Decisions under certainty

actions: outcomes

$$
\begin{array}{ccc}
a_1 & \longrightarrow & z_1 \\
a_2 & \longrightarrow & z_2 \\
\vdots & & \vdots \\
a_n & \longrightarrow & z_n
\end{array}
$$

choosing the best action =
choosing the best outcome

Hence need a ranking of the outcomes

Rationality requirements for preferences

$x \succsim y$ means that outcome x is at least as good as outcome y

(1) Transitivity:

if $x \succsim y$ and $y \succsim z$ then $x \succsim z$

if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

(2) Completeness: For every x and y

either $x \succsim y$ or $y \succsim x$ or both

either $(x, y) \in R$ or $(y, x) \in R$ or both

Set of outcomes: $W = \{a, b, c, d\}$

$R = \{(a,b), (a,d), (b,c), (c,d)\}$

Does R satisfy transitivity?

No: $(a,b) \in R$ and $(b,c) \in R$

but $(a,c) \notin R$

Does R satisfy completeness?

No: $(a,c) \notin R$ and $(c,a) \notin R$

REVEALED PREFERENCE

Z = {a, b, c, d, e, f}

Observations:

{a, c, f} $\longrightarrow$ **c**	**(1)**		**{a, d, b}** $\longrightarrow$ **a**	**(2)**
{d, e, f} $\longrightarrow$ **f**	**(3)**		**{a, f}** $\longrightarrow$ **f**	**(4)**

Assume also that his ranking is a strict ranking (no indifference)
Is there a complete and transitive strict ranking of Z that rationalizes the observations?

From (1) $c > a$ and $c > f$

From (3) $f > d$ and $f > e$ $\Rightarrow$ by transitivity $c > e$

From (2) $a > d$ and $a > b$ by transitivity $c > b$

by transitivity $c > d$

c is the best outcome

$Z = \{a, b, \cancel{c}, d, e, f\}$

Observations:

$\{a, c, f\} \longrightarrow c$ (1) $\qquad$ $\{a, d, b\} \longrightarrow a$ (2)

$\{d, e, f\} \longrightarrow f$ (3) $\qquad$ $\{a, f\} \longrightarrow f$ (4)

From (3) $f > d$ and $f > e$

From (4) $f > a$

Need $f > b$: $\underbrace{\text{From (4) } f > a \text{ From (2) } a > b}_{\text{by transitivity } f > b}$

f is the second best

best	c	c	c
	f	f	f
	a	a	⋮
	b	d	
worst	d	b	c

Decisions under **uncertainty**

state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9
a_4	z_{10}	z_{11}	z_{12}
a_5	z_{13}	z_{14}	z_{15}

state →	s_1	s_2	s_3
act ↓			
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9

best	z_8
	z_7, z_9
	z_1, z_3, z_4, z_6
	z_5
worst	z_2

- Compare a_3 and a_2: $z_7 \succ z_4$, $z_8 \succ z_5$, $z_9 \succ z_6$

  a_3 a_3 a_3

$\boxed{a_3 \text{ strictly dominates } a_2}$

- Compare a_2 and a_1: $z_4 \sim z_1$, $z_5 \succ z_2$, $z_6 \sim z_3$

  a_1 or a_2 a_2 a_1 or a_2

a_2 weakly dominates a_1

- Compare a_3 to a_1: $\boxed{a_3 \text{ strictly dominates } a_1}$

a_3 is a strictly dominant action.

Easier to see with utilities:

state $\rightarrow$	s_1	s_2	s_3			UTILITY
act $\downarrow$				best	z_8	4
a_1	z_1	z_2	z_3		z_7, z_9	3
a_2	z_4	z_5	z_6		z_1, z_3, z_4, z_6	2
a_3	z_7	z_8	z_9		z_5	1
				worst	z_2	0

state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a_1	2	0	2
a_2	2	1	2
a_3	3	4	3
a_4	2	1	2

	if s_1	if s_2	if s_3
Compare a_3 to a_2:	$3 > 2$ a_3 a_2	$4 > 1$ a_3 a_2	$3 > 2$ a_3 a_2
Compare a_2 to a_1:	$2 = 2$ a_2 a_1	$1 > 0$ a_2 a_1	$2 = 2$ a_2 a_1
Compare a_3 to a_1	$3 > 2$	$4 > 0$	$3 > 2$
Compare a_4 to a_2	$2 = 2$	$1 = 1$	$2 = 2$

a_2 and a_4 are EQUIVALENT

Act a ***weakly dominates*** act b if, for every state s, $a(s) \succsim b(s)$ and, furthermore, there is at least one state $\hat{s}$ such that $a(\hat{s}) \succ b(\hat{s})$.
Using utility, $U\left(a(s)\right) \geq U\left(b(s)\right)$ for every state s and there is at least one state $\hat{s}$ such that $U\left(a(\hat{s})\right) > U\left(b(\hat{s})\right)$.

state →	s_1	s_2	s_3
act ↓			
a_1	1	3	1
a_2	0	2	1
a_3	1	3	3

- a_1 weakly dominates a_2
- a_3 weakly dominates a_1
- a_3 strictly (and thus also weakly) dominates a_2.

a and b are ***equivalent***, if, for every state s, $a(s) \sim b(s)$ or, in terms of utility, $U\left(a(s)\right) = U\left(b(s)\right)$.

Act a is ***weakly dominant*** if, for every other act b, either a weakly dominates b or a and b are equivalent.

In the above example, …

Another example:

state →	s_1	s_2	s_3	s_4
act ↓				
a_1	1	3	3	2
a_2	0	2	1	2
a_3	1	3	3	2

a_1 is weakly dominant
a_1 is equivalent to a_3
a_1 weakly dominates a_2

a_3 is a weakly dominant act

You are bidding against a computer for an item that you **value at $30**. The allowed bids are $10, $20, $30, $40 and $50. The computer will pick one of these bids randomly. Let x be the bid generated by the computer. If your bid is greater than or equal to x then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than x then you get nothing and pay nothing.

computer's bid →	$10	$20	$30	$40	$50
your bid ↓					
$10					
$20					
$30					
$40					
$50					
pay	pay 10	pay 20	pay 30	pay 40	pay 50

Now same as above, but if you win the object and **pay your own bid**.

computer's bid →	$10	$20	$30	$40	$50
your bid ↓					
$10					
$20					
$30					
$40					
$50					