: States, outcomes, and acts.
Strict/weak dominance. Difference between “ a is a dominant act” and “ a dominates b ”. MaxiMin. Leximin.

state \rightarrow	s_1	s_2
act \downarrow		
a	4	8
b	3	7
c	2	5
d	5	0

state \rightarrow	s_1	s_2
act \downarrow		
a	4	8
b	3	7
c	2	5
d	4	0

state \rightarrow	s_1	s_2
act \downarrow		
a	4	8
b	3	7
c	2	5
d	4	0

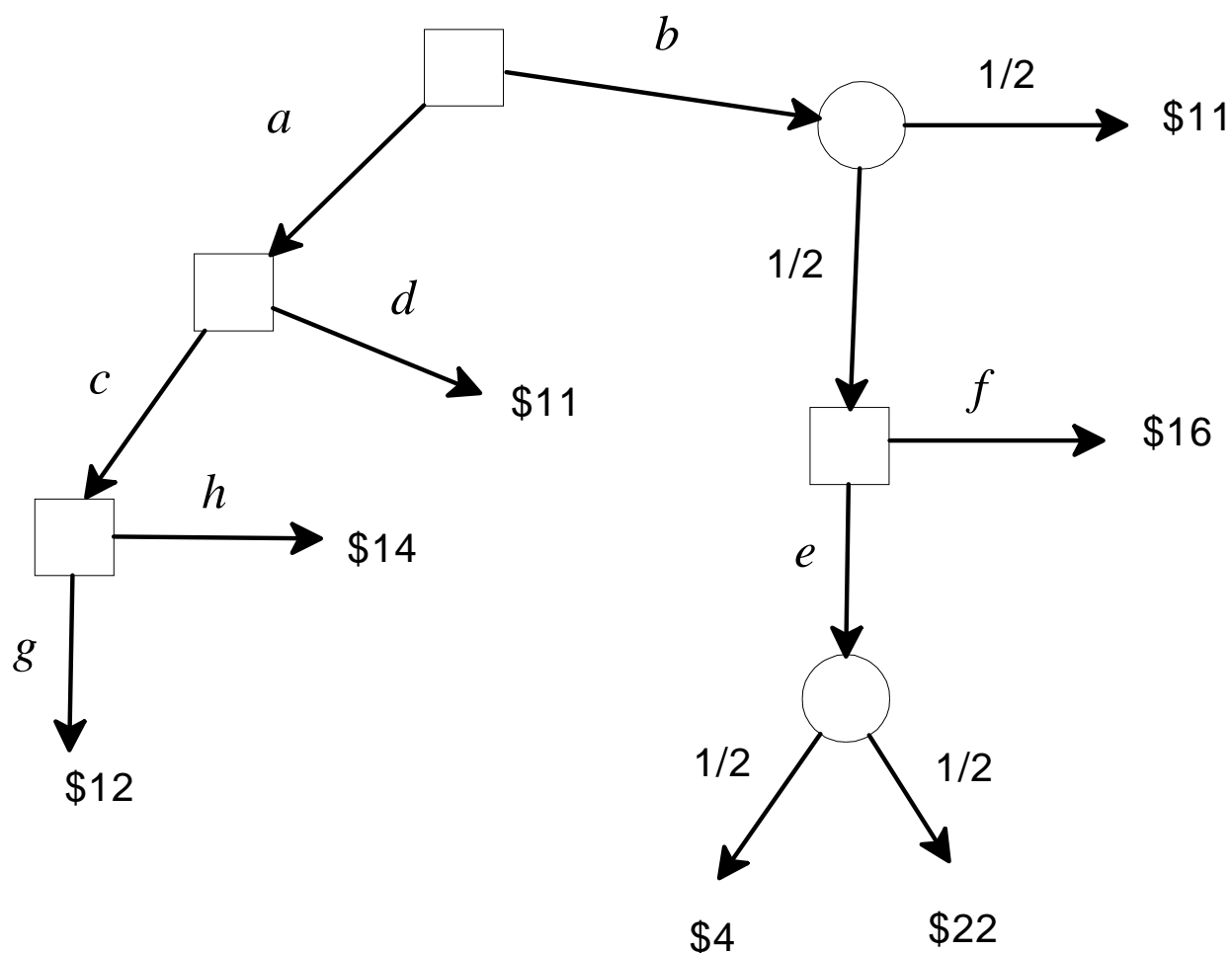
MaxiMin =

3. Attitudes to risk. Money lotteries, expected value and risk neutrality. Risk aversion. Risk love.

Ann prefers $A = \begin{pmatrix} \$15 \\ 1 \end{pmatrix}$ to $B = \begin{pmatrix} \$8 & \$20 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. What is her attitude to risk?

4. Decision trees. Sequential decisions. Backward induction.

Consider a money-loving individual who faces the following decision:



5. Expected utility: Part 1. von Neumann-Morgenstern utility functions. Normalization.

Suppose there are 6 basic outcomes. What is a utility function?

Suppose $Z = \{\$9, \$16, \$25, \$36\}$. Suppose the individual is indifferent between $A = \begin{pmatrix} \$16 \\ 1 \end{pmatrix}$ and $B = \begin{pmatrix} \$9 & \$36 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$. Construct a vNM utility function such that $U(\$9) = 3$ and $U(\$36) = 6$.

Is it the case that $U(\$x) = \sqrt{x}$?

Suppose $Z = \{\$9, \$16, \$25, \$36\}$. What is the **normalized** utility function of a risk neutral person?

6. Expected utility: Part 2. Decision trees again. MinMax Regret with cardinal utility.

		s_1	s_2	s_3			s_1	s_2	s_3
		a	9	2	1		a		
Utility:		b	6	2	2	Regret:	b		
		c	0	5	6		c		

	s_1	s_2	s_3	
a	9	2	1	
b	6	2	2	
c	0	5	6	Hurwicz index of pessimism α

$$H_\alpha(a) =$$

$$H_\alpha(b) =$$

$$H_\alpha(c) =$$

For example, if $\alpha = \frac{1}{3}$ then

7. Conditional probability. Bayes' formula: $P(E | F) = \frac{P(F | E) P(E)}{P(F)}$.

Bayes' theorem: $P(E | F) = \frac{P(F | E) P(E)}{P(F | E) P(E) + P(F | \neg E) P(\neg E)}$. A simple rule for updating a probability distribution over a finite set.

8. The value of information. Perfect information vs imperfect information. Does information have the potential to change your decision? What information should be chosen?

9. Intertemporal choice: (A) the discounted utility model.
Discounting and present value. Discount factor, discount rate. Time consistency.

10. Intertemporal choice: (B) hyperbolic discounting.
Conflict between current and future preferences. Time inconsistency.
Pre-commitment. Anticipating with time inconsistency: backward induction.

11. Group decision making: (A) social **preference** functions. Desirable properties (1. Freedom of expression, 2. Rationality, 3. Unanimity, 4. Independence of irrelevant alternatives, 5. Non-dictatorship). Arrow's theorem.

12. Group decision making: (B) social choice functions. Desirable properties (1. Unanimity, 2. Non-dictatorship, 3. Nonmanipulability). The Gibbard-Satterthwaite theorem.