People tend to be risk-averse towards gains, but risk-loving towards losses.

Can such an attitude be compatible with expected utility?

Choice between $A:\begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B:\begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Suppose that **she prefers** the sure gain: she prefers **A**. Then she displays **risk-aversion towards gains** (the expected value of these two options is the same). .

Choice between $C:\begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D:\begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Suppose that **she prefers** the risky prospect: she prefers **D**. **Then she is risk-loving towards losses** (the expected value of these two options is the same).

Is there a von Neumann-Morgenstern utility function that is consistent with these choices?

Suppose that her initial wealth is $100.

outcome *U* $200 $150 $100 $50 $0	$A = \begin{pmatrix} +\$50 \\ 1 \end{pmatrix} \succ B = \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (

Hence it is possible for an expected-utility maximizing individual to display risk aversion towards a gain and risk love towards a symmetric loss.

However, this cannot happen at every wealth level.

Beginning wealth: $200. Choice between $A:\begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$ and $B:\begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Beginning wealth: $200. Choice between $C:\begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$ and $D:\begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Can she prefer A to B and also D to C? Let's see.

<table>
<tr>
<td>

outcome	U
$200	1
$150	a
$100	b
$50	c
$0	0

</td>
<td>Since she prefers D to C, she prefers</td>
</tr>
</table>

Thus people who are consistently (that is, at every initial level of wealth) risk-averse towards gains and risk-loving towards losses cannot satisfy the axioms of expected utility. If those axioms capture the notion of rationality, then those people are irrational.

VALUE of INFORMATION

The general case (non-monetary outcomes)

probability $\rightarrow$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	z_1	z_2	z_3	z_4
b	z_5	z_6	z_7	z_8

suppose:

		utility
best	z_8	96
	z_4	80
	z_5	48
	z_1, z_2	32
	z_3, z_6	16
worst	z_7	0

then

probability $\rightarrow$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a				
b				

probability →	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	32	32	16	80
b	48	16	0	96

In the absence of further information.

$\mathbb{E}[U(a)] =$

$\mathbb{E}[U(b)] =$

Suppose now that the DM is offered **perfect information for free**.

probability →	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	32	32	16	80
b	48	16	0	96

- If told s_1 she chooses and gets utility
- If told s_2 she chooses and gets utility
- If told s_3 she chooses and gets utility
- If told s_4 she chooses and gets utility

Her expected utility under free perfect information is

Free perfect information means an **increase in expected utility** of

How to monetize the value of information in the general case

probability $\to$	q	$1-q$
state $\to$	s_1	s_2
act $\downarrow$		
a	y_1	y_2
b	y_3	y_4

To avoid triviality let us assume that it is not the case that one act dominates the other.

Assume that

$$U(y_1) > U(y_3) \text{ and } U(y_4) > U(y_2)$$

Not enough to tell which act the DM would choose. Assume that he would choose act ***a***:

$$qU(y_1) + (1-q)U(y_2) > qU(y_3) + (1-q)U(y_4)$$

What is the maximum price that the DM would be willing to pay for perfect information?

Each outcome y_i should be thought of a list of all the things that the DM cares about (wealth is just one of them). Separate from each y_i the wealth part and write the outcome as (z_i, W_i) where z_i is that part of y_i that does not refer to the DM's wealth and W_i is the DM's wealth in outcome y_i:

probability $\to$	q	$1-q$
state $\to$	s_1	s_2
act $\downarrow$		
a	(z_1, W_1)	(z_2, W_2)
b	(z_3, W_3)	(z_4, W_4)

Our assumption is that $U(y_1) > U(y_3)$ and $U(y_4) > U(y_2)$ thus

$$U(z_1, W_1) > U(z_3, W_3) \text{ and } U(z_4, W_4) > U(z_2, W_4)$$

What would he choose if, having paid \$*p* for perfect information, he were informed that the state was s_1? In general, we cannot infer from $U(z_1, W_1) > U(z_3, W_3)$ that $U(z_1, W_1 - p) > U(z_3, W_3 - p)$. Assume this, however and, similarly, $U(z_4, W_4 - p) > U(z_2, W_2 - p)$. Then if informed that s_1 the DM would choose and if informed that s_2 then he would choose . Thus with perfect information his expected utility would be

The maximum price the DM is willing to pay for perfect information is that value of *p* that solves the equation:

In Chapter 9 of the book (Section 9.3) there is a detailed (more complex) example along these lines.

Suppose now that the DM is offered, **for free, IMPERFECT information** of the form $\{\{s_1, s_2\}, \{s_3, s_4\}\}$.

probability →	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	32	32	16	80
b	48	16	0	96

Re-write the probabilities
in terms of a common denominator:

probability →	—	—	—	—
state →	s_1	s_2	s_3	s_4
act ↓				
a	32	32	16	80
b	48	16	0	96

- If told $\{s_1, s_2\}$ then

probability →		
state →	s_1	s_2
act ↓		
a	32	32
b	48	16

$\mathbb{E}[U(a)] =$

$\mathbb{E}[U(b)] =$

Thus would choose and expect a utility of

probability $\rightarrow$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{8}{16}$	$\frac{4}{16}$
state $\rightarrow$	s_1	s_2	s_3	s_4

- If told $\{s_3, s_4\}$ then:

probability $\rightarrow$		
state $\rightarrow$	s_3	s_4
act $\downarrow$		
a	16	80
b	0	96

$\mathbb{E}[U(a)] =$

$\mathbb{E}[U(b)] =$

Expected utility from free information is

Note: the same utility as under no information. Why?

Information is valuable only if it induces you to take a different action (than the action you would choose under no information), in response to at least one of the possible items of information.

See doctors' example in the textbook.