DECISIONS UNDER CERTAINTY

action	outcome
a_1	z_1
a_2	z_2
$\vdots$	
a_n	z_n

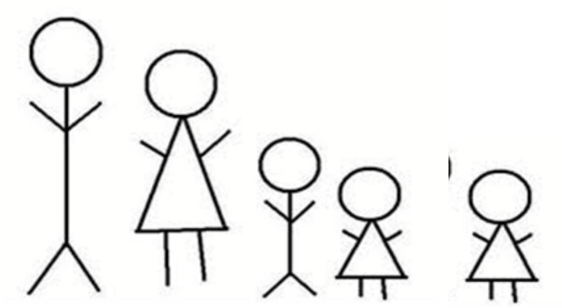
	action	outcomes	
	a_1	z_1	7
$\rightarrow$	a_2	z_2	10
	a_3	z_3	3
$\rightarrow$	a_4	z_4	10

Action a is rational if it gives a best outcome

" " " the highest utility

utility maximization

A binary relation R on a set Z is a set of ordered pairs (x,y) with both x and y elements of Z.



Z = {A, B, C, D}

ordered pair

R is the "taller than" relation: (x,y) means that x is taller than y

R = { (A,B), (A,C), (A,D), (B,C), (B,D), (C,D), (A,E), (B,E), (C,E) }

if (x,y) ∈ R and (y,z) ∈ R then (x,z) ∈ R transitivity

not complete because both (D,E) and (E,D) are missing.

S = (x,y) as "x is at least as tall as y"

(D,E) ∈ S also (E,D) ∈ S

x is just as tall as y if (x,y) and (y,x) both belong to S

$(x, y) \in W$ outcome x is at least as good as y

M, S, F, P

$\{ \underbrace{(F, M)}, (F, S), (F, P), \underbrace{(P, M), (M, P)} \}$

+ absence of (M, F)

F is better than M

individual is indifferent between M and P

not complete because neither (M, S) nor (S, M) belongs to W

transitivity: if (x, y) and (y, z) then (x, z)

completeness: either $(x, y) \in W$ or $(y, x) \in W$ or both

best	F
	P, M
worst	S

$\rightsquigarrow$ $\{ (F, S), (F, P), (F, M), (P, M), (M, P), (P, S), (M, S) \}$

Utility Function

ORDINAL UTILITY

S	M	P	F
-1	2	2	6
0	1	1	2
0	1	1	1,000

Example from the first class:

state → / act ↓	s_1: genetically predisposed	s_2: no genetic predisposition
smoke	get cancer enjoy smoking	no cancer enjoy smoking
not smoke	no cancer no enjoyment	no cancer no enjoyment

Act a ***strictly dominates*** act b if, for every state s, $a(s) \succ b(s)$.

state → / act ↓	s_1	s_2	s_3
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9

best	z_8
	z_5
	z_7, z_9
	z_1, z_6
	z_3, z_4
worst	z_2

$z_7 \succ z_4$ $\quad z_8 \succ z_5$ $\quad z_9 \succ z_6$

a_3 is strictly dominant

- a_3 strictly dominates a_2: if it gives a better outcome than a_2 in every possible state
- a_3 strictly dominates a_1:

Easier to see with utilities:

state → / act ↓	s_1	s_2	s_3
a_1	3	0	2
a_2	2	9	3
a_3	6	10	6

		Utility
best	z_8	10
	z_5	9
	z_7, z_9	6
	z_1, z_6	3
	z_3, z_4	2
worst	z_2	0

Act a is ***strictly dominant*** if it strictly dominates every other act.

In this example a_3 is a strictly dominant act.

Act a ***weakly dominates*** act b if, for every state s, $a(s) \succsim b(s)$ and, furthermore, there is at least one state $\hat{s}$ such that $a(\hat{s}) \succ b(\hat{s})$.
Using utility, $U\left(a(s)\right) \geq U\left(b(s)\right)$ for every state s and there is at least one state $\hat{s}$ such that $U\left(a(\hat{s})\right) > U\left(b(\hat{s})\right)$.

state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a_1	1	3	1
a_2	0	2	1
a_3	1	3	3

a_3 weakly dominates a_1

- a_1 weakly dominates a_2
- a_3 weakly dominates a_1
- a_3 strictly (and thus also weakly) dominates a_2.

a and b are ***equivalent***, if, for every state s, $a(s) \sim b(s)$ or, in terms of utility, $U\left(a(s)\right) = U\left(b(s)\right)$.

Act a is ***weakly dominant*** if, for every other act b, either a weakly dominates b or a and b are equivalent.

In the above example, …

Another example:

state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a_1	1	3	3	2
a_2	0	2	1	2
a_3	1	3	3	2