

MANIPULABILITY of the BORDA count

Four alternatives: a, b, c and d

Three voters

TRUE

	1	2	3	score
best	a	c	c	4
	d	b	b	3
	b	a	a	2
worst	c	d	d	1

a: $4 + 2 + 2 = 8$

b: $2 + 3 + 3 = 8$

c: $1 + 4 + 4 = 9$

d: $3 + 1 + 1 = 5$

Satisfies unanimity
and non-dictatorship

→ c

1 can manipulate

1 changes
to:

FALSE 1

	1	2	3	score
best	b	c	c	4
	a	b	b	3
	d	a	a	2
worst	c	d	d	1

a: $3 + 2 + 2 = 7$

b: $4 + 3 + 3 = 10$

c: $1 + 4 + 4 = 9$

d: $2 + 1 + 1 = 4$

→ b

MANIPULABILITY of the KEMENY-YOUNG method

The Kemeny-Young procedure is a social **preference** function. However, just like the Borda rule, it can be converted to a social **choice** function by picking the top-ranked alternative in the selected ranking.

Consider the following tie-breaking rule: if two or more rankings are selected by the Kemeny-Young procedure, then pick the one whose top alternative comes first in alphabetical order.

	voter 1	voter 2	TRUE voter 3	
best	A	C	B	
	B	A	C	→
worst	C	B	A	

by tie-breaking rule →

Ranking	Kemeny-Young score	
A > B > C	$\#(A \succ B) + \#(A \succ C) + \#(B \succ C) =$	$2 + 1 + 2 = 5$
A > C > B	$\#(A \succ C) + \#(A \succ B) + \#(C \succ B) =$	$1 + 2 + 1 = 4$
B > A > C	$\#(B \succ A) + \#(B \succ C) + \#(A \succ C) =$	$1 + 2 + 1 = 4$
B > C > A	$\#(B \succ C) + \#(B \succ A) + \#(C \succ A) =$	$2 + 1 + 2 = 5$
C > A > B	$\#(C \succ A) + \#(C \succ B) + \#(A \succ B) =$	$2 + 1 + 2 = 5$
C > B > A	$\#(C \succ B) + \#(C \succ A) + \#(B \succ A) =$	$1 + 2 + 1 = 4$

→ top alternative : A

If Voter 3 (for whom A is the worst alternative) lies and reports $C \succ B \succ A$ instead of the true $B \succ C \succ A$

	voter 1	voter 2	voter 3	
best	A	C	C	
	B	A	B	\rightarrow
worst	C	B	A	C

Ranking	Kemeny-Young score	
$A \succ B \succ C$	$\#(A \succ B) + \#(A \succ C) + \#(B \succ C) =$	4
$A \succ C \succ B$	$\#(A \succ C) + \#(A \succ B) + \#(C \succ B) =$	5
$B \succ A \succ C$	$\#(B \succ A) + \#(B \succ C) + \#(A \succ C) =$	3
$B \succ C \succ A$	$\#(B \succ C) + \#(B \succ A) + \#(C \succ A) =$	4
$C \succ A \succ B$	$\#(C \succ A) + \#(C \succ B) + \#(A \succ B) =$	6
$C \succ B \succ A$	$\#(C \succ B) + \#(C \succ A) + \#(B \succ A) =$	5

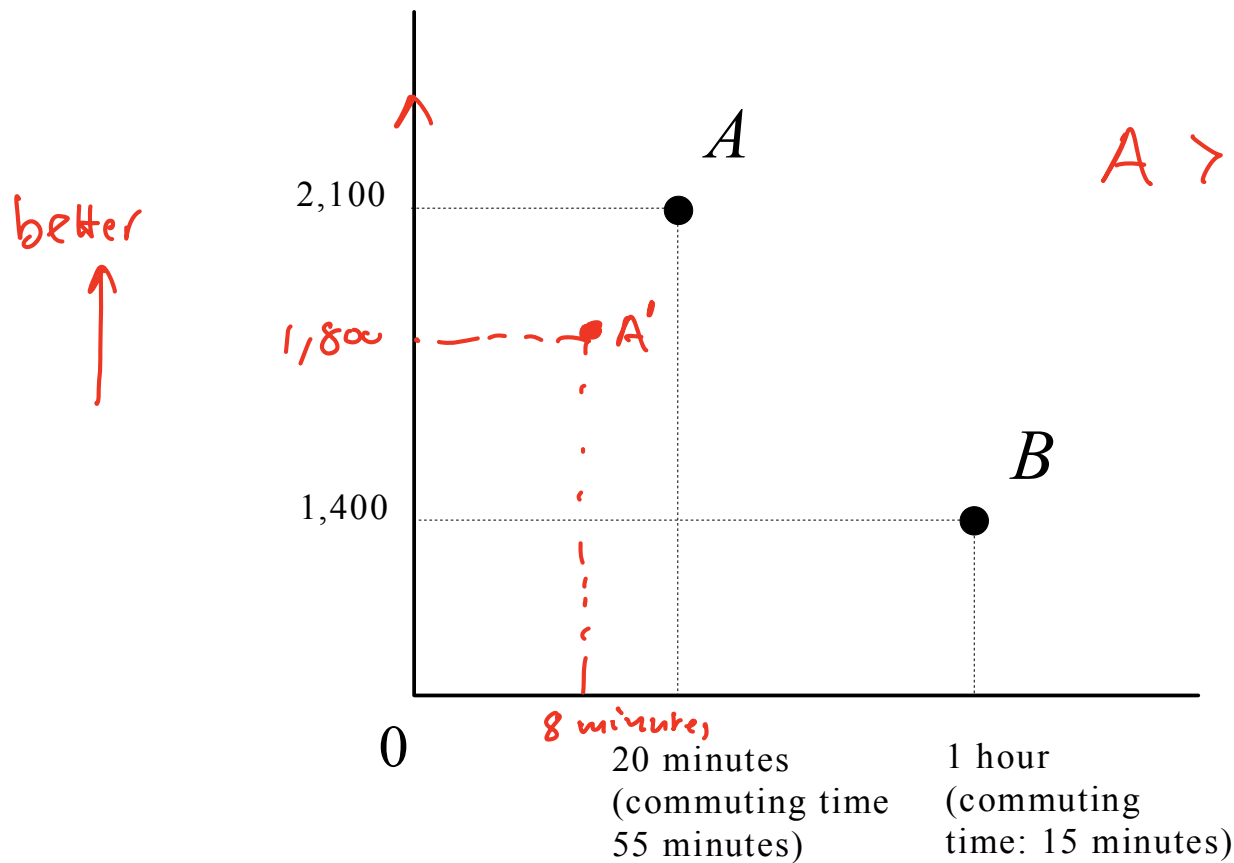
\rightarrow
top alternative: C

The Psychology of Decision Making

1. Manipulation of Choices Through Decoys

(voter 2: $A > B$)

square footage
of the house



Completeness
&
transitivity

time saved in commuting
relative to current
75 minutes

(voter 1:
 $B > A$)

→ better

GROUP 1. Choose one of the two:



Handsome A

50%



Handsome B

50%

GROUP 2. Choose one of the three:



Handsome A

75%



“uglified” version of A

0%



Handsome B

25%

GROUP 3. Choose one of the three:



Handsome A

25%



“uglified” version of B

0%



Handsome B

75%

2. Framing Effects: Gains versus Losses

I will give you \$200:



and then you will have to choose one of:

OPTION 1 : I give you an additional \$100:

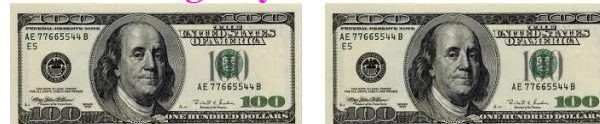


75%

OPTION 2 : I toss a coin



HEADS: I give you an additional \$200



TAILS: I give you no additional money

$$\begin{pmatrix} \$100 \\ 1 \end{pmatrix}$$

option 1

$$\begin{pmatrix} \$0 & \$200 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \text{ exp. value} = 100$$

option 2

risk averse

I will give you \$400:



and then you will have to choose one of:

OPTION 1 : You give me back \$100:

OPTION 2 : I toss a coin




HEADS: You keep the \$400

TAILS: You give me back \$200

75%


risk loving

I will give you \$200:





and then you will have to choose one of:


OPTION 1 : I give you an additional \$100:



OPTION 2 : I toss a coin

		<p>HEADS: I give you an additional \$200</p> 
		<p>TAILS: I give you no additional money</p>


I will give you \$400:



and then you will have to choose one of:

OPTION 1 : You give me back \$100:

OPTION 2 : I toss a coin

		<p>HEADS: You keep the \$400</p>
		<p>TAILS: You give me back \$200</p>

In both cases:

Option 1 = you end up with \$300

Option 2 = you face the uncertain prospect (lottery)

$$\left(\begin{array}{c|c} \text{You end up with \$400} & \text{You end up with \$200} \\ \hline \text{Probability } \frac{1}{2} & \text{Probability } \frac{1}{2} \end{array} \right)$$

Non-monetary example of effect of FRAMING in terms of GAINS vs LOSSES

You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table. Out of every 100 patients who chose surgery 90 survived the operation, 68 were alive after 1 year and 34 were alive after 5 years.
- Radiation. Out of every 100 patients who chose radiation 100 survived the treatment, 77 were alive after 1 year and 22 were alive after 5 years.

About 80% of experimental subjects chose surgery



You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table. Out of every 100 patients who chose surgery 10 died during the operation, 32 died after 1 year and 66 died within 5 years.
- Radiation. Out of every 100 patients who chose radiation none died during the treatment, 23 after 1 year and 78 died within 5 years.

About 50% of experimental subjects chose surgery.

Loss Aversion:

We are happy when we gain something, but

Twice unhappy when we lose it

The Pain of Paying

fMRI studies show that the pain centers of the brain light up when one has to part with one's cash.



The Pain of Paying

fMRI studies show that the pain centers of the brain light up when one has to part with one's cash.



Less pain with credit.



Functional magnetic resonance imaging (fMRI) is a procedure that measures brain activity by detecting associated changes in blood flow.

People tend to be risk-averse towards gains, but risk-loving towards losses.

Can such an attitude be compatible with expected utility?

Choice between $A: \begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B: \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Suppose that **she prefers** the sure gain: she prefers **A**. Then she displays **risk-aversion towards gains** (the expected value of these two options is the same).

Choice between $C: \begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D: \begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Suppose that **she prefers** the risky prospect: she prefers **D**. **Then she is risk-loving towards losses** (the expected value of these two options is the same).

Is there a von Neumann-Morgenstern utility function that is consistent with these choices?